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# Probabilistic Stratigraphy

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## ABSTRACT

Biostratigraphic resolution is currently limited by lack of a quantitative method of determining the most likely sequence of first and last occurrences of species or morphotypes and of expressing the probability that a particular assemblage belongs within certain limits in an established sequence. Fundamental to development of probability oriented stratigraphy is establishment of a suitable theoretical base allowing stratigraphic information to be fitted into a series of hypotheses which can be readily tested. The techniques suggested here will establish a new kind of stratigraphic system in which correlation is expressed as a probability. The probability of a correlation will be determined by comparison of the unknown sample with the sum of the available information on the sequence of appearance and disappearance of fossils or other criteria in diverse facies at different sites. It should result in an increase of stratigraphic resolution by at least one order of magnitude over that presently attainable.

## Introduction

The science of stratigraphy has developed into two major areas: lithostratigraphy and chronostratigraphy. Lithostratigraphy is concerned with the classification, description, and lateral tracing of rock units recognized by gross physical characters. The term "correlation" is sometimes used in discussion of lithostratigraphic units to indicate that original continuity of particular strata has been established or inferred as a result of mapping, well logging, or comparison of the sequence of rock units in separated sections. Principles of lithostratigraphy were among the first to be elucidated in geology, and are well understood by all experienced geologists. Chronostratigraphy is concerned with determining the age of rocks, using the fossils contained in them (biostratigraphy) or other suitable criteria such as radiometric age determinations, paleomagnetic reversals, etc. The principles of chronostratigraphy were developed in discontinuous, incomplete sections with significant facies changes. As a result, an incomplete theoretical basis developed, and this has resulted in considerable confusion so that experienced geologists have widely differing opinions as to the best methods for establishing age equivalence. Much of the uncertainty regarding methodology in chronostratigraphy has been a result of disagreements among paleontologists or biostratigraphers using fossils to determine the age of the rocks in which they occur.

Fossils have been used for almost 150 years to subdivide and correlate series of rock. Stratigraphic units based on the occurrences, abundances, or absences of

fossils have been termed "biostratigraphic units". The American Commission on Stratigraphic Nomenclature in its Code of Stratigraphic Nomenclature (1961) recognized biostratigraphic units as a category of equivalent rank to chronostratigraphic units (see also HEDBERG 1970), but most biostratigraphers consider biostratigraphic units to be a special kind of chronostratigraphic unit and some, such as SEITZ (1958) and WIEDMANN (1968), do not recognize any basic difference between chronostratigraphic and biostratigraphic units. ARKELL (1933), SHAW (1964), the American Commission on Stratigraphic Nomenclature (1961) and GEORGE et al. (1969) have discussed biostratigraphic methods and terms. The American Code refers to "taxon or taxa" in discussions of biostratigraphic units; the British "Recommendations for stratigraphic usage" (GEORGE et al. 1969) refers to "fossil group" in the same context. This is an important difference because the American Code, perhaps unintentionally, seems to limit biostratigraphic definitions to formally described fossils with linnean names, while the British Recommendations allows definitions in any convenient terms.

The basic biostratigraphic unit is the Zone. Except for the Assemblage Zone and Acme Zone, biostratigraphic zones are based on the appearance or disappearance of the fossil groups used to define their limits. The Assemblage Zone is defined by the American Code and by GEORGE et al. as a body of strata characterized by a certain assemblage of fossils without regard to their ranges. Because consideration of the ranges of fossils is specifically excluded from the definitions, the Assemblage Zone must be regarded as primarily a reflection of environmental conditions. It may have an age connotation or significance, but this is necessarily vague and imprecise. The Peak Zone of the American Code, equivalent to the Acme Zone of GEORGE et al., is a biostratigraphic unit based on the exceptional abundance of a single fossil group, and is independent of the range of the fossil group. The Range Zone of the American Code, or Total-Range Zone of GEORGE et al., is defined as the body of strata comprising the total horizontal and vertical range of occurrence of a specified fossil group. The Local Range Zone of the American Code, or Local-Range Zone of GEORGE et al., is defined as a body of strata in a specific geographical section characterized by the occurrence of a specified fossil group. The Concurrent-Range Zone, the only biostratigraphic unit other than Assemblage Zone to have exactly the same name in both the American Code and GEORGE et al., is defined as a body of strata characterized by the overlapping ranges of specified fossil groups. The Partial-Range Zone is defined by GEORGE et al. as a body of strata within the range of a fossil group above the last appearance of the preceding fossil group and below the first appearance of the succeeding fossil group; no equivalent category is suggested in the American Code. A special case of partial range Zone is indicated by GEORGE et al.: the Consecutive-Range Zone, defined as a body strata within the range of a fossil group such that it forms the first part of the range of that fossil group before the appearance of its immediate evolutionary descendent. In practice, most modern zonations based on first and last occurrences of fossil groups employ combinations of Concurrent-Range Zones, Total-Range Zones and Partial-Range Zones.

