Magic and Mathematics at the Court of Rudolph II

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Charles B. Thomas was born near London in 1938, and received his university education in Cambridge and Heidelberg. He has held positions in various universities in Europe and the USA. At present he is Cayley Lecturer in the Department of Mathematics at the University of Cambridge. His mathematical interests include cohomological methods in finite group theory and the interplay between algebraic topology and differential geometry. His interest in history is long-standing; he claims at times to envy the professional historian! One of his greatest moments of satisfaction was overhearing one of his children being asked for his nationality in California and replying "European".

The government I cast upon my brother,
And to my state grew stranger, being transported
And rapt in secret studies.
<div align="right">The Tempest, I.2.74–76.</div>

Die Geistesgeschichte verbindet mit Recht den Beginn der modernen Naturwissenschaft an der Wende zum 17. Jahrhundert mit den Namen von Kepler und Galilei. Neben dem (angeblichen) Ausspruch Galileis "Eppur si muove!" bilden die drei Keplerschen Gesetze der Planetenbewegung dafür ein bekanntes und gern zitiertes Beweisstück. Die herausragende Leistung Keplers verdankt ihr Zustandekommen einer einzigartigen Konstellation von günstigen Umständen am Hofe Rudolf II in Prag. Diese lassen sich allerdings erst richtig würdigen, wenn man sie fächerübergreifend betrachtet. Charles Thomas versucht in seinem Beitrag den dafür notwendigen weiten Bogen zu spannen: Nicht nur von Astronomie, Physik und Mathematik ist hier die Rede, sondern auch von Shakespeare's *Der Sturm*, vom *Corpus Hermeticum* und von den Anfängen der wissenschaftlichen Medizin. Klar wird auch, dass nicht allein die wissenschaftliche Neugier, die Harmonie der Welt zu ergründen, die epochemachenden Erkenntnisse über die Planetenbewegung ermöglicht haben. Am Hofe Rudolf II spielten auch handfeste praktische Gründe eine Rolle. Dazu gehörte insbesondere der Wunsch, genauere Mondtabellen zu erarbeiten, um der eigenen, d.h. spanischen Flotte einen Navigationsvorteil zu verschaffen. — Vielleicht kann dieser sehr weit gespannte Beitrag von C.B. Thomas Anstoss sein, das Thema in der Schule fächerübergreifend darzustellen. Geschichte, Chemie, Physik, Mathematik, Geographie, Latein und Englisch hätten alle etwas Wesentliches zum Verständnis der Zusammenhänge beizutragen. *ust*

The evidence suggests that Shakespeare's last play, from which the lines above are taken, was first performed in 1611. At least some among the audience on that occasion must have noticed the parallel between the fictional events in Milan and the very real events which had recently taken place in central Europe. Here the Emperor Rudolph II had been deprived of the Kingdom of Bohemia by his brother Matthias, retaining little more than his imperial title, and was struggling to retain even this when he died in January 1612. Indeed when *The Tempest* was revived in 1613 on the occasion of the marriage of Elizabeth Stuart to Friedrich V of the Kurpfalz, the "politically aware" could not have helped but recall Rudolph's swing towards the Protestant Union in the last months of his life, identifying Rudolph with the magician Prospero and Elizabeth with his daughter Miranda. Who was this strange Emperor-Magus, who conceivably was being celebrated by Shakespeare in this way, and why did he make such an impact on his contemporaries?

Rudolph was a great nephew of the Emperor Charles V, who had succeeded to the imperial title in 1576, and more importantly to the direct rule of the "lands of the Bohemian crown". He established his capital in Prague, a cosmopolitan city of 50,000 inhabitants of widely varying religious beliefs. The politics of his reign do not concern us directly, except to note that he shared his great uncle's "universalist" conception of the imperial office. This is illustrated by the attempts early in his reign to mediate in the Netherlands conflict, before the battle and confessional lines hardened with the renunciation of Philip II's sovereignty by the United Provinces in 1581. What does concern us however is the Emperor's patronage of late sixteenth century science — a heady mixture of sympathetic magic, search for religious concord and genuine technological innovation. A systematic foundation for these studies was provided by the *Corpus Hermeticum*, a collection of manuscripts from hellenised Egypt, compiled between 100 and 300 AD, but purporting to contain a system of ancient wisdom associated with Hermes Trismegistos, imagined to be a contemporary and perhaps teacher of Moses.

