THE MATHEMATIGAL TRAINING OF THE PHYSICIST IN THE UNIVERSITY

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DEUXIÈME SÉANCE

Lundi 26 août, à 3 heures.

Présidence de Sir J.-J. THOMSON (Cambridge).

Ordre du jour :

La préparation mathématique des physiciens à l'Université, rapport de M. RUNGE, professeur de mathématiques appliquées à l'Université de Gœttingue. — Discussion.

THE MATHEMATICAL TRAINING OF THE PHYSICIST IN THE UNIVERSITY

Report presented by

C. RUNGE (Göttingen)

The international commission on the Teaching of Mathematics requested me to make an inquiry into the state of the mathematical studies of the students of physics at the universities of different countries. A circular letter was sent out during the winter of 1911-1912 to collect the necessary information, and a number of answers from Italy, Austria, Germany, Switzerland, Holland, England and the United States were received. It is however extremely difficult to form an adequate idea of the state of things in another country with which you are not familiar, because the same term describing the mathematical education may mean very different things when used by different men, so that a description does not convey a very definite idea to the reader. Some countries have developped their educational conditions in so widely different a manner, that it is almost impossible to understand them except by visiting the country and seeing for yourself. In the announcement of his lectures on mathematical physics HELMHOLTZ used to invite the students by saying no other previous knowledge was wanted besides the elements of the calculus. But his so-called elements were by no means considered elementary by his students. In the same way the terms « analytic geometry », « integral and differential calculus », no doubt have a different meaning with my correspondents from different countries. Though vague the report may however help us to get acquainted with the

state of affairs. But I beg to consider it, incomplete and imperfect as it is. only as an introduction leading up to the discussion, which is to follow. I shall follow here the order of the questions in my cicular letter. The original text is given in the notes.

I[1]. — The subjects of mathematical lectures attended by students of physics in their regular course of studies seem to be very much the same almost everywhere. They pass through a fairly complete course of instruction in mathematics, which however has no particular connection with the special work that a student of physics as such ought to do. Analytic geometry, differential and integral calculus, elements of ordinary differential equations, dynamics are taught, the mathematical professors in general paying litte attention to the particular needs of physical students. No distinction is made between students of experimental physics and of theoretical physics and there are on these mathematical subjects no special mathematical courses for physicists.

The lectures on dynamics are almost always given by mathematical professors and sometimes also certain other lectures on subjects of mathematical physics. Some of my correspondents bitterly complain of the mathematical training of students of physics in consequence of the professors of pure mathematics ignoring some mathematical theorems and methods, that are of greatest importance to the physicist. Green's and Stokes' theorem for instance ought to be taught in the integral calculus and Fourier's series ought to be treated in a more practical manner and not only as furnishing interesting instances of conditions of convergence. Vector analysis ought to be taught quite regularly by the mathematical professors, so that students should be perfectly familiar with it in their studies in dynamics and mathematical physics. Instead of that some of the mathematical professors spend too much time on the logical foundations of the calculus. Their subtilty is lost on the student who is only beginning to grasp the power of the calculus and ought to be trained to use it. This applies equally to the student of pure mathematics. On the whole there seems to be no need for special mathematical courses for students of physics nor does it seem necessary to compel them to attend more mathematical lectures. But a need seems to be strongly felt for mathematicians and physicists to draw closer together. The spirit of the mathematical teaching should be

[1] *Question* I : Welche mathematischen Fächer gehören zum regelrechten Studium eines Physikers? Wird in den Anforderungen an die mathematische Ausbildung der Physiker ein Unterschied gemacht zwischen Physikern einer mehr experimentellen und einer mehr theoretischen Richtung ? Wird von den Mathematik-Professoren besondere Rücksicht auf die Bedürfnisse der Physiker genommen ? Sind besondere mathematische Kurse für Physiker eingerichtet ? Wie weit und in welchem Sinne beteiligen sich die Mathematik-Professoren an den Vorlesungen *a)* über Mechanik *b)* über sonstige insbesondere moderne Gebiete der mathematischen Physik ?

altered, so as to make it more practical and easier to apply to physical problems. At present the gap is very wide and is not tending to close up.

