Kleine Mitteilungen

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Natürlich gewinnt man aus (2) analog zu Mortini durch Integration die Ungleichung

$$\left| \int_{-1}^{1} f(x)\, dx \right| \leqq \int_{0}^{1} \max_{\theta} |g(x^2 e^{i\theta})|\, dx\,, \tag{8}$$

falls das letztere Integral endlich ist. Diese Ungleichung ist aber nicht mehr scharf, wenn nicht über g spezielle Voraussetzungen gemacht werden. Die Beschränktheit des rechten Integrals in (8) folgt für eine konvexe Funktion, deren Bildgebiet weder eine Halbebene noch ein Parallelstreifen ist, aus der Tatsache, dass das Bildgebiet in diesem Fall in einem Sektor mit Öffnungswinkel $p\pi$ mit $p < 1$ liegt. Also gilt nach einer geeigneten Transformation der Form $g \mapsto Ag + B$ die Beziehung $g \prec \left(\frac{1+z}{1-z}\right)^p$ und folglich

$$\int_{0}^{1} \max_{\theta} |g(x^2 e^{i\theta})|\, dx \leqq \int_{0}^{1} \left(\frac{1+x^2}{1-x^2}\right)^p dx \leqq 2^p \int_{0}^{1} \frac{dx}{(1-x)^p} < \infty\,.$$

Prof. Pfluger möchte ich für seine zahlreichen wertvollen Bemerkungen und Anregungen danken.

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LITERATURVERZEICHNIS

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An upper bound for a sequence of cevian inequalities

1. In El. Math. Vol. 35/6, the sequence

$$\sum h_a \leq \sum g_a \leq \sum w_a \leq \sum m_a \leq \sum n_a$$

of inequalities for the altitudes, Gergonne cevians, internal angle bisectors, medians, and Nagel cevians respectively of a triangle ABC is given. The well-known inequality $9r \leq \sum h_a$, where r is the inradius, [1] p. 61, provides a lower bound for the sequence; in this paper we derive an upper bound.

2. In figure 1, $AN_a = n_a$, $AM_a = m_a$, $AG_a = g_a$, denote respectively, the Nagel cevian, the median, and the Gergonne cevian to side BC of a given triangle ABC. Since the Nagel cevian $n_a = AN_a$ is the join of the vertex A and the point of contact N_a of the corresponding excircle with side BC, it follows immediately that $AB + BN_a =$

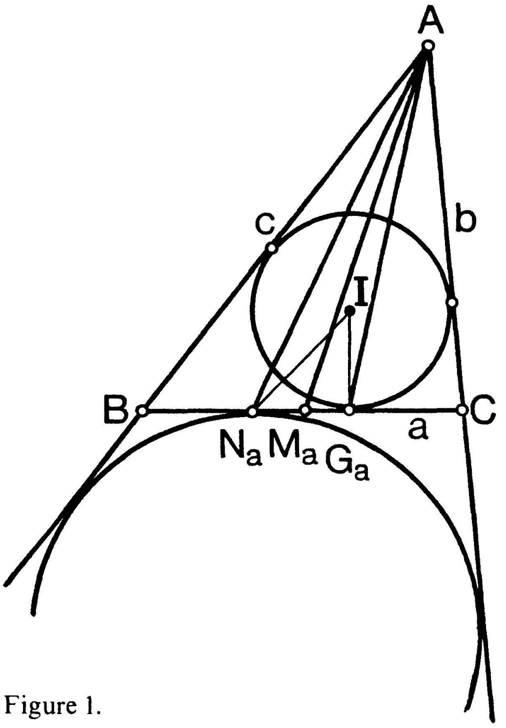

Figure 1.