The interests of Rudolph's circle can be seen from the list of contents of the library of the President of the Hofkammer, Ferdinand Hofmann von Grunpichl und Strechau, known to be acquainted with and well-disposed towards Kepler. The library ·can be assumed to be typical of many, and was preserved intact until recently at Nikolsburg. Rudolph's own library and collections seem to have been largely broken up during the Thirty Years War. It contained a wide theological collection, books on geography, travel and antiquities, medicine and natural science. There were texts in Czech, Hebrew and Arabic, and most significantly an edition of the *Corpus Hermeticum* (published in Cracow) and the *Magna Alchemia* of Leonhard Thurneysser.

As a mathematician one is struck by the fact that the Hermetic texts, or at least their final version, are contemporaneous with the last flowering of Alexandrian mathematics (both pure and applied). For example the Hermetic writers are much concerned with infusing the presence of the Egyptian gods in their statues. Here perhaps we have an echo of the use of steam power, and other mechanical means, to operate temple doors and to make the statue of a god move in such a way as to indicate acceptance of the suppliant's offering.[1] But tricks of this kind apart the writers of the Hermetic texts see the physical world as a reflection of ideas in the divine mind. Since the human mind has something of the divine in it, by understanding the workings of his own mind (essentially by internal reflection)

the Magus could understand and even influence the structure of the material world. This teaching, along with much else, had been rediscovered in renaissance Florence, and strongly influenced thinkers throughout Europe up to the beginning of the Thirty Years War. For example it pervades the natural magic of Marsilio Ficino[2], who taught that certain combinations of colours, animals, plants, scents and sounds could influence human behaviour. Later that extraordinary man Giordano Bruno (who spent part of his wandering life in Prague) clearly believed that he could use magic to augment his own undoubted personal charm to influence and even control the action of princes. As in Alexandria these teachings intrigue the creative mathematicians — an early example is Girolamo Cardano, famous for at least contributing to the algorithm for solving cubic equations, but also a highly regarded physician and the author of books with a "hermetic" flavour such as *De Subtilitate Rerum*.

Confident in the Emperor's patronage such "wizards, alchemists, kabbalists and the like" (to quote a hostile report to Matthias and the other Archdukes in 1606) journeyed to Prague, either to visit (like Bruno), or to settle (like the physician Michael Maier, and crucially for the development of science, Johannes Kepler). And here we meet one vital difference between Alexandria and Prague — the artisan is not held at a distance. The magician may start by trying to understand the world by internal reflection, but he is now ready to test his ideas against experiment, and is always looking for better methods for doing this. Thus the Italian Mordante designs an improved compass (much to his compatriot Bruno's scorn!), the Swiss Bürgi[3] constructs improved clocks as an aid to astronomical observation, and as is well-known, Kepler comes to Prague to exploit the planetary data of the Dane Tycho de Brahe. In part this new concern with accuracy, this desire to make the theory fit the facts, grows out of the sixteenth century's navigational needs. We remember Kepler for the three laws of planetary motion; his contemporaries were as interested in the compendium of astronomical information contained in the *Rudolphine Tables*. This mixture of the weirdly theoretical and the down to earth practical is well illustrated by the career of the English magician/scientist John Dee, yet another visitor to Rudolphine Prague (1583–1589)[4]. Dee's interests were so encyclopaedic, that he still needs an adequate biographer; at one extreme he sought to communicate with angels (aided by Edward Kelley, who in his turn claimed to Rudolph to be able to transmute base metals into gold), at the other he wrote an introduction to the first English edition of Euclid's Elements, emphasising the practical applications of geometry. On an earlier journey through central Europe Dee had attended the coronation in Pressburg/Bratislava of Rudolph's father Maximilian II as King of Hungary, and had dedicated a book describing his *Monas Hieroglyphica* (see below) to him. In the Hermetic system this must be regarded as a "universal talisman', and since I hope to show that this notion is central to Kepler's way of thinking, some background is necessary.