II[1]. — Modern graphical methods seem to be very slightly introduced except in France and a few other universities. On the other hand descriptive geometry is now provided for, nearly everywhere. In some countries it is even cultivated to such an extent that opposition seems to be awakening. Indeed the work in descriptive geometry has a tendency to overgrow other mathematical branches, if it is not carefully pruned down. Though exceedingly useful in its way as training the power of geometrical intuition the student may become too fond of it and neglect the calculus. Besides there is scarcely any development going on in descriptive geometry, so that little research work is done in connection with it. Whatever the value of descriptive geometry may be, there can be little doubt that graphical methods showing how to represent functions of one and of more than one variable and how to handle them, have at least the same importance for any students of natural philosophy. The physicist in a great many cases has to deal with empirical functions that are conveniently and with sufficient accuracy represented graphically. To approximate them by means of analytical expressions becomes in many cases so tedious that it is out of the question. When these functions enter into mathematical considerations, all operations are best carried on graphically. Engineers have long been in the habit of doing so. But it is only since MASSAU (Gand) and d'OCAGNE (Paris) worked out the methods systematically, that they have become a branch of mathematics. It appears that except in France teachers of mathematics have not yet paid sufficient attention to this remarkable growth of our mathematical power. The methods of graphical integration of any given function of a real or complex variable, of ordinary differential equations and of some partial differential equations should be considered essential parts of the integral calculus. Due attention should be paid to them in every course on the calculus. And there is this second advantage, that it makes the calculus easier. The difficulty of performing the analytical integration of a given function has led to an undue preponderance of differentiation in the teaching of the calculus. The graphical methods restore the natural order. With an empirical function the process of integration is performed easily and accura-

[1] *Question* II : Wie weit sind auf den Universitäten die modernen graphischen Methoden über graphische Integration und Nomographie verbreitet ? Lernen die Physik-Studierenden darstellende Geometrie, numerisches Rechnen, numerische Auflösung von Gleichungen, numerische Auflösung von Differentialgleichungen, Methode der kleinsten Quadrate ? Lernen sie den Gebrauch mathematischer Apparate, des Rechenschiebers, der Rechenmaschine, des Planimeters ? Geschieht dies in besonderen mathematichen Vorlesungen oder Uebungen oder nur beiläufig im physikalischen Praktikum ?

tely, while the process of differentiation is much more uncertain. The methods of nomography, if not taught in a special course of lectures might be included in the lectures on coordinate geometry; but it will not do to leave them out altogether.

The *numerical* methods of handling an empirical function are older than the graphical methods and in consequence better known. Nevertheless they are also too much neglected in the mathematical education of the student of physics. The method of least squares is generally taught, but greater stress ought to be laid on the calculus of finite differences, on the numerical solution of equations, on numerical calculation of integrals and the numerical solution of differential equations. The difficulty is that many professors of mathematics have never been in the habit of calculating numerically and seem to have an aversion to teaching their students. Many are not familiar with the handling of mathematical instruments, with the slide rule, the integraph, the planimeter, the calculating machine, and little mention is made of them in the mathematical lectures. The student thus forms a wrong idea of the possibility of carrying out a mathematical operation. As long as he is only interested in mathematical theorems, this does not much matter. But a physicist or an engineer cannot be satisfied with existence-theorems, he wants the actual numerical result for the given data, he has before him. Of course he may learn to calculate in his pratical work in the laboratory, and his teachers in physics will to a certain extent take care of this part of his mathematical education. Nevertheless a decided advance could be made, if the mathematical teachers better understood the wants of their students in physics and did not rest satisfied with the proof of a system of theorems. The solution of a mathematical problem is not complete unless it supplies an answer to any intelligent question raised in connection with the problem. It is not sufficient that the answer may be found « by a finite number of operations »; but one must be able to find it without an unreasonable expenditure of work.

