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Kleine Mitteilungen

On a Problem of Rotkiewicz on Pseudoprime Numbers

A composite number k is called pseudoprime with respect to $a > 1$ if $k \mid a^k - a$. A pseudoprime number with respect to 2 is called simply a pseudoprime number. Let P_n be the n-th pseudoprime number. K. Szymiczek [3] proved that the series $\sum 1/P_n$ is convergent. A. Rotkiewicz in his survey [2] asked (problem 47) whether the series $\sum 1/\log P_n$ is convergent. Below we prove that the series $\sum 1/\log P_n(a)$, where $P_n(a)$ is the n-th pseudoprime number with respect to a, is divergent.

M. Cipolla [1] proved that if p is an odd prime number not dividing $a^2 - 1$ (e.g. $p > a^2$), then $(a^{2p} - 1)/(a^2 - 1)$ is a pseudoprime with respect to a. Therefore

$$\sum \frac{1}{\log P_n(a)} \geq \sum_{p > a^2} \frac{1}{\log (a^{2p} - 1)/(a^2 - 1)} \geq \sum_{p > a} \frac{1}{\log a^{2p}} = \frac{1}{\log a^2} \sum_{p > a^2} \frac{1}{p},$$

but the last series is divergent (see, e.g. E. Trost [4], theorem 23, p. 49), which completes the proof.

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On the Equation $\varphi(n + k) = 2\varphi(n)$

W. Sierpiński [1] proved that the equation $\varphi(n + k) = a\varphi(n)$ for $a = 1$ and for every positive integer k has at least one solution (φ denotes the Euler totient function). Here we prove the analogous result for $a = 2$.

Indeed, if $(k, 6) = 1$ we can take $n = 2k$; if k is even: $k = 2^l u$ ($l \geqq 1$, u odd), we can take $n = 2^l u$; if k is odd and divisible by $3: k = 6l + 3$, we can take $n = 2l + 1$.

The proof follows from the equalities

$$\varphi(2k + k) = \varphi(3k) = 2\varphi(k) = 2\varphi(2k) \text{ for } (k, 6) = 1,$$

$$\varphi(2^l u + 2^l u) = \varphi(2^{l+1} u) = 2^l \varphi(u) = 2\varphi(2^l u)$$

$$\varphi(2l + 1 + 6l + 3) = \varphi(4(2l + 1)) = 2\varphi(2l + 1).$$

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