Among the various Hermetic texts is the *Picatrix*, a manual of practical magic, which first became known in the West in the form of an Arabic translation from the original Greek. A translation into Spanish was made for King Alfonso the Wise of Castile in the thirteenth century, but this seems to have been lost. But in one version or another this work was known to Marsilio Ficino, who based his own cautious magical system on it. I have already referred to his belief that one can construct "talismans" to influence human

Fig. 1 John Dee, *Monas hieroglyphica*

behaviour — from this it is a short step to the attempted construction of a universal such object, through the contemplation of which Man can understand the workings of the universe itself. Ficino is unclear in his description of such a *Figura Mundi*, although he seems to think that it will take the form of an elaborate jewel. Thus it should be made of brass combined with gold and silver, its colours should be green (for Venus), gold (for the Sun) and blue (for Jupiter). Its construction should be begun when the sun enters Aries, suspended over the sabbath, and completed on a Friday.[5] What should be emphasised here is that Ficino is attempting to give a model for the cosmos, one arrived at by internal reflection rather than external observation, but scientific in the sense that its maker is trying to understand the world through a model of it. Note also the emphasis given to the sun, as the physical embodiment of the creator's mind.

Bruno and Dee continued the search for the *Figura Mundi*, the latter's candidate (already mentioned) is illustrated in Figure 1. Although Dee describes it as a "mathematical"
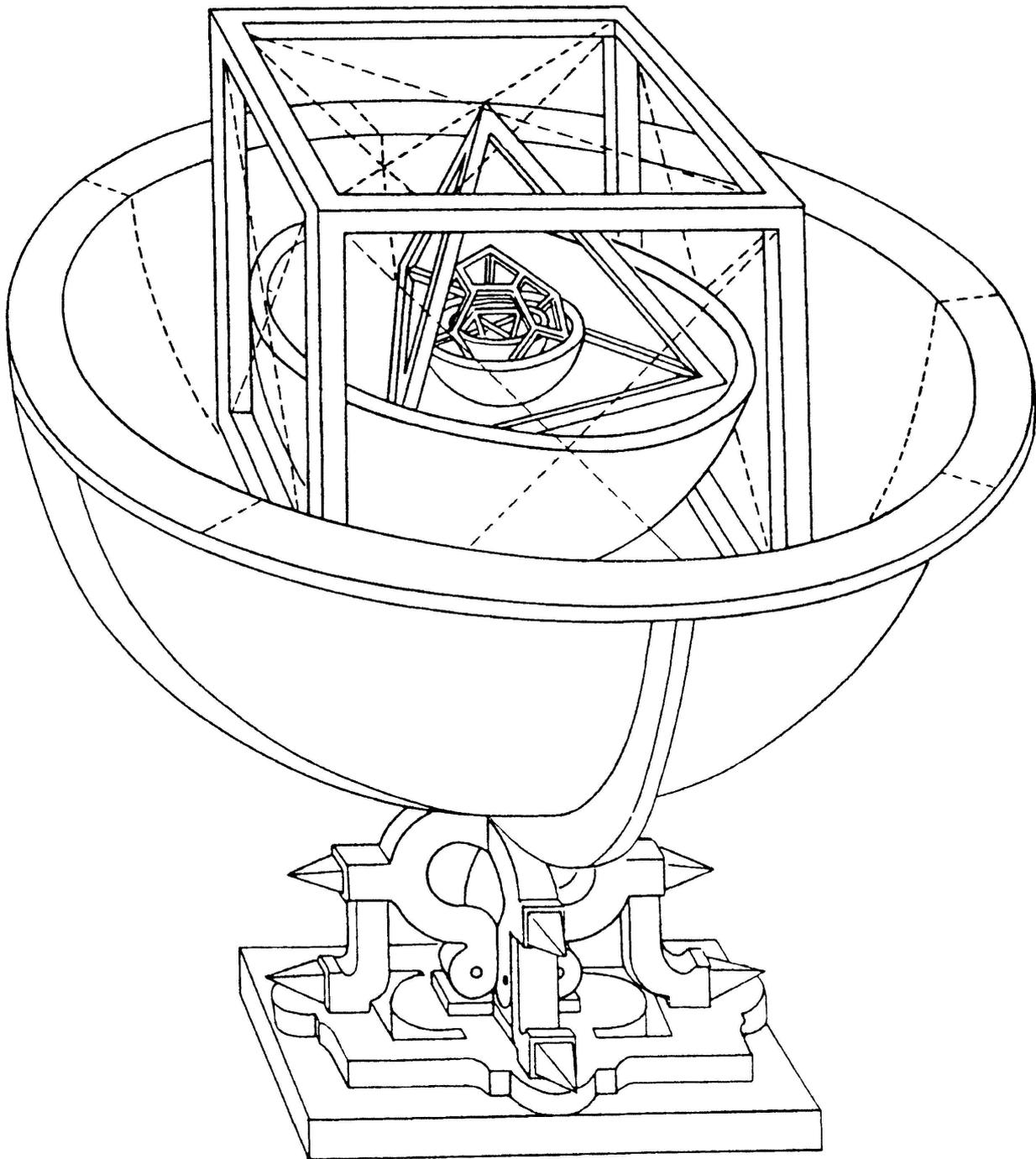
Fig. 2 Kepler's nested polyhedrons and planetary spheres (*Mysterium cosmographicum*), 1596

figure, it could clearly be cast as an enammeled jewel along the lines suggested by Ficino, and built as it is out of the traditional planetary signs, it can be regarded as a model for the solar system.