The use of the slide rule is commonly learned incidentally in the physical laboratory or in engineering courses and the calculating machine or planimeter may be learned in the same way. But if they were handled in the mathematical courses it would react favorably on the teaching and the students of pure mathematics would profit by it as well.

III[1]. — This however is only possible with individual teaching,

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[1] *Question* III : Wie sind die mathematischen Uebungen der Physik-Studierenden beschaffen? Werden sie in der Weise von Laboratoriumsübungen abgehalten durch Ausführen der Aufgaben an Ort und Stelle und steht der Docent oder seine Assistenten in persönlicher Beziehung zu den einzelnen Studierenden ?

which in most of the universities and in most of the mathematical courses is not insisted upon. The organisation of the mathematical exercises ought to be on a similar plan as the exercises in a chemical or physical laboratory. As to descriptive geometry this is recognized everywhere. Nearly everywhere special rooms are provided for the exercises on the drawing board and the professor enjoys the help of assistants to look after each student individually to correct his drawing and to discuss with him his difficulties. The same plan ought to be carried out for exercises in numerical and graphical calculation and for general mathematical exercises. The difficulty that a student finds in his mathematical studies is not so much that of understanding the proof of a mathematical theorem but more that of grasping the contents of it, of seeing its application in a variety of cases, of knowing how to make use of it. For the student of physics or of engineering the power to use his mathematics is of primary importance. Without it he might almost as well dispense with the knowledge of mathematical theorems altogether. They will be an unnecessary burden to his intellect. This power can only be acquired by exercise. Mathematics cannot be learned by lectures alone, no more than pianoplaying can be learnt by listening to a player. It does not seem advisable to leave the exercises to the physical student himself, although he will himself feel the necessity of them, and although his physical studies will bring him in contact with problems where his mathematics are wanted. The better pedagogic no doubt is to combine the theorem with its application, to help the student to find his way back from the generalisation to the single case. It is the single case that he wants to understand, the abstraction has its value only as a means to understand the single case.

In mathematical exercises organized after the usual plan of laboratory work, the teacher is able to help his students in a far more efficient manner than by lectures alone. Each student has his particular way of looking at things and should not be forced to another way unless for good reasons. Written solutions of problems need not be excluded, but the personal intercourse, the discussion of individual difficulties ought to be insisted on. Besides graphical work and the use of mathematical instruments must be shown to each student separately and must be practised for some time under the eye of the teacher.

IV[1]. — Most of my correspondents agree in considering this

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[1] *Question* IV: Was sind Ihre eignen Ansichten über die Zweckmässigkeit des gegenwärtigen Unterrichts? Schlagen Sie Aenderungen vor in Bezug auf Ausdehnung oder Einschränkung des mathematischen Unterrichts oder in Bezug auf Unterscheidung der Physik-Studierenden nach Gruppen oder in Bezug auf Unterrichtseinrichtungen?

the only way for the organisation of mathematical exercises. Some express their opinion that the difficulty is not that the students of physics do not learn enough mathematics but rather that they do not learn to apply their mathematics to concrete problems. « There often exists a curious disproportion of which they are well aware between their mathematical ability and the depth and wideness of their mathematical studies. This evil can hardly be remedied by lectures but only by prolonged practise ». Among the exercises the more elementary ones are naturally of greater importance than the more advanced. While only a small minority of students of physics will draw advantage from the highest mathematical studies, all of them will profit by the full possession of the graphical and numerical methods and the ability to use mathematical instruments. These exercises ought to accompany the mathematical lectures of the first two years and ought to constitute an integral part of the mathematical education of the physicist.

To recapitulate ,the general opinion seems to be that the mathematical instruction for physical students is very much in need of reform. Large portions of the theoretical matter intended for the pure mathematician might be left out and the attention concentrated on those subjects which are of continual application in mathematical physics. Improvement is to be looked for in the progressive adaptation of the teaching of applied mathematics to the more modern views of Physics on Electricity and Matter. As to the breaking up of students into groups it seems that it is both impracticable and undesirable. There is a limit to the indefinite multiplication of classes owing to the limited numbers of teachers and the limited amount of funds available.

