

**Zeitschrift:** Elemente der Mathematik  
**Herausgeber:** Schweizerische Mathematische Gesellschaft  
**Band:** 62 (2007)

**Artikel:** A short proof of Morley' theorem  
**Autor:** Hashimoto, Yoshitake  
**DOI:** <https://doi.org/10.5169/seals-98918>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 17.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

---

---

## A short proof of Morley's theorem

---

---

Yoshitake Hashimoto

Yoshitake Hashimoto received his D.Sc. from University of Tokyo in 1990. He then had a postdoctoral position at University of Tokyo. Since 1994 he has a position at Osaka City University, where he is now associate professor in the Department of Mathematics. His main fields of research are topology and differential geometry.

We present a proof of the following:

**Morley's theorem** (1899) *In any triangle, the three points of intersection of the adjacent angle trisectors form an equilateral triangle.*

*Proof.* Let  $\alpha, \beta, \gamma$  be arbitrary positive angles with  $\alpha + \beta + \gamma = 60^\circ$ . For any angle  $\eta$  we put  $\eta' := \eta + 60^\circ$ .

Let  $\triangle DEF$  be an equilateral triangle, and  $A$  [resp.  $B, C$ ] be the point lying opposite to  $D$  [resp.  $E, F$ ] with respect to  $EF$  [resp.  $FD, DE$ ] and satisfying  $\angle AFE = \beta'$ ,  $\angle AEF = \gamma'$  [resp.  $\angle BDF = \gamma'$ ,  $\angle BFD = \alpha'$ ;  $\angle CED = \alpha'$ ,  $\angle CDE = \beta'$ ]. Then  $\angle EAF = 180^\circ - (\beta' + \gamma') = \alpha$ , and similarly  $\angle FBD = \beta$ ,  $\angle DCE = \gamma$ . By symmetry it is enough to show that  $\angle BAF = \alpha$  and  $\angle ABF = \beta$  as well.

The perpendiculars from  $F$  to  $AE$  and  $BD$  have the same length  $s$ . If the perpendicular from  $F$  to  $AB$  has length  $h < s$ , then  $\angle BAF < \alpha$  and  $\angle ABF < \beta$ . If, on the other hand,  $h > s$ , then  $\angle BAF > \alpha$  and  $\angle ABF > \beta$ . Since

$$\angle BAF + \angle ABF = \alpha' + \beta' + 60^\circ - 180^\circ = \alpha + \beta,$$

we see that necessarily  $h = s$  and  $\angle BAF = \alpha$ ,  $\angle ABF = \beta$ . □

Yoshitake Hashimoto  
Department of Mathematics  
Graduate School of Science  
Osaka City University  
3-3-138, Sugimoto  
Sumiyoshi-ku  
Osaka, 558-8585 Japan  
e-mail hashimot@sci.osaka-cu.ac.jp