As such it is an ancestor of Kepler's far more complex model of the planetary orbits, elaborated during his first years at Graz, and published as the solution to the *Mysterium Cosmographicum* in 1597. We shall return to Kepler's model as the starting point for his genuinely mathematical investigations in a moment, but first I think it is important to recognise that the model, although immensely fruitful in a way undreamt of by earlier

magi, stands at the end of a tradition. Let us look at its construction, see Figure 2, more carefully. Kepler starts with the five regular solids, and nests them in such a way that the orbits of the known planets fit between them. Thus we have the sequence:

Saturn-CUBE-Jupiter-TETRAHEDRON-Mars-DODECAHEDRON-Earth-
ICOSAHEDRON-Venus-OCTAHEDRON-Mercury.

And I think we can be sure that this model would have appealed to Ficino as the realisation of the programme which he found sketched in the *Picatrix*. Indeed, with the earth replacing the sun in position three, he might have argued as follows: green, gold and blue, corresponding to planets Venus, Earth and Jupiter, are to be thought of as the domain of the white magician. What more natural than to associate the dark planets, Saturn and Mars, with their mathematically dual solids? (Read inwards for this identification, and note the ambiguous stature of Jupiter, associated with the self-dual tetrahedron.) Now we shall never know exactly what motivated Kepler in the construction of his model, but a hint that he regarded it as a "universal talisman" in the Hermetic sense is provided by the project (never carried out) of realising it as a table ornament-cum-drinking fountain for the ducal court in Wurttemberg. Furthermore, even after the announcement of the first two planetary laws, which in a scientific sense render the model obsolete, Kepler continues to be fascinated by it. Thus in the *Harmonice Mundi* (1619), in which by trial and error he comes upon the third law, the law which will abolish the distinction between the macro- and microcosm, he can write: geometry ... is coeternal with the mind of God ...; geometry provided God with a model for the Creation, and was implanted into Man, together with God's own likeness — and not merely conveyed to his mind through the eyes. Here Hermes Trismegistos himself could be speaking.

But of course Kepler does not stop with his magical model of the solar system, but moves to Prague, first in the hope of using Tycho de Brahe's data to confirm his original ideas, and then to construct a totally new kind of model for planetary behaviour. How and why does he do this?[6] Right from the start of his investigations he places the sun rather than the centre of the earth's orbit at the centre of the solar system, and thinks of the planets as being driven in their orbits by a force emanating from the sun. Both these assumptions are natural for someone steeped in sixteenth century hermeticism. After all Copernicus himself had come close to a direct quotation from the *Asclepius*, another text from the hermetic canon: *in medio vero omnium residet sol ...*

