Zeitschrift: Elemente der Mathematik

Herausgeber: Schweizerische Mathematische Gesellschaft

Band: 34 (1979)

Heft: 4

Rubrik: Kleine Mitteilungen

gleichzeitig eine Existenzaussage für die Lösung der diophantischen Gleichung, und dies auch im Falle von Primzahlen $\equiv 1$ oder 9 mod20 (siehe oben Satz 3). Die Überlegungen sollen an anderer Stelle publiziert werden. Peter Wilker, Bern

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Kleine Mitteilungen

Some equations involving the sum of divisors

Pomerance [2] considered the sets $S_k(a)=\{n:\sigma(n)=kn+a\}$ $(a,k\in\mathbf{Z})$. He observed that the sets $S_{\sigma(m)/m}(\sigma(m))$ if $m\mid\sigma(m)$ and $S_2(-1)$ are infinite and wrote: "We know of no other example." Below we give other examples of infinite $S_k(a)$.

Proposition 1. *If m is a positive integer not divisible by a prime number p and such that $\sigma(m)=(p-1)m$ then $p^k m\in S_p(-m)$ for natural k.*

Proof: $\sigma(p^k m)=\sigma(m)(p^{k+1}-1)/(p-1)=(p^{k+1}-1)m=p\cdot p^k m-m$.
For instance we have $3^k\cdot P\in S_3(-P)$, where P is a perfect number not divisible by 3, e.g. $P=28$ or $2^{19936}(2^{19937}-1)$. Similarly, $5^k\cdot Q\in S_5(-Q)$, where $\sigma(Q)=4Q$ and $5\nmid Q$, e.g. $Q=2^9\cdot 3^2\cdot 7\cdot 11\cdot 13\cdot 31$ or $2^{13}\cdot 3^2\cdot 7\cdot 11\cdot 13\cdot 43\cdot 127$ (R. Descartes).
Eleven numbers of the set $S_2(2)$ were listed in the paper [1]. We generalize the result stated there (case $b=1$).

Proposition 2. *If 2^a-2b-1 is a prime number $(b\in\mathbf{Z})$ then $2^a-2b-1)\in S_2(2b)$.*

Proof: $\sigma(2^{a-1}(2^a-2b-1))=(2^a-1)(2^a-2b)=2^{2a}-2^{a+1}b-2^a+2b$
$=2\cdot 2^{a-1}(2^a-2b-1)+2b$.

Andrzej Makowski, Institute of Mathematics, University of Warsaw

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