DARWIN's theory of evolution has provided a comfortable basis for understanding why the limits between zones of this sort should not be ambiguous; indeed, many systems of zonation are based upon supposed evolutionary sequences in particular

groups of fossils. The resolution of a zonation based on evolutionary sequences is fixed by the rate at which the members of the lineage evolved and by the number of morphological types into which the lineage can be divided. Biostratigraphic correlation generally involves the comparison of assemblages of fossils. Although the degree of similarity of two assemblages may be expressed statistically, the number obtained cannot be construed as an indication of the probability that the assemblages were contemporaneous. Because the occurrences of fossils are a function of the original biogeographic distribution of the species, the local environmental conditions where the organisms lived, vagaries of transportation and sorting or solution by the transporting agent, solution and/or alteration during the period of burial, and of the nature of the available samples as well as a function of their occurrence in time, the number expressing the statistical correlation between assemblages represents a complex variable which may be markedly influenced by any of these factors.

Chronostratigraphic units are defined by type sections. GEORGE et al. (1969) suggest a set of new "Stratometric Standard Terms" which are essentially standard "Chronostratigraphic Units" in the sense of the American Commission on Stratigraphic Nomenclature (1961). GEORGE et al. have proposed the term "Standard Chronozone" as the basic unit for their scheme; the "Standard Chronozone" is "defined by reference to marker points in type sections" (p. 148).

The American "Code of Stratigraphic Nomenclature" and British "Recommendations on stratigraphical usage" have formalized the chaos of time-rock stratigraphy, but have done little to end the confusion inherent in the traditional methods. The sad situation which exists today seems to be due in large part to the lack of an underlying theoretical basis for time-rock stratigraphy amenable to mathematical manipulation and analysis in terms of probability. It therefore seems appropriate to consider development of a new kind of time-rock stratigraphy involving: 1. specification of a method for handling and analyzing stratigraphic information on the distribution and relative abundances of fossils; 2. determination of the most likely sequence of widespread physical events and of first and last occurrences for as many species or morphotypes as possible; and 3. establishment of an expression of correlation as a probability based on three factors: a) the probability that the events defining the stratigraphic interval have been detected in the section studied, b) the probability that the sequence of events is correctly known, and c) the probability that the fossil groups or physical criteria used to define time increments have been correctly identified.

### **Determination of the most probable sequence and probability of correlation**

In spite of the important advances made by SHAW (1964), much still needs to be done in biostratigraphy to take full advantage of the fossil record as a base for the subdivision of the geologic record. Of prime importance is the development of a theoretical base for establishing the sequence of first and last occurrences of species. Once this has been accomplished, the way is open to development of an expression for biostratigraphic correlation in terms of probability.

*Definition 1:* A stratigraphic event is an occurrence of some importance in stratigraphy. (The term "stratigraphic event" can be used with appropriate modifiers,

such as "local stratigraphic event" or "regional stratigraphic event" or "global stratigraphic event", to indicate the areal extent of the occurrence.)

*Definition 1a:* A biostratigraphic event is the lowest or highest stratigraphic occurrence of a fossil group.

