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**Autor:** Blatter, Christian / Wanner, Gerhard  
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## Note from the Editorial Board

### Note on rectangles with vertices on prescribed circles

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Christian Blatter and Gerhard Wanner

Inspired by [2], Ionascu and Stanica considered the following problem: The four subsequent vertices of a rectangle  $R$  in the plane are at distances  $r_1, r_2, r_3, r_4$  from the origin. Given these data, what can be said about the area  $A$  of  $R$ ? In a recent paper [1] they proved

**Theorem 1.** (a) For given  $r_i \geq 0$ , rectangles  $R$  of the described kind exist iff

$$r_1^2 + r_3^2 = r_2^2 + r_4^2. \quad (1)$$

(b) The areas of these rectangles lie between the bounds

$$A_{\min} = |r_2r_4 - r_1r_3|, \quad A_{\max} = r_2r_4 + r_1r_3.$$

The authors call their problem “unusual because of its surprisingly simple answer in spite of our rather laborious solution” (which takes 10 pages). In this note we shall prove Theorem 1 in a simple way, making use of no more than the Pythagorean theorem and the formula for the derivative of a product.

After cyclic reordering, and neglecting special or degenerate cases, we may assume that  $0 < r_1 < r_i$  ( $2 \leq i \leq 4$ ). We may also assume that the sides of the rectangle are parallel to the axes and that the point  $P_1 = (x, y)$  on the circle of radius  $r_1$  is the lower lefthand vertex of  $R$  (Fig. 1). From  $P_1$  we draw a horizontal to the right and obtain the vertex  $P_2 = (\bar{x}, y)$  on the circle of radius  $r_2$ ; in a similar way, going from  $P_1$  vertically upwards, we obtain the vertex  $P_4 = (x, \bar{y})$  on the circle of radius  $r_4$ . So far we have

$$\begin{aligned} x^2 + y^2 &= r_1^2, \\ \bar{x}^2 + y^2 &= r_2^2, \quad \bar{x} > 0, \\ x^2 + \bar{y}^2 &= r_4^2, \quad \bar{y} > 0. \end{aligned} \quad (2)$$

The upper righthand vertex of  $R$  is  $P_3 = (\bar{x}, \bar{y})$ , and it lies on the circle of radius  $r_3$  iff

$$r_3^2 = \bar{x}^2 + \bar{y}^2 = r_2^2 - y^2 + r_4^2 - x^2 = r_2^2 + r_4^2 - r_1^2,$$

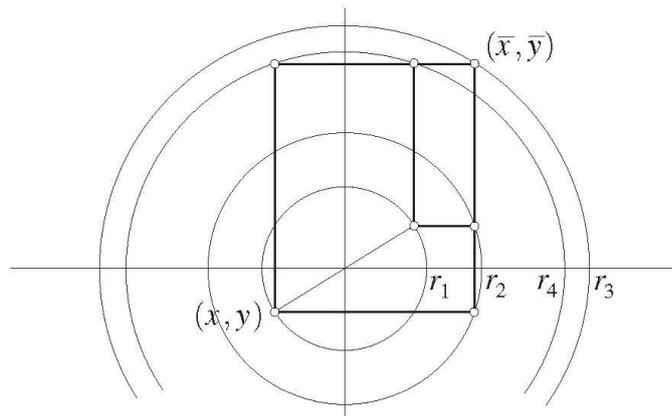


Figure 1

i.e., iff (1) holds. Since the above construction is always possible, part (a) of the theorem follows.

**Remark.** The midpoint of  $R$  bisects both diagonals. It follows that (1) can be seen as an instance of the so-called Pappus-Fagnano-Legendre formula  $a^2 + b^2 = (c^2 + z^2)/2$  where  $z/2$  is the length of the median through  $C$  of a triangle with sides  $a, b, c$ .