The main danger is the gap between physicists and pure mathematicians that seems to be widening. Mathematics is suffering from overspecialisation and is cutting itself off from Natural Philosophy and Experimental Science. But very little can be done by regulations. What is really wanted is that Mathematical teachers should understand the problems and the needs of the Physicist.

Questionnaire. — Nous croyons utile de rappeler, dans son texte français, le questionnaire distribué l'hiver dernier par M. Runge au nom de la *Sous-commission B*, composée de MM. Klein (Gœttingue), président, Bourlet (Paris), Fehr (Genève), Sir G. Greenhill (Londres), Coialovich (St-Pétersbourg), Levi-Civita (Padoue),

RUNGE (Gœttingue), TIMERDING (Braunschweig), WEBSTER (Worcester, E. U.), WIRTINGER (Vienne).

(*Extrait de la circulaire* de M. le prof. C. RUNGE.) — Objet : *Les mathématiques dans les études universitaires des physiciens.*

1. — Quelles sont les branches mathématiques qui appartiennent à un enseignement régulier destiné au physicien ? Dans la préparation mathématique des physiciens fait-on une différence entre les étudiants qui suivent une direction plutôt expérimentale et ceux qui suivent une voie plus théorique ?

Les professeurs de mathématiques tiennent-ils particulièrement compte des besoins des physiciens ?

Y a-t-il des cours de mathématiques spécialement destinés aux physiciens ?

Dans quelle mesure et à quel point de vue les mathématiciens participent-ils aux cours *a)* de mécanique ; *b)* à d'autres cours et particulièrement à ceux qui se rattachent au domaine moderne de la physique mathématique ?

2. — Jusqu'à quel point les méthodes graphiques modernes d'intégration et de nomographie sont-elles répandues dans les universités ?

Les étudiants en physique sont-ils appelés à apprendre la géométrie descriptive, le calcul numérique, la résolution numérique des équations différentielles et la méthode des moindres carrés ?

Apprennent-ils le maniement d'instruments mathématiques tels que la règle à calcul, la machine à calculer et les planimètres ?

Y a-t-il des cours ou des exercices spéciaux à cet effet ou cet enseignement se fait-il dans les travaux pratiques de physique ?

3. — Quelle est l'organisation des exercices mathématiques destinés aux physiciens ? Ces exercices ont-ils lieu suivant le mode habituel des travaux de laboratoire ? Le professeur ou ses assistants entrent-ils en relation personnelle avec les différents étudiants ?

4. — Quelle est votre opinion personnelle sur l'opportunité de l'organisation actuelle de cet enseignement ?

Avez-vous des propositions à faire au sujet d'une extension ou d'une réduction de l'enseignement mathématique ou au sujet d'une distinction des étudiants en physique en divers groupes ou encore pour ce qui concerne l'organisation de l'enseignement ?

DISCUSSION

Sir J.-J. THOMSON remercie M. Runge de sa belle conférence. Elle a été suivie d'une discussion d'un grand intérêt à laquelle ont pris part MM. P. STÆCKEL, C. BOURLET, F. ENRIQUES, Sir G. GREENHILL, A.-G. WEBSTER, E. BOREL, Sir J. LARMOR, C. BIOCHE, A.-E.-H. LOVE, E.-W. HOBSON, G.-A. GIBSON, Sir J.-J. THOMSON et C. RUNGE. En outre, afin de ne pas allonger le débat, M. LANCHESTER nous a adressé par écrit les remarques qu'il comptait présenter à la séance.