The accuracy of the determination of lowest and highest stratigraphic occurrences depends on two factors: 1. the abundance of the fossil group in the total population, and 2. the size of the sample available for study. Although biostratigraphers are almost universally loathe to admit it, biostratigraphy depends as much on the absence as it does on the presence of certain fossil groups. Determining the probability of a fossil group being absent from a population is very important in biostratigraphy; the Partial-Range Zone, already in common use, relies on establishing the absence of the fossil group defining the next higher or next lower zone. Figure 1 is a graph from DENNISON and HAY (1967) from which the probability of *not* finding a fossil group present in a population at a given level of abundance may be read directly. The abundance of a fossil group in a population at the time of its origin or extinction may be assumed to be vanishingly small, and it may also be assumed that it is virtually impossible to determine the absolute level of its lowest or highest occurrence. It is possible, however, to give a figure which indicates the probability that a fossil group is absent in a population, based on its known abundance in samples elsewhere and on the number of specimens in the sample investigated. For the system to be developed here, in contrast to that of SHAW (1964), it is not so important to know the exact level in given section at which a fossil group has its lowest or highest occurrences, but it is necessary to determine the sequence of such occurrences. If it is advantageous to know the level of lowest or highest occurrence of a fossil group in a given section, this level may be determined with any desired degree of precision by using the figures for the probability of absence of the fossil group in samples and splitting the sampling interval as needed.

*Definition 1b:* A physical stratigraphic event is an occurrence of stratigraphic importance not based on fossils.

Geomagnetic reversals and fluctuations of isotopic composition, such as the  $^{36}\text{O}/^{38}\text{O}$  ratio, which might be plotted and unusual layers, such as ash or coal beds, are examples of physical stratigraphic events. Physical stratigraphic events are by their nature recurrent, in contrast to biostratigraphic events which are unique.

*Definition 2:* The stratigraphic distance between two stratigraphic events is a stratigraphic increment.

*Assumption 1:* No two stratigraphic events ever occurred at the same moment; i.e. every pair of stratigraphic events are separated by a finite time increment.

If it is assumed that absolute simultaneity does not exist in nature, it follows that it might be possible to separate pairs of stratigraphic events, and to determine a sequence of stratigraphic events, provided a sufficiently complete record is found. If the dispersal rates for fossils of a particular group were very high in comparison with their evolutionary rates, it follows that the sequence of biostratigraphic events (appearances and disappearances of species) should be everywhere the same. On this basis, the biostratigraphic record would be divisible into a number of increments equal to twice the number of distinguishable fossil groups minus one. Obviously, most groups of fossils do not satisfy the requirements of occurrence and distribution