Kepler began his investigations still looking for a circular orbit. He tried de Brahe's data for the planet Mars, used the result that a circle is determined by three points on its circumference, predicted the planet's position at a fourth point, and found that this gives a small but non-negligeable conflict with observation. Motivated by the potential use of a "good', i.e. predictive, model as an aid in calculating entries in the Rudolphine tables, the *Imperial Mathematicus* could not accept this discrepency. Hence, at least for the time being, he had to abandon his *Figura Mundi* of 1597, and look for a mathematical model which fitted all the observed facts. And it is at this moment that the Renaissance Magus becomes recognisable as a scientist. By 1602 uniform motion in a circular orbit has been replaced by the second law (K2) that a line joining the planet to the sun sweeps out
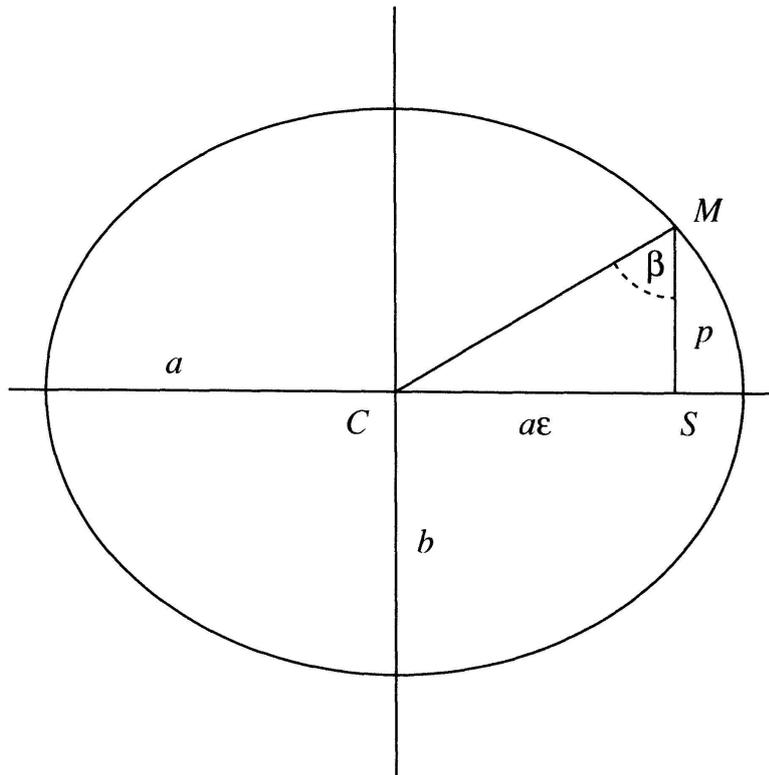
Fig. 3

equal areas in equal times, using an argument showing that the author senses the need to develop what we would call the integral calculus. Then came the real struggle: what is the mathematical expression for the "oval" traced by the moving planet with such regularity? The clue was provided by the number 0.00429, which occurs in two places in Kepler's calculations: (i) the difference between the major and the minor semi-axes of the future ellipse, the former beeing normalised to 1, and (ii) the reciprocal of the cosine of the angle β between the semi-latus rectum (MS) and the radial line $(MC$ — see Figure 3). Elementary coordinate geometry shows that if we assume that M is moving on an elliptical orbit with unit major semi-axis and eccentricity ϵ, then the difference (i) and $1/\cos\beta - 1$ are both approximated by $(1/2)\epsilon^2$. Kepler himself, predating Descartes, could not use this argument, and had to grope his way using little more than elementary trigonometry to his identification of the "oval" with an ellipse having the sun at one focus (K1). The third law will be considered below, but all three must be considered the intellectual triumph of Rudolphine Prague. The formulation of the problem needed the magical tradition of the sixteenth century, coupled with the heliocentric hypothesis. But its solution depended on accessible accurate data — without Rudolph's invitation to Tycho de Brahe to settle at the Bohemian court, an invitation motivated by his Spanish cousins' need for more accurate navigational tables, Kepler's work would have been inconceivable.

The publication of the *Astronomia Nova* in Frederick V's capital Heidelberg in 1609 marks a sea change in scientific thought. The magical constructions of the sixteenth century — however important they may have been in providing motivation — are now obsolete. Indeed magic was now an impediment to further progress. One can go further
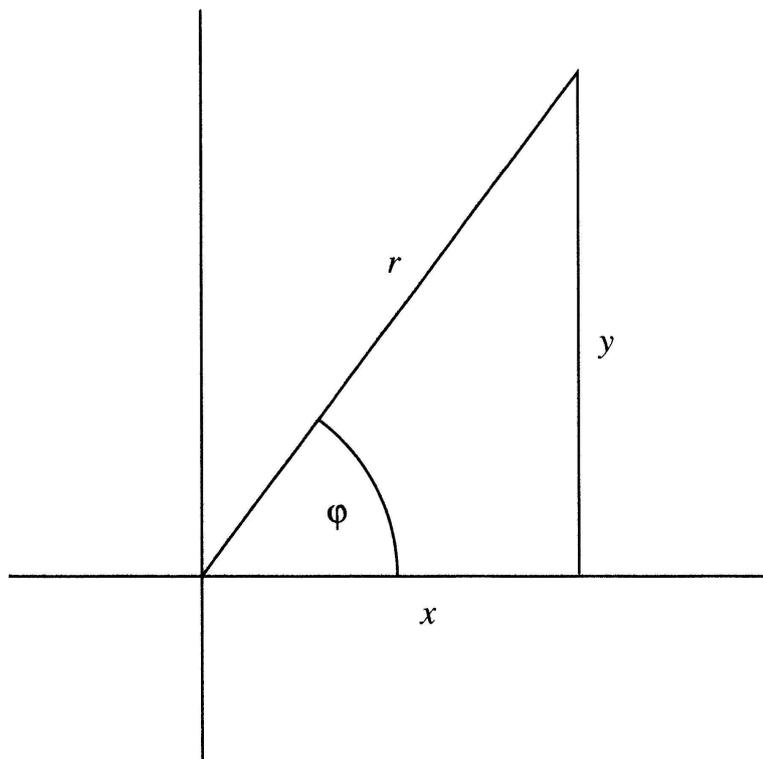
Fig. 4