Quelques orateurs ont insisté sur les dangers qu'il y aurait de négliger le côté logique dans l'enseignement supérieur, même si celui-ci s'adresse uniquement à des physiciens, ce qui n'est généralement pas le cas. Tout en reconnaissant la nécessité d'accorder une juste place au côté pratique, à l'aide de problèmes bien choisis, ils estiment que le côté utilitaire ne doit pas prédominer. D'autres font remarquer que les étudiants en mathématiques eux-mêmes retireraient un grand bénéfice d'un enseignement par lequel on les initierait davantage aux applications d'ordre pratique. C'est de ce côté qu'on devrait diriger l'effort sans porter préjudice à l'enseignement théorique bien approprié. Le temps nécessaire se trouvera aisément; on pourrait le prendre sur les heures de laboratoire de la physique, dont le nombre, au dire même des physiciens, est parfois exagéré. Il suffirait de renoncer à des manipulations souvent sans intérêt et sans aucune portée théorique ou pratique.

On lira ci-après le résumé de la seconde partie de cette séance qui a été l'une des plus intéressantes du Congrès.

Prof. Staeckel (Karlsruhe) ist überzeugt, dass die Vorschläge Runges durchgeführt zu werden verdienen, und möchte nur eine gewisse Schwierigkeit beleuchten, die darin liegt, dass die Ausbildung der Physiker und Mathematiker zum Teil gemeinsam erfolgt und dafür gesorgt werden muss, dass die Pflege der logischen Seite der Mathematik nicht vernachlässigt wird.

M. Bourlet (Paris) présente les excuses de la Sous-commission française de n'avoir pas répondu au questionnaire de M. le Prof. Runge. La raison est que, devant une question aussi vaste, la Sous-commission a décidé de faire plus que de donner des renseignements sommaires, et qu'elle publiera tout un volume.

M. Bourlet indique à grands traits ce que sera cet ouvrage.

Comme il l'a déjà expliqué dans des réunions antérieures, l'enseignement est donné en France dans un esprit très général dans les classes de *Mathématiques spéciales* et dans les *Cours de Mathématiques générales* des Universités. En France, on essaie de donner aux jeunes gens qui se destinent aux sciences mathématiques, physiques et naturelles, une éducation générale commune. C'est ainsi que les élèves de l'Ecole normale Supérieure, suivaient jadis, en première année *tous* les *mêmes* cours.

En dehors de l'étude de l'état actuel de l'enseignement oral, la Commission étudiera les ouvrages en usage pour donner un tableau complet de ce qui se fait en France.

Mais M. Bourlet estime que le travail ne devrait pas s'arrêter là et que le rôle de la Commission internationale est de chercher des directions communes au moyen d'une enquête parmi les mathématiciens, les physiciens et les ingénieurs. Cette enquête pourra porter sur deux points : les *matières* qu'il faut enseigner, les *méthodes* qu'il y a lieu d'employer. M. Bourlet estime que c'est surtout le choix des matières qu'il y a lieu de faire avec soin ; quant à la méthode, il est d'avis qu'il ne peut pas y en avoir deux : une pour les mathématiciens et l'autre pour les physiciens, et que l'enseigne-

ment des mathématiques aux physiciens doit conserver son caractère de rigueur et de logique. On n'enseignera pas les fonctions monogènes de M. Borel, mais ce que l'on enseignera on devra le faire avec toute la précision nécessaire. La Commission internationale devant se réunir vraisemblablement à Pâques 1914 à Paris, M. Bourlet espère que le volume sera prêt quelques mois d'avance.

M. F. Enriques (Bologne), remarque que M. Runge s'est placé à un point de vue utilitaire en se demandant quels instruments il faudra donner au physicien. Il y a lieu de poser une autre question qui n'a pas moins un intérêt pratique, c'est-à-dire comment on peut attirer les jeunes gens à l'étude de la physique.

M. Enriques remarque que, dans plusieurs pays au moins, il y a une tendance des étudiants les *mieux doués* à préférer l'étude des questions de mathématiques pures à celles de physique. Cela dépendrait-il de ce que l'on néglige le plus souvent de présenter aux élèves les côtés de la physique théorique qui sont les plus beaux et les plus intéressants, en expliquant le but même de la science qui consiste à mettre d'accord les faits avec les représentations de notre esprit ?