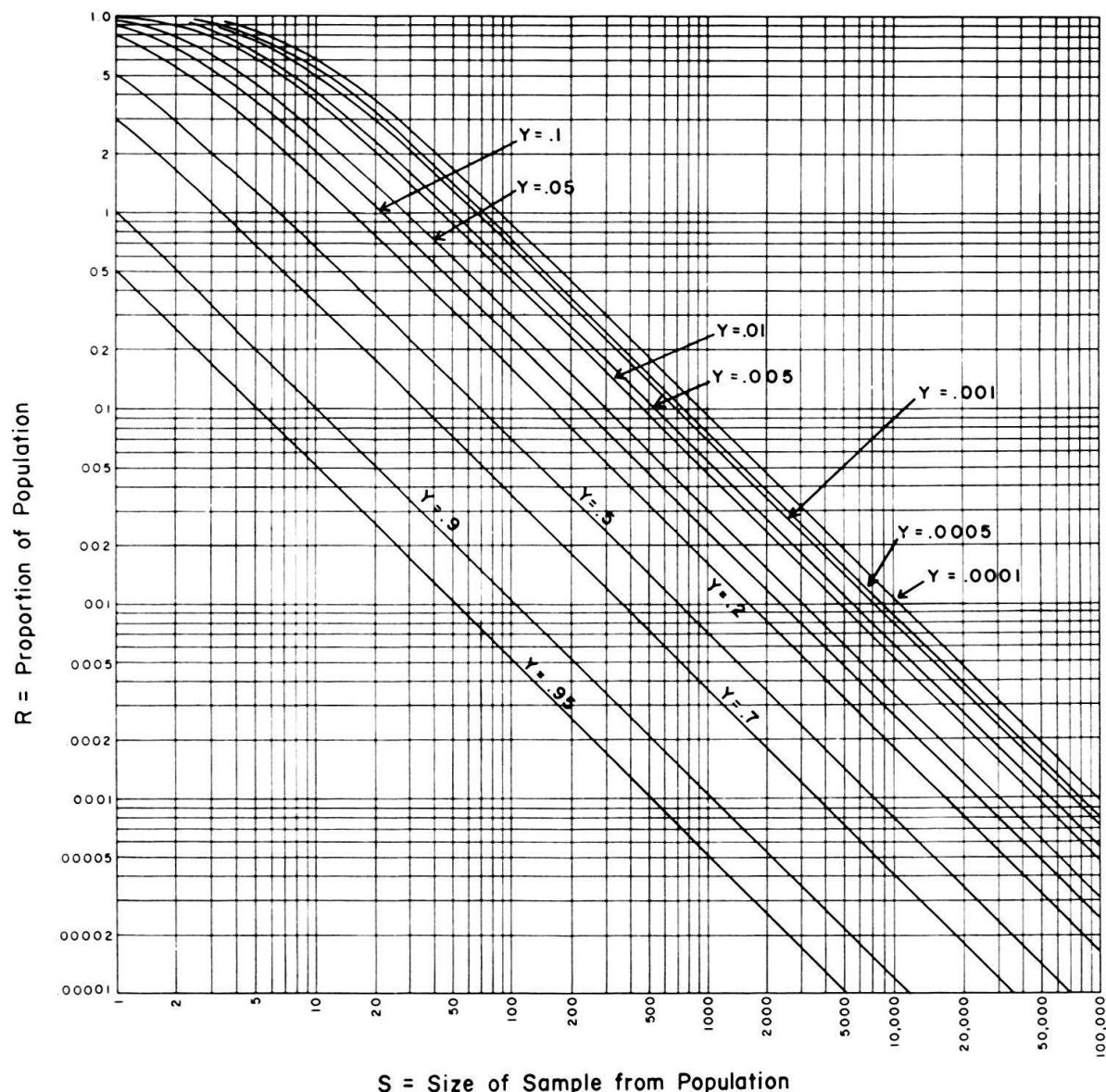


Fig. 1. Graph of the function  $Y = e^{-SR}$  derived from the Poisson distribution.  $Y$  = probability of failure to detect a fossil group.  $R$  = average proportional abundance of the fossil group in representative populations.  $S$  = size of the sample of the population being investigated. The probability that the fossil group is not present in the population being investigated is  $1 - Y$  (after DENNISON and HAY 1967).

to permit them to be used for the very fine subdivision suggested here, but one group, the calcareous nannofossils, can feasibly be analyzed for this purpose at this time. As discussed further by HAY et al. (1967):

“Calcareous nannofossils have several peculiarities which make them uniquely suitable for use as biostratigraphic indicators: 1. they are extremely abundant in many samples; 2. a large number of species have worldwide distribution; and 3. many groups have evolved very rapidly.”

*Definition 3:* A continuous section is a section which can be sampled in such a way that no two local stratigraphic events occur in the same sample.

In other words, for a section to be considered to be continuous, it must be possible to determine the sequence of all events which occur within it. Sections may be discontinuous for a number of reasons: 1. the strata might be present and might contain the necessary fossils or physical criteria, but adequate sampling may be impossible due to outcrop conditions, loss of core, or poor sample recovery; 2. the strata may be unfossiliferous at the critical level due to a change in facies, leaching, recrystallization, etc.; or 3. a hiatus may exist.

In actual practice, it is found that long continuous sections, as defined here, are exceedingly rare. Some of the best sections containing calcareous nannofossils have been discussed by HAY et al. (1967). In preparing the range charts presented in HAY et al., the "noise" in the original data was eliminated, with only the most well defined, widely recognized species included. Undoubtedly, much information was also eliminated, but this information can only be made useful through analysis of the sort proposed below. Consecutive samples in which more than one species had its first occurrence, or more than one species had its last occurrence, were separated by a broad strip, and the nature of the discontinuity indicated by such terms as "covered", "barren interval", "paraconformity", "impoverished assemblages", etc. (The discontinuities were inadvertently omitted from Figure 7 of HAY et al., but exist between samples A-7088 and A-7061, A-7061 and A-7052, and A-7038 and A-7009. Each of the gaps represents a long interval of unfossiliferous rock.) If the range charts are examined in detail, it will be noted that none of the sections yet described is continuous over intervals of more than four nannofossil zones, but these are some of the best sections that have been found after a number of years of research involving many samples, many sections, and many investigators. It can now be safely concluded that long continuous sections are rare, even in the deep sea, but short continuous sections are common. Because only short sections have generally been available, it has been an almost impossible task to piece them together in the right order; indeed this is the reason why comprehensive nannoplankton zonation schemes could not be suggested until 1967.