For our extremal problem we have to consider admissible variations  $dx, dy, d\bar{x}, d\bar{y}$  of the quantities  $x, y, \bar{x}, \bar{y}$ . To this end we differentiate the equations (2) and obtain

$$x dx + y dy = 0, \quad \bar{x} d\bar{x} + y dy = 0, \quad x dx + \bar{y} d\bar{y} = 0 \quad (3)$$

from which we easily infer

$$d\bar{x} = \frac{x}{\bar{x}} dx, \quad d\bar{y} = \frac{y}{\bar{y}} dy. \quad (4)$$

The area of  $R$  is  $A = (\bar{x} - x)(\bar{y} - y)$ ; it is maximal or minimal iff

$$dA = (d\bar{x} - dx)(\bar{y} - y) + (\bar{x} - x)(d\bar{y} - dy) = 0.$$

Using (4) we get

$$dA = \frac{x - \bar{x}}{\bar{x}} (\bar{y} - y) dx + (\bar{x} - x) \frac{y - \bar{y}}{\bar{y}} dy = -\frac{A}{\bar{x}\bar{y}} (\bar{y} dx + \bar{x} dy).$$

So the condition  $dA = 0$  reduces to  $\bar{y} dx + \bar{x} dy = 0$ . Combined with the first equation (3) this shows that in the stationary situation we necessarily have

$$\frac{y}{x} = \frac{\bar{x}}{\bar{y}}. \quad (5)$$

In order to determine the maximal and minimal areas of the rectangle we argue as follows: From (5) and (2) we get

$$\frac{\bar{x}^2}{\bar{y}^2} = \frac{y^2}{x^2} = \frac{r_2^2 - \bar{x}^2}{r_4^2 - \bar{y}^2}, \quad \text{whence} \quad \bar{x}^2(r_4^2 - \bar{y}^2) = \bar{y}^2(r_2^2 - \bar{x}^2),$$

and after cancelling terms we see that

$$\frac{\bar{y}}{\bar{x}} = \frac{r_4}{r_2}, \quad \frac{y}{x} = \frac{r_2}{r_4},$$

the latter using (5) again.

We now know the ratios of these coordinates as well as their Pythagorean sums, whence they must be

$$x = \pm \frac{r_4}{\sqrt{r_2^2 + r_4^2}} r_1, \quad y = \pm \frac{r_2}{\sqrt{r_2^2 + r_4^2}} r_1, \quad \bar{x} = \frac{r_2}{\sqrt{r_2^2 + r_4^2}} r_3, \quad \bar{y} = \frac{r_4}{\sqrt{r_2^2 + r_4^2}} r_3.$$

Inserting these values into the formula for  $A$  we obtain, using (1):

$$A = \frac{(r_2 r_3 \mp r_4 r_1)(r_4 r_3 \mp r_2 r_1)}{r_2^2 + r_4^2} = \frac{r_1^2 + r_3^2}{r_2^2 + r_4^2} r_2 r_4 \mp r_1 r_3 = r_2 r_4 \mp r_1 r_3.$$

This leads to

$$A_{\min} = r_2 r_4 - r_1 r_3, \quad A_{\max} = r_2 r_4 + r_1 r_3,$$

as stated. It is easily seen that for the maximum the origin lies in the interior of  $R$  whereas for the minimum it is in the exterior.

## References

- [1] Ionascu, E.J.; Stanica, P.: Extremal values for the area of rectangles with vertices on concentric circles. *Elem. Math.* 62 (2007), 30–39.
- [2] Zahlreich Problems Group: Problem 11057. *Amer. Math. Monthly* 111 (2004), 64. Solution: *ibid.* 113 (2006), 82.

Christian Blatter  
 Albertus-Walder-Weg 16  
 CH–8606 Greifensee, Switzerland  
 e-mail: christian.blatter@math.ethz.ch

Gerhard Wanner  
 Université/Mathématiques  
 Case postale 64  
 CH–1211 Genève 4, Switzerland  
 e-mail: Gerhard.Wanner@math.unige.ch