Il n'est pas douteux que les progrès de la physique exigent de se tenir près des faits d'expérience et que l'expérience demande un long apprentissage. Cependant on ne peut méconnaître que certains esprits élevés ne sauraient se soumettre à l'effort de cet apprentissage s'ils ne voient d'abord l'intérêt des questions qu'on est appelé à résoudre par cette voie. D'autres aussi qui n'auront jamais la faculté de fournir eux-mêmes un travail expérimental utile, pourraient apporter une contribution aux questions théoriques grâce à une intuition heureuse.

M. Enriques pense que l'enseignement actuel de la physique mathématique n'est pas fait assez dans le sens de développer cette intuition et parfois même qu'il tend à repousser les esprits qui ont une plus grande intuition par un excès de développement des algorithmes de l'analyse où le moyen semble être pris comme une fin.

Sir G. Greenhill (London). — The Report we have had in our hand for preliminary study, and the exposition of it by Prof. Runge, has covered the whole ground of the state of mathematical training of the student of Physics.

But a participator in the discussion must confine himself to some detail, where his experience encourages him to offer some remarks.

In my own case I endorse heartily what is said of the overgrowth of Descriptive Geometry (see Question II).

The subject is too often easy and soft, and the student grows too fond of it, and cannot be brought back to harder study of the Calculus.

We found this at Coopers Hill College, where I was one of the original staff. Reports from India complained of the young engineer we sent out; that he knew no mathematics, but he could *draw*.

Give him a task on the drawing board with his instruments and he is pleased to work for ever. In fact his instructors at Coopers Hill used to groan at his output of drawings.

So although we envy Prof. Gutzmer his Mathematical Institute at Halle, there is one point of his equipment we do not admire so much- the elaborate laboratory for drawing and descriptive Geometry.

For my part I would rather see a Mathematical Workshop in its place, with a carpenters bench and tools, an anvil and some large hammers, and other simple apparatus for illustrating elementary dynamical principles, and making them familiar to the muscular sense.

Prof. WEBSTER (Clark University, Etats-Unis). — I find myself in general agreement with most of the gentlemen who have spoken, especially with the ideas of Professor Runge, as expressed in his admirable report. I regret very much the distance that often separates the mathematician from the physicist, especially the lack of perception that often exists of what the physical student needs. I should like to point out a very important practical consideration in the words « Ars longa, vita brevis ». In other words so much time must be taken for the education of the physicist that it is totally wrong to waste precious time on what is not useful. I fully agree that the teaching should be thorough- the best is none too good, but it is foolish to spend time in pointing out to the student imaginary difficulties that he will never encounter. As an example, the physicist will need to integrate many functions, but he will not require a rigid discussion of the conditions of integrability of a function, since all the functions that he will meet will probably be integrable. He will have to make frequent use of Fouriers series, but it would be wrong to devote several months of his time to the study of all sorts of convergence and non-convergence of series in general. I believe that every mathematician should know the main principles of physics, just as much as the physicist should be acquainted with the leading methods of mathematics. It is also quite possible to illustrate mathematical ideas, by physical examples which are interesting, illuminating and perfectly logical. I need only suggest the notions of velocity and acceleration, and the employment of such physical ideas as are the possession of everyone.

Sir Joseph LARMOR (Cambridge) said that what he had heard induced him to ask them to consider the position of the mathematicians. In mathematical teaching they ought not to be expected to give up everything that was interesting, and confine themselves to routine work and to providing mere tools to be used in more attractive occupations. What we were suffering from was overspecialisation. In his view mathematics included the whole of theoretical physics ; there was no essential difference between reasoning on the doctrine of thermodynamics and reasoning on the doctrine of limits. Mathematics was not all technical algebraical analysis, any more than physics was all experimental dexterity.