*Definition 4:* Stratigraphic resolution is the smallest stratigraphic increment which can be distinguished at a given level of probability.

Up to present, the stratigraphic resolution attainable with calcareous nannoplankton fossils is about 1-2 million years. The existing zonations are still very crude because only the most obvious species are utilized. Nevertheless, the zonation based on calcareous nannoplankton is almost as refined as that based on planktonic foraminifera, a group that has received much more careful study. The number of species used to produce the most modern zonation (MARTINI and WORSLEY 1970, MARTINI, 1970) was very small in comparison with the total number of species known so that a much more refined zonation must be possible.

#### Correlation expressed as a probability

*Definition 5:* Stratigraphic correlation is an expression of the degree of probability that samples from two different sections occupy the same level in the known sequence of stratigraphic events. (It should be noted that the term "correlation" is used in lithostratigraphy in a different sense, an expression of proof of original continuity of

a stratum. Ambiguity would be eliminated if the use of the term "correlation" were restricted in geology to the definition presented here and the term "continuity" were adopted for lithostratigraphy.)

Consider first the special case of biostratigraphic correlation. As has been noted by HAY and ČEPEK (1969), the probability of a biostratigraphic correlation depends upon 1. the probability that the biostratigraphic events defining a biostratigraphic increment have been detected (this value is readily determined from the table of DENNISON and HAY 1967, provided that an estimate of the abundance of the fossil group is known); 2. the probability that the true sequence of stratigraphic events is known; and 3. the probability that the species which define the biostratigraphic increments have been correctly identified.

### STRATIGRAPHIC INFORMATION

I	II	III	IV	V	VI	VII	VIII	IX	Δ
				W					W
			Δ	Θ					Θ
			W	Θ	Δ			W	Θ
			V	V	W	W			Θ
W			W	δ	□	□V	ΔV	Θ	Θ
V			<	□	□	Θ	<Θδ	W	δ
□			W	δ	<	Θδ	□Θ	Δ<δ	□<
<Θ□ΘδΘ	Θ□□ΘδΘΔ	Θ	Θ	Θδ	<□	□	□	Θ	Θ

Fig. 2. Stratigraphic information from a series of sections in California studied by SULLIVAN (1965, I-VIII) and BRAMLETTE and SULLIVAN (1961, IX). The vertical columns represent individual sections: I, Vaca Valley area, Solano County; II, Pacheco Syncline area, Contra Costa County; III, Tres Pinos area, San Benito County; IV, upper Reliz Creek area, Monterey County; V, New Idria area, San Benito County; VI, Media Agua Creek area, Kern County; VII, upper Canada de Santa Anita area, Santa Barbara County; VIII, Las Crecus area, Santa Barbara County; IX, Lodo section (Lodo Formation) and Tunney Gulch (Domingene Sandstone), Fresno County. The symbols represent biostratigraphic events:  $\delta$  = lowest occurrence of *Coccolithites gammation*;  $\Theta$  = lowest occurrence of *Coccolithus cribellum*;  $\Theta$  = lowest occurrence of *Coccolithus solitus*; V = lowest occurrence of *Discoaster cruciformis*; < = lowest occurrence of *Discoaster distinctus*; □ = lowest occurrence of *Discoaster germanicus*; □ = lowest occurrence of *Discoaster minimus*; W = highest occurrence of *Discoaster tribrachiatus*; Δ = lowest occurrence of *Discolithus distinctus*;  $\Theta$  = lowest occurrence of *Rhabdosphaera scabrosa*. Two or more symbols on the same level in a section indicate that the events they represent cannot be separated. The column of symbols in the box at left is a hypothetical sequence of events suggested by inspection of some of the more complete sections.

