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Erdős-Mordell-type inequalities¹

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Zhiqin Lu graduated from the Courant Institute of New York University in 1997. He was a Ritt Assistant Professor at Columbia University before joining the faculty of the University of California at Irvine in 2000. His field of research is differential geometry.

The famous Erdős-Mordell inequality states that, if P is a point in the interior of a triangle ABC whose distances are p, q, r from the vertices of the triangle and x, y, z from its sides, then

$$p + q + r \geq 2(x + y + z).$$

In the paper by Satnoianu [1], some generalizations of the above inequality were given. His proof depends heavily on the geometry of the triangle ABC . In this note, we give a more algebraic proof of the Erdős-Mordell inequality.

Theorem. *Let $p, q, r \geq 0$ and let $\alpha + \beta + \gamma = \pi$. Then we have the inequality*

$$p + q + r \geq 2\sqrt{qr} \cos \alpha + 2\sqrt{rp} \cos \beta + 2\sqrt{pq} \cos \gamma. \quad (1)$$

Proof. We consider the following quadratic function of x :

$$x^2 - 2(\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma)x + q + r - 2\sqrt{qr} \cos \alpha. \quad (2)$$

Then a quarter of the discriminant is

$$\frac{1}{4}\Delta = (\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma)^2 - (q + r - 2\sqrt{qr} \cos \alpha).$$

Since $\alpha + \beta + \gamma = \pi$, we have

$$\cos \alpha = -\cos(\beta + \gamma) = -\cos \beta \cos \gamma + \sin \beta \sin \gamma.$$

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Using the above identity, the discriminant can be simplified as

$$\Delta = -(\sqrt{r} \sin \beta - \sqrt{q} \sin \gamma)^2 \leq 0.$$

Thus the expression (2) is always nonnegative. Letting $x = \sqrt{p}$, we get (1). \square

Corollary. *Let x' , y' , z' be the length of the angle bisectors of $\angle BPC$, $\angle CPA$, and $\angle APB$, respectively. Then we have*

$$p + q + r \geq 2(x' + y' + z').$$

Proof. We have

$$\begin{aligned} x' &= \frac{2qr}{q+r} \cos \gamma \leq \sqrt{qr} \cos \gamma, \\ y' &= \frac{2pr}{p+r} \cos \beta \leq \sqrt{pr} \cos \beta, \\ z' &= \frac{2pq}{p+q} \cos \alpha \leq \sqrt{pq} \cos \alpha. \end{aligned}$$

The corollary follows from the theorem. \square

Remark. Since $x' \geq x$, $y' \geq y$ and $z' \geq z$, the corollary implies the Erdős-Mordell inequality

$$p + q + r \geq 2(x + y + z).$$

References

- [1] Satnoianu, R.: Erdős-Mordell-type inequalities in a triangle. *Amer. Math. Monthly* 110 (2003) 8, 727–729.

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