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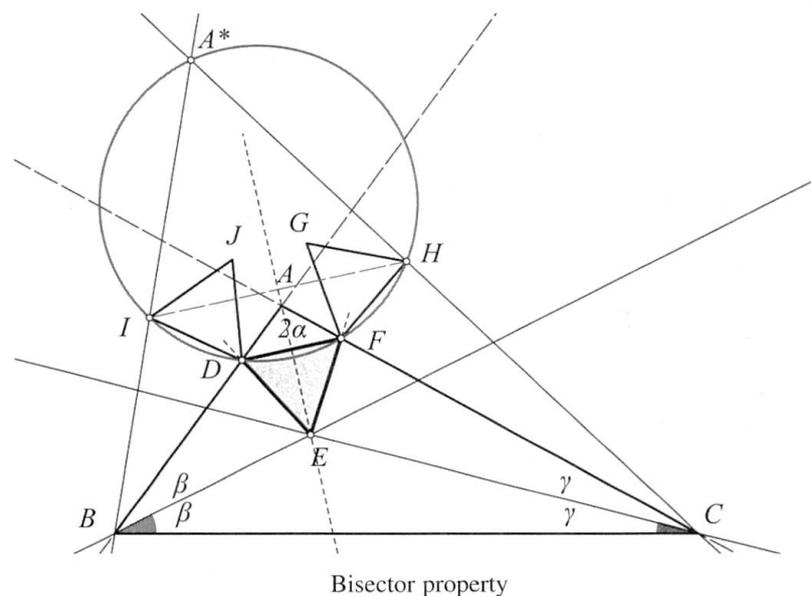
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## Short note    A short proof of Morley's theorem

Paris Pamfilos

Among the dozens of proofs of Morley's Theorem, the proofs of J.M. Child [J.M23] and Naraniengar [HC67, p. 47] are particularly neat. In this paper, we simplify them by stressing the symmetry of the equilateral and the symmetries created by two reflections.<sup>1</sup>



For this, we start with a trivial property of the bisectors of a triangle  $ABC$  with angles  $\{2\alpha, 2\beta, 2\gamma\}$ .

**Lemma.** *From the incenter  $E$  of triangle  $ABC$  and on both sides of  $AE$  draw two lines inclined to it by  $30^\circ$  and intersecting the other sides, respectively at  $D, F$ . Then  $DEF$  is equilateral and  $DF$  is orthogonal to  $AE$ .*

Having that, reflect  $BE$  and  $DEF$  on  $AB$  to obtain  $BI$  and the equilateral  $DIJ$ . Do the same on the other side, i.e., reflect  $CE$  and  $DEF$  on  $CF$  to obtain  $CH$  and the equilateral  $FHG$ . By the symmetry with respect to  $AE$  the quadrilateral  $IDFH$  is an isosceles

<sup>1</sup>I would like to express my gratitude to the referee, for pointing out to me the proof by J.M. Child and making several useful remarks, helping me towards a better and more concise exposition.

trapezium with three equal sides, hence cyclic, and its angle  $\widehat{IHF}$  is easily computed:

$$\begin{aligned}\widehat{IDF} &= \widehat{IDJ} + \widehat{JDF} = 60^\circ + 2(90^\circ - \alpha) = 60^\circ + 180^\circ - 2\alpha = 60^\circ + 2\beta + 2\gamma \Rightarrow \\ \widehat{IHF} &= 180^\circ - \widehat{IDF} = 120^\circ - 2\beta - 2\gamma.\end{aligned}$$

This, essentially, finishes the proof, since the arcs  $(IDF)$  and  $(IDFH)$  are in ratio 2:3 and the angle at the intersection point  $A^*$  of lines  $BI$  and  $CH$  will be then of measure  $\widehat{A^*} = 180^\circ - 3\beta - 3\gamma$ . This shows that  $A^*$  is on the circumcircle  $\kappa$  of  $IDFH$  and  $\{A^*D, A^*F\}$  are the trisectors of  $\widehat{A^*}$ , implying obviously that, “for a triangle  $A^*BC$  with base angles  $\{3\beta, 3\gamma\}$  the adjacent trisectors intersect at the vertices of an equilateral triangle”, as required by the theorem of Morley.

## References

- [HC67] S. Greitzer, H. Coxeter. *Geometry Revisited*. Mathematical Association of America, Washington D.C., 1967.
- [J.M23] J.M. Child. Proof of “Morley's theorem”. *Mathematical Gazette*, 11:171, 1923.

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