and argue that Kepler had taken the original programme, and stood it on its head. The magus seeks to understand the universe internally, and to this end uses number in an attempt to understand the creative Mind. The scientist on the other hand is no less interested in constructing a "universal talisman', but it must be testable against observation. His use of mathematics is to measure the conformability of the model with external nature. Kepler saw the distinction clearly in his dispute with the Hermeticist Robert Fludd, whom he accused of using numerical and geometric arguments to set up an analogy between the micro- and macrocosm, rather than using them to study the heavens in themselves.

Kepler may well have had some inkling that his first two laws quantified the force emanating from the sun and that the his third law might show that celestial and terrestrial bodies move according to the same physical laws, but he lacked the mathematical tools to prove any such claim. This had to wait for Newton, whose argument may be outlined as follows[7].

In Figure 4 we have the usual relation between Cartesian and polar coordinates:

$$x = r \cos\phi, \quad y = r \sin\phi \ .$$

Differentiating twice according to time and collecting terms gives

$$J^2 = \dot{x}^2 + \dot{y}^2 = (\dot{r} - r\dot{\phi}^2)^2 + (2\dot{r}\dot{\phi} + r\ddot{\phi})^2 \ .$$

Assume that all motion takes place in the (r, ϕ)-plane. Then (K2) is equivalent to the statement that the acceleration of the planet M is directed towards the sun S. Mathematically (K2) states that

$$\frac{1}{2} r^2 \dot{\phi} = c, \quad c \text{ constant.}$$

If this holds, substituting $2\dot{r}\dot{\phi}+r\ddot{\phi}=0$ into the expressions for $\ddot{x}$ and $\ddot{y}$ gives $\ddot{y}/\ddot{x}=\tan\phi$, which implies central acceleration.

Conversely an easy manipulation shows that the relation

$$(\ddot{r}-r\dot{\phi}^2)\sin\phi+(2\dot{r}\dot{\phi}+r\ddot{\phi})\cos\phi=\tan\phi\left((\ddot{r}-r\dot{\phi}^2)\cos\phi-(2\dot{r}\dot{\phi}+r\ddot{\phi})\sin\phi\right)$$

can only hold if $2\dot{r}\dot{\phi}+r\ddot{\phi}=0$.

The next step is to show that (K1) and (K2) together imply that the magnitude J of the acceleration is proportional to $1/r^2$. For this we start with the equation of the ellipse

$$r=\frac{p}{1+\epsilon\cos\phi}$$

with $a^2=b^2+a^2\epsilon^2$, where p denotes the semi-latus rectum, $p=a(1-\epsilon^2)$.

The law (K2) implies that $J^2=(\ddot{r}-r\dot{\phi}^2)^2$, so that $J=\ddot{r}-(4c^2/r^3)$ with c as above. Write the equation of the ellipse in (K1) as $\epsilon\cos\phi/p=1/r-1/p$, differentiate twice and use the relation $r^2\dot{\phi}=2c$. We obtain

$$\ddot{r}=\frac{4\epsilon\,c^2}{p}\frac{}{r^2}\cos\phi.$$

The equation $J=\ddot{r}-r\dot{\phi}^2$ plus the equation of the ellipse finally give

$$J=\frac{4c^2}{p}\frac{1}{r^2}.$$

Conversely the inverse square law states that

$$\ddot{r}-r\dot{\phi}^2=-\frac{\gamma}{r^2}.$$

As above planar motion plus central acceleration imply that $r^2\dot{\phi}=2c$, so that

$$\ddot{r}=\frac{4c^2}{r^3}-\frac{\gamma}{r^2}.$$

Replacing a differential equation for r in terms of t by one for r in terms of ϕ means applying the chain rule to determine $\ddot{r}$ in terms of dervitatives according to ϕ. Doing this and again using (K2) (twice) in the form $\dot{\phi}^2=2c/r^2$ we obtain

$$\ddot{r}=\frac{-4c^2}{r^2}\left(-\frac{d^2r}{d\phi^2}\cdot\frac{1}{r^2}+\frac{dr}{d\phi}\cdot\frac{2}{r^3}\right)=\frac{4c^2}{r^2}\cdot\frac{d^2(1/r)}{d\phi^2}.$$