Determining the probability that the true sequence of events is known can be accomplished as follows:

Figure 2 presents information on biostratigraphic events from 9 different sections, taken from data of SULLIVAN (1965). Each of the symbols represents either the lowest or the highest occurrence of a taxon. Two or more symbols at the same level indicate that the occurrences are in a single sample, i.e. that the section is discontinuous at that level. It is very difficult to guess by inspection of the data the most probable order of the biostratigraphic events. By inspection of the three most complete sections, the sequence indicated in the rectangle at the right was suggested, and this can serve as a working hypothesis. Using the suggested sequence as a base, a matrix may be con-

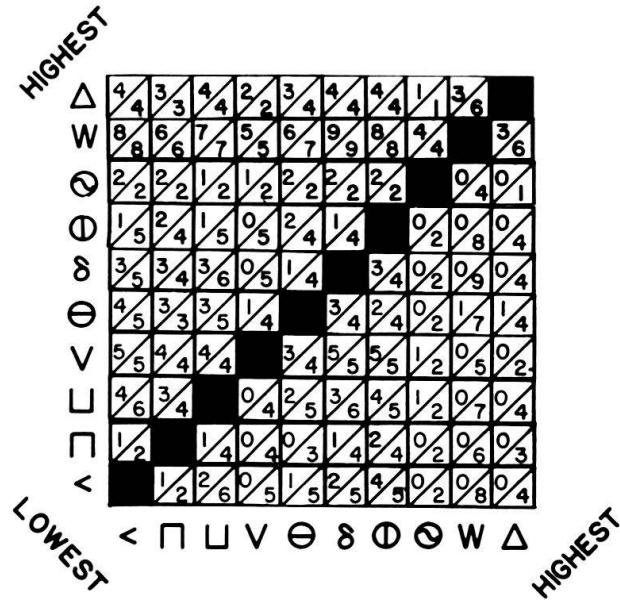


Fig. 3. Matrix showing the relations of biostratigraphic events in Figure 2. The number ( $N$ ) in the lower right of each square is the number of sections in which the pair of events is separable. The number ( $n$ ) in the upper left of each square is the number of times the event on the bottom row occurs below the event on the left side. The sequence from lowest to highest on the bottom and left side of the matrix is that suggested by inspection of the more complete sections in Figure 2.

structed to determine the number of times one biostratigraphic event occurs below another, and conversely, the number of times the order is reversed (Fig. 3). In Figure 3, the number in the upper left half of each box indicates the number of sections in which the biostratigraphic event indicated on the bottom of the matrix occurs below the biostratigraphic event indicated on the left side of the matrix. The number in the lower right half of each box indicates the number of sections in which the two biostratigraphic events are separated (no matter whether they occur in the supposed correct or in reversed order). The most likely sequence is determined by re-evaluating the relations of all pairs which show a fraction greater than  $1/2$  in the lower right half of the matrix.

In other words, inspection of the matrix reveals that the relation of  $V$  and  $\Theta$  should be reversed, the number in the appropriate square in the upper left hand part of the matrix being  $1/4$ , less than  $1/2$ . After making this correction, the new order of the symbols would read, from lowest to highest:  $< \sqcap \sqcup \Theta V \delta \Theta \sqcap W \Delta$ . Next, it will be discovered that  $\delta$  should come below both  $\Theta$  and  $V$ , the relationships being expressed by the fractions  $0/5$  and  $1/4$  respectively; making this correction, the order from lowest to highest becomes:  $< \sqcap \sqcup \delta \Theta V \Theta \sqcap W \Delta$ . Finally, it is evident that the position of  $\Pi$  in the sequence needs to be changed because its relation to  $\delta$  is  $1/4$ , to  $V$  is  $0/5$ , to  $\sqcup$  is  $1/5$ , and to  $<$  is  $1/5$ . It must come below any of these symbols, and does in fact become the lowest event in the revised order:  $\Pi < \sqcap \sqcup \delta \Theta V \Theta \sqcap W \Delta$ . No further fractions less than  $1/2$  can be found in the matrix, and the most likely sequence of events, determined from all the stratigraphic information available from the nine sections, is, from lowest to highest:  $\Pi < \sqcap \sqcup \delta \Theta V \Theta \sqcap W \Delta$ , the last revision of the sequence. Figure 4 shows the rearranged matrix; all

