

Zeitschrift: Elemente der Mathematik
Herausgeber: Schweizerische Mathematische Gesellschaft
Band: 36 (1981)
Heft: 1

Rubrik: Kleine Mitteilungen

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 06.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

vectors defining any level curve can be constructed in the Euclidean sense. In practice, a combination of both geometric and arithmetic methods is most convenient to sketch any level curve quickly yet accurately.

Duane W. DeTemple and Donald G. Iverson, Washington State University

REFERENCES

- 1 D.W. DeTemple: A Geometric Method of Phase Plane Analysis. Am. Math. Monthly, 87, 102–112 (1980).
- 2 D.W. DeTemple: A Graphical Analysis of 2×2 Matrices. Math. Notes from Washington State University, vol.22, No.1 (1979).
- 3 Howard Eves: An Introduction to the History of Mathematics, 4th ed. Holt, Rinehart and Winston, New York 1976.
- 4 John Wallis: Algebra, 1673 (an English translation of the relevant chapters appears in: D.E. Smith: A Source Book in Mathematics. McGraw-Hill, New York 1929).

Kleine Mitteilungen

An identity involving Ramanujan's sum

Let f be an arithmetical function and let $f' = \mu * f$ denote the Dirichlet convolution of f and the Möbius function μ :

$$f'(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right), \quad n \geq 1.$$

A Ramanujan series is a series of the form

$$\sum_{q=1}^{\infty} a_q c_q(n),$$

where $c_q(n)$ is Ramanujan's sum,

$$c_q(n) = \sum_{\substack{h=1 \\ (h,q)=1}}^q \exp\left(2\pi i \frac{hn}{q}\right),$$

and where

$$a_q = \sum_{m=1}^{\infty} \frac{f'(mq)}{mq}.$$

H. Delange proved [1] the following.

Theorem. If $\sum_{n=1}^{\infty} 2^{\omega(n)} |f'(n)| / n < \infty$, where $\omega(n)$ is the number of distinct prime divisors of n , then $\sum_{q=1}^{\infty} |a_q c_q(n)| < \infty$ for every n , and $\sum_{q=1}^{\infty} a_q c_q(n) = f(n)$.

In his proof, Delange used the inequality

$$\sum_{d|k} |c_d(n)| \leq 2^{\omega(k)} n$$

(see [1], p. 263).

In this note we establish the following identity:

For positive integers k and n ,

$$\sum_{d|k} |c_d(n)| = 2^{\omega\left(\frac{k}{(k,n)}\right)} (k, n). \quad (*)$$

Proof: For any fixed positive integer n , the arithmetical function $g(k) = (k, n)$ is multiplicative:

$$(k l, n) = (k, n)(l, n) \quad \text{if } (k, l) = 1.$$

Hence if F is any multiplicative arithmetical function, then (for any fixed $n \geq 1$), $F(k/(k, n))$ is multiplicative in k :

$$F\left(\frac{k l}{(k l, n)}\right) = F\left(\frac{k}{(k, n)}\right) F\left(\frac{l}{(l, n)}\right) \quad \text{if } (k, l) = 1.$$

It follows that the right hand side of $(*)$ is multiplicative in k , for any fixed integer $n \geq 1$.

So is the left hand side, since $c_k(n)$, and therefore also $|c_k(n)|$ has this property ([2], theorem 67).

Thus in order to prove $(*)$, it suffices to verify it when $k = p^a$, a prime power. And indeed in this case, each side is equal to 2 if $p \nmid n$; if $p^\beta \parallel n$ for some $\beta \geq 1$, then each side is equal to $2 p^\beta$ if $1 \leq \beta < a$, and to p^a if $\beta \geq a$.

Aleksandr Grytczuk, Zielona Góra, Poland

REFERENCES

- 1 H. Delange: On Ramanujan Expansions of Certain Arithmetical Functions. *Acta Arith.* 31, 259–270 (1976).
- 2 G.H. Hardy and E.M. Wright: An Introduction to the Theory of Numbers, 4th ed. Oxford University Press, 1962.