M. E. BOREL (Paris) se déclare entièrement d'accord avec M. Runge. Nous devons tendre à rendre plus simple l'enseignement des mathématiques. Puis il donne des renseignements sur ce qui se fait à l'Ecole normale supérieure. En première année l'enseignement est commun aux physiciens et aux mathématiciens, sauf de rares exceptions. La méthode et le choix des matières dépendent souvent des questions d'actualité, de mode et de la personnalité du maître.

M. Ch. BIOCHE (Paris) estime que les physiciens ou naturalistes doivent avoir quelques connaissances générales des mathématiques, en dehors des connaissances pratiques qu'ils utilisent journellement, de façon à ce qu'ils puissent savoir qu'à l'occasion de telle question les intéressant ils trouve-

ront auprès d'un mathématicien professionnel les indications qui leur seraient utiles.

Lorsqu'on veut traiter un problème d'ordre pratique il ne suffit pas d'en avoir une solution théorique ; il faut en avoir une solution commode. Il importe que, dans l'enseignement mathématique, on se préoccupe de discuter les solutions des problèmes au point de vue de la simplicité dans les applications

Prof. Love (Oxford) wished to associate himself with Sir J. Larmor's view. The ideal thing would be, every mathematician a physicist and every physicist a mathematician. To gain time for the necessary training it would be desirable to discard obsolete things in mathematics and to devise some scheme by which a general knowledge of physics could be brought within the reach of all students of mathematics. In regard to the proposed choice of mathematical matter, he thought there might be a tendency to make the subject too narrow, with too little regard to the possible future application of branches of analysis which are now studied for their own sake.

Prof. Hobson (Cambridge) said that perhaps too large a share of the blame for the undoubted evils of the present divorce of Mathematical and Physical teaching had been put upon the Mathematicians. If the mathematical teacher spent too much time on matters which would be better omitted and in the teaching of merely analytical expertness, it might be said that the Physicist spent too much time on the merely manipulative work of the Laboratory. Mathematical teachers were expected to teach only what was directly useful to the Physicist, and also what was directly useful to the Engineer, or to the Chemist. The main object of the Mathematical teaching ought to be that of developing habits of clear and logical thinking in the pupils, and that could not be hampered by a too close adhérence to what was required only for application in other departments. Dexterity in such matters as the use of the slide rule was not part of what the Mathematician should be expected to teach. He would lose his self-respect if he became a mere provider of tools for use in subjects outside his own.

Sir J.-J. Thomson (Cambridge), l'illustre physicien, reconnaît que de nos jours on exagère parfois le temps accordé aux travaux pratiques dans l'enseignement supérieur où ils prennent une trop grande place. Ce serait nuisible de donner aux physiciens un enseignement mathématique spécialisé, par contre il faut qu'ils soient familiarisés avec les applications pratiques.

Prof. Runge. — I believe that the introduction of numerical and graphical exercises do not interfere in any way with the logical deduction of pure mathematical theorems. My idea is that practical mathematics are indispensable not only to the physicist but also to the pure mathematician in so far as they enable him to work out examples for the application of pure mathematical theorems, and in this way enlighten him and aid his logical deductions.

Remarques de M. F. W. Lanchester (Birmingham). — The question of the mathematical training of the physicist in the university has perhaps two aspects — that is from the point of view of the mathematician, and from the standpoint of the work which the student in after life finds it incumbent upon him to undertake. Whether trained primarily as enginer or physicist a large number of those who are trained in applied mathematics either en-

ter by intention or by accident into the engineering profession. The following remarks may be taken to relate more particularly to this class of student.