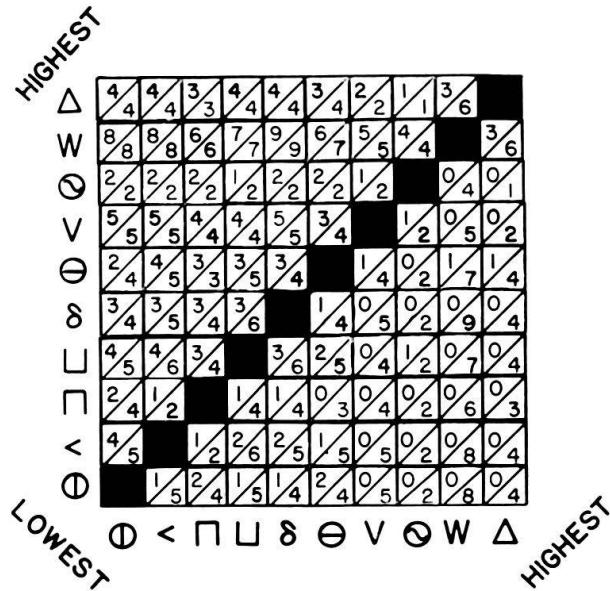


Figure 4. Revised matrix in which the ratio  $n/N$  has been rearranged so that all values greater than  $1/2$  are in the upper left part of the matrix. The lowest-highest sequence along the bottom and left side of the matrix now represent the most probable sequence of events.

fractions greater than  $1/2$  are in the upper left half of the matrix, and all fractions less than  $1/2$  are in the lower right part of the matrix. Now that the most likely sequence has been established, it is desirable to determine the probability that each pair of biostratigraphic events is separable and known in true sequence. This can best be done by assuming that the events are not sequential, but that the relations of all pairs are random; then the probability of this being untrue can be determined. If the sequence of a pair of biostratigraphic events is random, the probability of one biostratigraphic event preceding the other is  $1/2$ .

If the biostratigraphic events occur  $n$  times in the order predicted by the analysis above, in a total of  $N$  sections in which the events occur sequentially, then the probability  $P$  that  $n$  would occur, assuming a random distribution, is

$$P = \frac{N!}{n! (N-n)!} \left(\frac{1}{2}\right)^N.$$

The probability  $P$  that the observed arrangement is caused by random distribution is

$$P = \sum_{r=n}^N \frac{N!}{r! (N-r)!} \left(\frac{1}{2}\right)^{N-1}.$$

where  $r$  is a variable integer running from  $n$  to  $N$ . The probability that the sequence is nonrandom is derived by subtracting the probability that it is random from 1.

<sup>1)</sup> Note: Using the above formula, probability values for  $n/N$  values of  $1/2$ ,  $2/4$ ,  $3/6$ ,  $4/8$ , etc. are greater than 1.0. This is a peculiarity inherent in the formula; for practical purposes all of these values can be reduced to 1.0 (indicating complete randomness).

Table 1. Probability that the sequence of a pair of biostratigraphic events is described by random distribution.  $N$ =number of sections in which the pair of events is separable.  $n$ =number of sections in which the events occur in a predicted order.  $P$ =probability that this arrangement is caused by random distribution.  $1-P$ =probability that this arrangement is nonrandom, or can be repeated in the next section investigated.

$N$	$n$	$P$	$1-P$	$N$	$n$	$P$	$1-P$
2	2	0.5000	0.5000	11	11	0.0010	0.9990
	1	1.0000	0.0000		10	0.0117	0.9883
3	3	0.2500	0.7500		9	0.0654	0.9346
	2	1.0000	0.0000		8	0.2266	0.7734
					7	0.5488	0.4512
4	4	0.1250	0.8750	12	6	1.0000	0.0000
	3	0.6250	0.3750		12	0.0005	0.9995
					11	0.0063	0.9937
5	5	0.0625	0.9375	13	10	0.0386	0.9614
	4	0.3750	0.6250		9	0.1460	0.8540
	3	1.0000	0.0000		8	0.3877	0.6123
6	6	0.0312	0.9688		7	0.7744	0.2256
	5	0.2187	0.7813	14	13	0.0002	0.9998
	4	0.6875	0.3125		12	0.0032	0.9968
7	7	0.0156	0.9844		11	0.0222	0.9778
	6	0.1250	0.8750		10	0.0920	0.9080
	5	0.4531	0.5469		9	0.2666	0.7334
	4	1.0000	0.0000		8	0.5808	0.4192
8	8	0.0078	0.9922	15	7	1.0000	0.0000
	7	0.0703	0.9297		14	0.0001	0.9999
	6	0.2891	0.7109		13	0.0018	0.9982
	5	0.7266	0.2734		12	0.0129	0.9871
9	9	0.0039	0.9961		11	0.0537	0.9463
	8	0.0391	0.9609		10	0.1796	0.8204
	7	0.1797	0.8203		9	0.4240	0.5760
	6	0.5078	0.4922		8	0.7905	0.2095
10	10	0.0020	0.9980	16	15	0.0001	0.9999
	9	0.0215	0.9785		14	0.0010	0.9990
	8	0.1094	0.8906		13	0.0074	0.9926
	7	0.3438	0.6562		12	0.0352	0.9648
	6	0.7539	0.2461		11	0.1185	0.8815
					10	0.3018	0.6982
					9	0.6072	0.3928
					8	1.0000	0.0000