Substituting in the original expression for $\ddot{r}$ and rearranging terms gives

$$\frac{d^2(1/r)}{d\phi^2}=-\frac{1}{r}+\frac{\gamma}{4c^2}.$$

If we take the cosine solution of this second order equation, i.e. possibly introduce a change of phase, we obtain

$$\frac{1}{r} = \frac{\gamma}{4c^2} + \delta \cos(\phi - \alpha),$$

the required ellipse. The argument so far has shown that an inverse square law for planetary acceleration is equivalent to (K1 and K2), and that we have

$$\gamma = 4 \left(\frac{dA}{dt}\right)^2 \frac{1}{p}$$

where A equals the area swept out by the planet M. What is the significance of the third law (K3), stating that the square of a planet's period is proportional to the cube of the major axis of its orbit? Kepler states this in a late work, the *Harmonice Mundi*, in which he returns to his original hermetic motivation. His aim was to relate the class of constructible polygons with musical concord/discord, and to show that the planetary orbits are arranged as they are so as to reflect perfect harmony. (In passing and with another imaginative leap he comes close to suggesting that there can be only finitely many Fermat prime numbers.) From all this the third law emerges as a result of trial and error, the true significance of which was again shown by Newton. As above write $c = dA/dt$, so that, if $2b$ denotes the minor axis of the orbit,

$$ab\pi = cT.$$

From our previous arithmetic it follows that

$$\gamma = \frac{4a^4(1 - \epsilon^2)\pi^2}{a(1 - \epsilon^2)T^2} = 4\pi^2 \frac{a^3}{T^2},$$

showing that γ is independent of the planet M. This independence is confirmed by an examination of the Jovan satellite system, and also by the three body system consisting of the earth, moon and an arbitrary falling body. Hence the moon and the falling body obey the same physical laws.

It is not only in this abolition of the distinction between the microcosm and the macrocosm that Newton refers back to work done around 1600 in Prague. In a remarkable reference to the Hermetic tradition he claims to be doing no more than rediscover truths known to the ancients. And to think of Newton as a mathematician and a physicist is to take him out of the context of his own time. It has been said that he was as much the last of the magicians as the first of the scientists, and to see him whole we must take account of his theological and alchemical writings. Let us first consider the latter, where, in contrast to astronomy, progress had been impeded by fascination with the search for the philospher's stone and the means to purify "base" metals into gold. But among the Prague alchemists one does find the same mixture of the mystical and the practical, as we have already noted in Kepler. Thurneysser's *Magna Alchemia* contains much practical information on minerals and the extraction of metals from ores, but whereas Kepler's