The weakness commonly exhibited by the engineering student, who having graduated in a university takes a post as an engineer or in an engineering work, is found more often than not to lie in his inability to deduce from the complex maze of facts with which he has to deal, a definite problem in what may be termed « examination-question » form[1]. In my experience there is usually ample knowledge of mathematical principles and mathematical method, but there is a very obvious deficiency in the development of the mathematical student's reasoning powers, which are the essential counterpart to any purely mathematical training, before it can serve him usefully in the responsible carrying out of engineering work. It is naturally very questionable whether it is possible by any modification or supplementing of the university education to make good this deficiency entirely; in fact it stands to reason that for the elastic application of his mathematical training the latter will require to be supplemented by considerable actual experience before it can be fully utilized. Notwithstanding this, I consider that there are certain subjects (which in the discussion of the paper were belittled by some of the speakers present) which a little reflection will show to be of vital importance. As an example (and in my opinion a very important example) it is impossible to overrate the utility from an educational point of view of descriptive geometry. There is no subject that teaches a student to *think in three dimensions* so well as descriptive geometry, and the inability to think in three dimensions is one of the most common failings of the university mathematically trained engineer. This aspect of descriptive geometry is quite apart from its obvious utility in training a man to read drawings of a complicated character such as are constantly before him in the course of his duties. I would call particular attention incidentally to the difference between a study such as descriptive geometry which is a real educating factor and enlarges the capacity of the mind, and those mathematical studies which cannot be regarded in the same light, but of which one must think as analagous to tools in the hands of a workman. I do not wish to suggest that all mathematical work is merely a matter of providing a mental tool equipment, but only that a great deal of the ordinary mathematical curriculum can be looked upon from this point of view, rather than be considered as a real educative factor.

I am convinced that the mathematical investigation of three dimensional problems is in many cases detrimental to the expansion of the mind, inasmuch as the mathematician rather than think in three dimensions employs a machine process to evade mental gymnastics; for instance in fluid dynamics he thinks once and for all in three dimensions in an imaginary infinitesimal unit cube of the fluid. From this he makes equations and from these equations he obtains his results. From the time these equations are formulated he may forget all about three dimensional space until he comes to the interpretation of his results. I think that in the mathematical training of a young engineer such dangerous methods should not be introduced until

[1] It is as if some-one accustomed to dining off a well cooked and nicely served steak were shown the carcase of a freshly slaughtered ox and told to get their dinner out of that!

he has had a very thorough grounding in descriptive geometry. There is truly the risk pointed out by Professor Greenhill that after he has devoted a great deal of time to descriptive geometry he will be liable to shirk, or ignore, or neglect the more rigid mathematical method ; but even if this undesirable consequence results (which in nowise appears necessary) I think that it may be taken as the lesser of two evils.

Briefly from a more general point of view I think that in the discussion of the circular there has been too much tendency to dwell on detail questions as to what should be taught and what should not be taught, and to ignore the principle that true education ought to be a process of developing and expanding the mind in those directions in which such development and expansion is most likely to serve the ends in view, and that the provision of a mental tool equipment is, or should be considered as merely an incidental.

TROISIÈME SÉANCE

Mardi 27 août, à 9 heures et demie du matin.

Présidence de M. R. Fujisawa (Tokio) et C. Godfrey (Osborne)

Ordre du jour :

I. — *Intuition and experiment in mathematical Teaching in the Secondary schools* (l'intuition et l'expérience dans l'enseignement mathématique des écoles moyennes), rapport présenté par M. D.-E. Smith (New-York). — Discussion.

II. — Remarques sur une biblographie de l'enseignement mathématique, par M. C. Goldziher (Budapesth).

III. — Prolongation du mandat de la Commission, Les travaux pendant la prochaine période.

I. — INTUITION AND EXPERIMENT IN MATHEMATICAL TEACHING IN THE SECONDARY SCHOOLS[1]

Report presented by

David Eugene SMITH (New-York)

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1. — Method of Investigation.

In the year 1911 the Central Committee appointed a subcommittee known as « Subcommittee A » charged with the duty of investigating the rôle of intuition in the teaching of

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[1] The German topic as assigned was « Anschauung und Experiment im mathematischen Unterricht der höheren Schulen ». This was translated into French as « L'intuition et l'expérience dans l'enseignement mathématique des écoles moyennes ». The translation is not