

**Zeitschrift:** Elemente der Mathematik  
**Herausgeber:** Schweizerische Mathematische Gesellschaft  
**Band:** 63 (2008)

**Artikel:** Erdős-Mordell-type inequalities  
**Autor:** Lu, Zhiqin  
**DOI:** <https://doi.org/10.5169/seals-99059>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 17.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

---

---

## Erdős-Mordell-type inequalities<sup>1</sup>

---

---

Zhiqin Lu

Zhiqin Lu graduated from the Courant Institute of New York University in 1997. He was a Ritt Assistant Professor at Columbia University before joining the faculty of the University of California at Irvine in 2000. His field of research is differential geometry.

The famous Erdős-Mordell inequality states that, if  $P$  is a point in the interior of a triangle  $ABC$  whose distances are  $p, q, r$  from the vertices of the triangle and  $x, y, z$  from its sides, then

$$p + q + r \geq 2(x + y + z).$$

In the paper by Satnoianu [1], some generalizations of the above inequality were given. His proof depends heavily on the geometry of the triangle  $ABC$ . In this note, we give a more algebraic proof of the Erdős-Mordell inequality.

**Theorem.** *Let  $p, q, r \geq 0$  and let  $\alpha + \beta + \gamma = \pi$ . Then we have the inequality*

$$p + q + r \geq 2\sqrt{qr} \cos \alpha + 2\sqrt{rp} \cos \beta + 2\sqrt{pq} \cos \gamma. \quad (1)$$

*Proof.* We consider the following quadratic function of  $x$ :

$$x^2 - 2(\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma)x + q + r - 2\sqrt{qr} \cos \alpha. \quad (2)$$

Then a quarter of the discriminant is

$$\frac{1}{4}\Delta = (\sqrt{r} \cos \beta + \sqrt{q} \cos \gamma)^2 - (q + r - 2\sqrt{qr} \cos \alpha).$$

Since  $\alpha + \beta + \gamma = \pi$ , we have

$$\cos \alpha = -\cos(\beta + \gamma) = -\cos \beta \cos \gamma + \sin \beta \sin \gamma.$$

---

<sup>1</sup>Partially supported by the NSF CAREER award DMS-0347033.

Using the above identity, the discriminant can be simplified as

$$\Delta = -(\sqrt{r} \sin \beta - \sqrt{q} \sin \gamma)^2 \leq 0.$$

Thus the expression (2) is always nonnegative. Letting  $x = \sqrt{p}$ , we get (1).  $\square$

**Corollary.** *Let  $x'$ ,  $y'$ ,  $z'$  be the length of the angle bisectors of  $\angle BPC$ ,  $\angle CPA$ , and  $\angle APB$ , respectively. Then we have*

$$p + q + r \geq 2(x' + y' + z').$$

*Proof.* We have

$$\begin{aligned} x' &= \frac{2qr}{q+r} \cos \gamma \leq \sqrt{qr} \cos \gamma, \\ y' &= \frac{2pr}{p+r} \cos \beta \leq \sqrt{pr} \cos \beta, \\ z' &= \frac{2pq}{p+q} \cos \alpha \leq \sqrt{pq} \cos \alpha. \end{aligned}$$

The corollary follows from the theorem.  $\square$

**Remark.** Since  $x' \geq x$ ,  $y' \geq y$  and  $z' \geq z$ , the corollary implies the Erdős-Mordell inequality

$$p + q + r \geq 2(x + y + z).$$

## References

- [1] Satnoianu, R.: Erdős-Mordell-type inequalities in a triangle. *Amer. Math. Monthly* 110 (2003) 8, 727–729.

Zhiqin Lu  
 Department of Mathematics  
 University of California Irvine  
 Irvine, CA 92697, USA  
 e-mail: zlu@math.uci.edu