mathematical tools were just strong enough to provide a mathematical substitute for the *Mysterium Cosmographicum*, the same could not be true for alchemy. Metals differ because of their atomic structure, and the base-to-pure schemes from the sixteenth century were too weak a model for even the feeblest approximation to the periodic table. But there is evidence that an alternative line of alchemical research was being pursued — namely medicine. Two case histories are those of Jan Jesensky and Michael Maier. The former was well-known for his anatomical studies and dissections, and was among those executed after the suppression of the Bohemian revolt in 1620. The latter was Rudolph's personal physician, later spent time in Heidelberg, and vanished in Magdeburg in 1632, at the time of its sack. Both men were thus direct victims of the Thirty Years War — however it is also possible that a whole potential chemical tradition died with them. Maier's published Heidelberg work has been described by Frances Yates as "Rosicrucian" in that it sees alchemy as a means for achieving spiritual renewal and religious concord.[8] But here may be hidden the dream of a disappointed man for combining medical and alchemical research. This would only come much later, and in a very mechanistic form — but given a chemical Tycho de Brahe with a passion for collecting exact data, and a continuation of the Rudolphine tradition of intellectual curiosity in Prague after 1612, it is possible that a version of organic chemistry, motivated by medical need, could have evolved. The consequences of the Thirty Years War for the "theology" of the Hermetic tradition are far less conjectural, and are well-illustrated by the parallel careers of Newton and Leibniz. The Hermetic texts had been studied in a confessionally divided world, and in the sixteenth century this led to various plans for religious peace. Here Giordano Bruno is an extreme case — his attitude to much traditional Christianity was extremely hostile, and he urged a return to the pure Egyptian religion of Hermes Trismegistos. More usually compromise was proposed inside the existing Christian framework, as for example in the *Confessio Bohemica* of 1575, which achieved surprisingly wide assent. Individually too the thinkers around Rudolph's court were open in their attitude — Kepler's Lutheranism was so heterodox as to exclude his return to Wurttemberg, and John Dee seems to have conformed to Catholic, or perhaps Utraquist, practice while he was in Prague. Such openness must have appealed to the Emperor, conscious as he was of his duty of trying to maintain religious unity. And although visionary it was still possible for men of goodwill to believe in such a programme in 1600. In contrast, after the Thirty Years War, Leibniz was an exceptional figure, emphasising what is common to Catholic and Protestant, and seeking ways to overcome hurdles such as the character, universal or otherwise, of the Council of Trent. How different is Newton — the apologist for the self-sufficient English nation state, defining itself by its own brand of protestantism and virulent anti-popery.

Among the various strands of the Rudolphine tradition, and by a happy sequence of circumstances, the most fruitful was the astronomical, which marks the beginning of modern science. But as Goethe — himself very much in the magical/Hermetic tradition — noted, with the dispersal of the Rudolphine community of scholars and the Emperor's collections in the general misery of war, a humanist component in science was lost. In Kepler's *Mysterium Cosmographicum* and Maier's mystical alchemy, the "two cultures" have not yet separated. And while it is certainly true that the formulation by Kepler of his three laws of planetary motion marks a new departure in human thought, subsequent

developments were coloured by the changed intellectual environment of the later seventeenth century. Perhaps only now, as in trying to repair the damage caused by Prospero's myopic apprentices, we realise how interdependent are the parts of our world, can we also fully appreciate the "magical" concerns of late sixteenth century Prague.

Notes

1. See Peter Brown, *Alexander to Actium* (University of California Press, 1990), for a lively if controversial discussion of Hellenistic science.

2. M. Ficino, *De Vita Coelitus Comparanda, Libri de Vita III*, first published 1489. This is first and foremost a medical text, with Ficino advising the reader to omit the more magical parts, if he disapproves.

3. Jost Bürgi (1552–1632) had a career which deserves serious attention. To assist de Brahe in his observations he constructed clocks combining the advantages of weight and spring driven mechanisms. He should also be regarded with Napier as a codiscoverer of logarithms, and his improvements to arithmetical notation may well have been of real assistance to Kepler.

4. It is a very interesting question to ask to what extent some of the thinkers we are discussing were involved in Sir Francis Walsingham's secret service. It has been suggested that both John Dee's trips to central Europe had some kind of "diplomatic" purpose. And very recently John Bossy in his book *Giordano Bruno and the Embassy Affair* (Yale University Press, 1991) has argued that Bruno was recruited to spy on the French Ambassador in the early 1580's.

5. In my discussion of the "universal talisman" I have followed Frances Yates in *Giordano Bruno and the Hermetic Tradition* (Routledge and Kegan Paul, London, 1964).

6. For a more detailed discussion of Kepler's mathematical arguments, without the Hermetic motivation, see the still marvellously stimulating book by Arthur Koestler, *The Sleepwalkers* (Hutchison, London, 1959), especially IV.6, *The Giving of the Laws*.

7. The discussion given here of the mathematical relation between Kepler's laws and gravity is based on O. Toeplitz, *The Calculus — A Genetic Approach* (University of Chicago Press, 1963), pages 150–172. This is a translation of the German text published by Springer Verlag in 1949.

8. There is a discussion of Michael Maier's alchemical publications in Frances Yates' *The Rosicrucian Enlightenment* (Routledge and Kegan Paul, London, 1972). See in particular Chapter VI, *The Palatinate Publisher*. Yates is always stimulating, but perhaps because of her primarily literary training, some of her conclusions need to be treated with caution. In the case of alchemy for example, I think that she neglects the aborted *scientific* potential of Maier's work.

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This article is a preliminary version of one chapter of a book being written by C.B. Thomas on the intellectual background of the scientific revolution.