Table 1 presents values for  $1-P$ , the probability that the sequence is nonrandom for values of  $N$  (number of sections) up to 15. If 0.9900 is chosen as an arbitrary limit to select biostratigraphic events to be used, inspection of Table 1 will reveal that this is a stringent limit. The sequence must be known from at least 8 sections and must be invariable within those. However, it should be noted that if 12 sections are known and the sequence is reversed in one of them, the calculated value still exceeds 0.9900. If 15 sections are known, the sequence may be reversed in two of them before the

calculated value of  $1-P$  falls below 0.9900. In practice biostratigraphers commonly work with values of  $1-P$  as low as 0.9000 or even 0.8000. A matrix of values of  $1-P$  for the data of SULLIVAN (1965) is presented in Figure 5. Two stratigraphic levels can be defined at the  $> 0.9900$  limit, the lower defined by  $\delta$ ,  $\Theta$  or  $<$  and the upper by  $W$ . Three levels can be recognized at the  $> 0.9000$  limit, the lowest defined as  $\Theta$ ,  $<$ , or  $\delta$ , the intermediate level defined as  $V$  and the upper level defined as  $W$ .

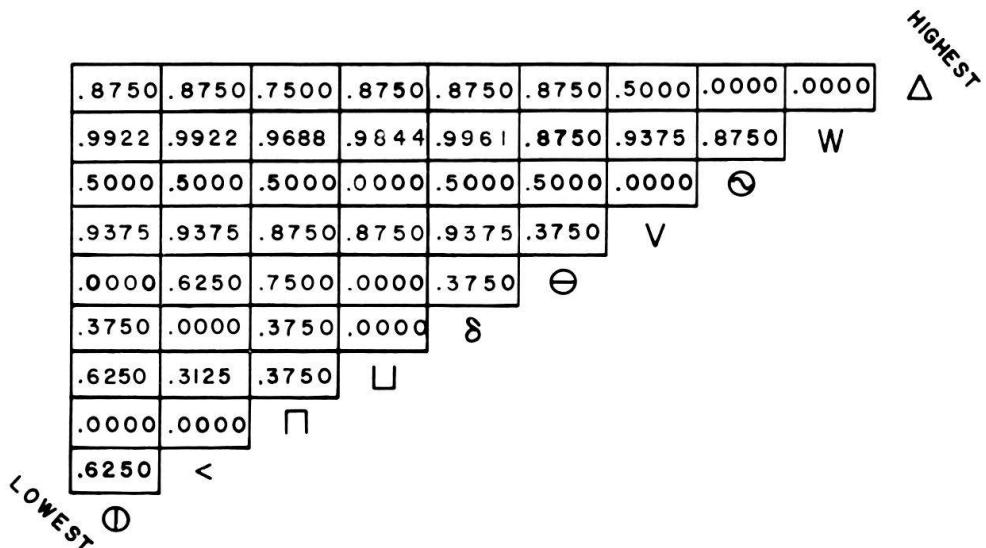


Fig. 5. Values of  $1-P$ , the probability that the sequential relation between two events is nonrandom, taken from Table 1. This may be considered an expression of the probability that the pair of events will occur in the predicted order in the next section examined.

The third factor in stratigraphic correlation, the probability that the critical species used to define biostratigraphic increments have been correctly identified, is perhaps the most difficult factor to quantify. To begin with, a simple number expressing the opinion of an experienced investigator may suffice, but it is expected that other techniques may evolve. For example, any stratigraphic event which differs markedly in its position in the sequence from section to section is open to suspicion; such stratigraphic events would be readily detected as they would bear consistently low probability values when paired with other stratigraphic events.

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