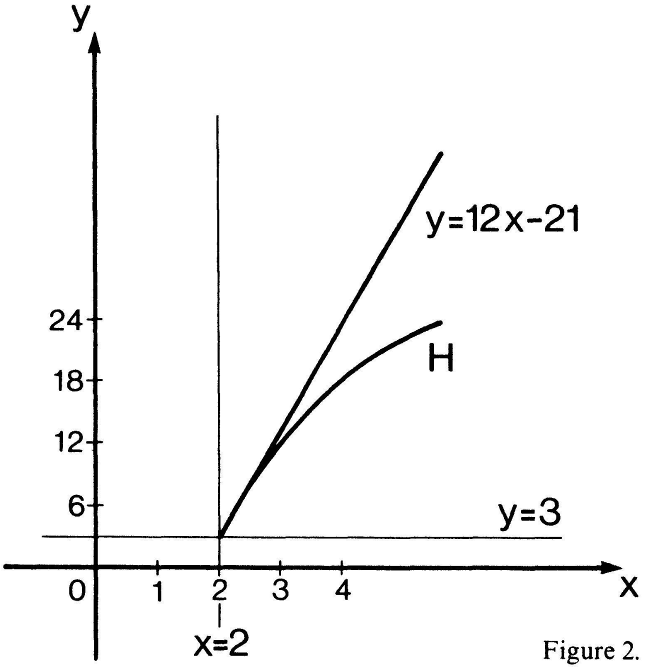

Figure 2.

$N_aC+CA=s$, hence $BN_a=s-c$, where s represents the semiperimeter of ABC. Also, since n_a and g_a are isotomic lines, [33, pp. 158, 184], $BN_a=G_aC=s-c$, hence $N_aG_a=|b-c|$.

Let I denote the incenter, then from figure 1,

$$\sum n_a \leq \sum AI + \sum IN_a. \tag{1}$$

Also, $IN_a=\sqrt{(b-c)^2+r^2}$, with similar expressions for IN_b and IN_c, consequently,

$$\sum IN_a = \sum \sqrt{(a-b)^2+r^2}. \tag{2}$$

Applying the inequality $x+y+z\leq\sqrt{3(x^2+y^2+z^2)}$, [1] p. 11, to (2), we obtain the inequality

$$\sum IN_a \leq \sqrt{6\sum a^2 - 6\sum bc + 9r^2}. \tag{3}$$

To (3), we now apply the inequalities $\sum a^2 \leq 8R^2+4r^2$ and $\sum bc \geq 4r(5R-r)$, where R denotes the circumradius, [1] p. 53, and obtain the inequality

$$\sum IN_a \leq \sqrt{48R^2 - 120Rr + 57r^2}. \tag{4}$$

Next, we put the right hand side of (4) equal to y and make the substitutions $R=x$, $r=1$, this yields the hyperbola

$$H\colon\; 3(4x-5)^2 - y^2 - 18 = 0, \tag{5}$$

the relevant portion of which is sketched in figure 2 above.

To establish an upper bound of the form $H \leq ax+b$, with equality for equilateral triangles which occurs when $x=2$, it is necessary only to determine the equation of the tangent line at the point (2, 3), and this equation is $y=12x-21$: we now have

$$\sum IN_a \leq 12R-21r. \tag{6}$$

If (6) and the known inequality $\sum AI \leq 2R+2r$, [1] p. 103, is applied to (1), we obtain the inequality

$$\sum n_a \leq 14R-19r \tag{7}$$

and this is an upper bound for the sequence as desired. We now have $9r \leq \sum h_a \leq \sum g_a \leq \sum w_a \leq \sum m_a \leq \sum n_a \leq 14R-19r$ with equality throughout when the given triangle is equilateral.

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Aufgaben

Aufgabe 929. Die Folge (a_n) sei definiert durch

$$a_0=2, \quad a_{n+1}=a_n+\frac{3+(-1)^n}{2} \quad (n \geq 0).$$

Man ermittle

$$s=\sum_{n=0}^{\infty} \frac{(-1)^{[(n+1)/2]}}{a_n^2-1} \quad ([\ ]\text{: Ganzteilfunktion}).$$

M. Vowe, Therwil

Lösung: Mit vollständiger Induktion zeigt man, dass $a_{4n}=6n+2$, $a_{4n+1}=6n+4$, $a_{4n+2}=6n+5$, $a_{4n+3}=6n+7$ für alle $n \in \mathbf{N}_0$.

$$s=\sum_{n=0}^{\infty}\left[\frac{1}{(6n+2)^2-1}-\frac{1}{(6n+4)^2-1}-\frac{1}{(6n+5)^2-1}+\frac{1}{(6n+7)^2-1}\right].$$