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Kleine Mitteilungen

On Regular Right Duo Semigroups

Let S be a semigroup[1]. We shall say that S is *right duo* if every right ideal R of S is two-sided. Analogously can be defined the left duo semigroup. A semigroup is said to be a *duo* semigroup if it is both left and right duo. S is called *regular* if to every element a of S there exists an element x in S such that $a = a\,x\,a$. For example, the full transformation semigroup of a set of 2 elements is a regular right duo semigroup. Another example is given by the following multiplication table:

	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	1	2	3
3	0	1	2	3

In this short note some ideal-theoretic characterizations of regular right (and left) duo semigroups will be proved.

Theorem 1. *A semigroup S is a regular right duo semigroup if and only if the condition*

$$B \cap R = RB \tag{1}$$

holds for every bi-ideal B of S and every right ideal R of S.

Proof. Let S be a regular right duo semigroup. Then every bi-ideal B of S can be represented in the form

$$B = IL\,, \tag{2}$$

where L is a left ideal and I is a two-sided ideal of S (cf .[3]). Therefore (1) is implied by (2) and the Kovács-Iséki regularity criterion (see [1], p. 34).

Conversely, suppose that S is a semigroup admitting property (1) for every bi-ideal B and every right ideal R of S. Then, for any right ideal R of S, (1) implies

$$R \cap S = SR\,. \tag{3}$$

Hence S is right duo. But a right duo semigroup S is regular if and only if $I \cap L = IL$ for every two-sided ideal I and every left ideal L of S, which is implied by (1).

The following criterion can similarly be proved.

Theorem 2. *A semigroup S is a regular right duo semigroup if and only if the relation*

$$Q \cap R = RQ \tag{4}$$

holds for every quasi-ideal Q and every right ideal R of S.

---

[1] For the undefined notions and notations we refer to [1].

Next we give necessary and sufficient conditions for a right duo semigroup S to be a semilattice of groups.

Theorem 3. *For a right duo semigroup S the following conditions are pairwise equivalent:*

(A) *S is a semilattice of groups.*

(B) *$B \cap I = BI$ for every bi-ideal B and every two-sided ideal I of S.*

(C) *$I \cap Q = QI$ for any quasi-ideal Q and any two-sided ideal I of S.*

(D) *$I \cap L = LI$ for every left ideal L and every two-sided ideal I of S.*

Proof. (A) implies (B). It is known (cf. [2], [5]) that for any two bi-ideals B_1, B_2 of a semigroup S that is a semilattice of groups, the condition

$$B_1 \cap B_2 = B_1 B_2 \tag{5}$$

holds. This implies (B).

Evidently (B) implies (C) and (C) implies (D), because every left ideal is a quasi-ideal, and every quasi-ideal is a bi-ideal.

(D) implies (A). In case of $I = S$, (D) implies

$$L \cap S = LS \tag{6}$$

for every left ideal L of S, i.e. S is a left duo semigroup. Therefore condition (D) implies

$$L(a) \cap R(a) = R(a)\ L(a)\,, \tag{7}$$

that is,

$$J(a) = J^2(a)\,, \tag{8}$$

for every element a of S. (8) implies $a \in a^2 \cup a\,S\,a$ ($\forall\ a \in S$), whence S is regular. Thus S is a regular duo semigroup, that is a semilattice of groups (cf. [4], Theorem 2).

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A Contour for the Poisson Integral

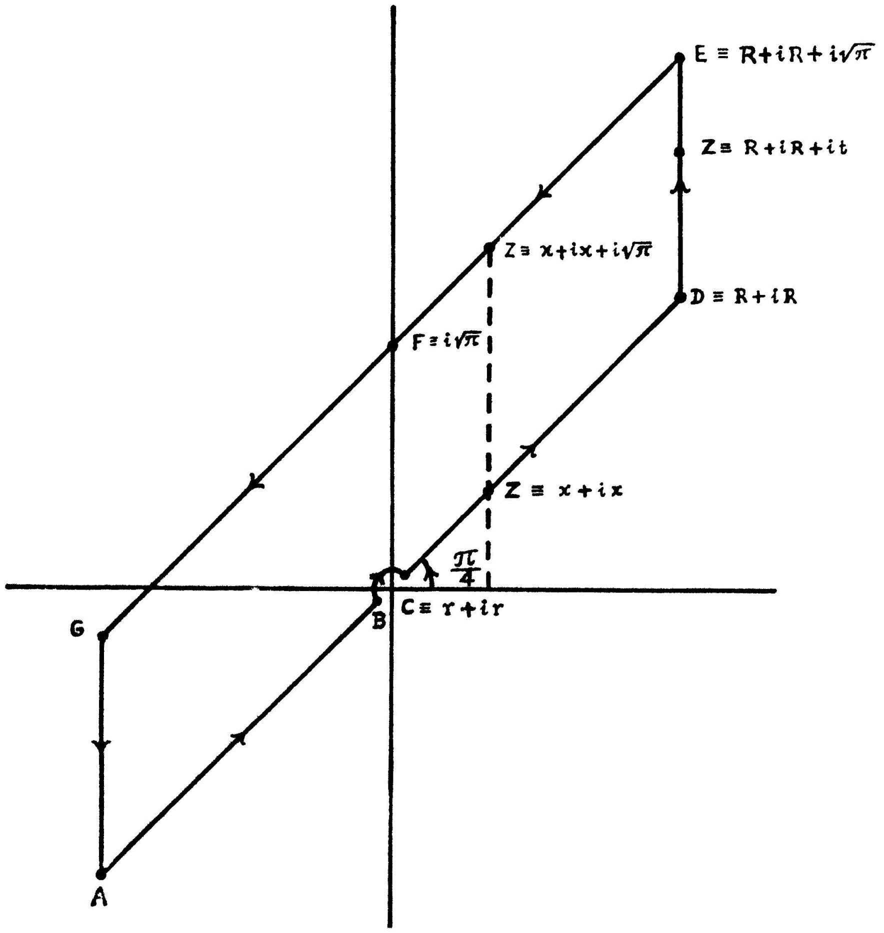
A convenient contour for the direct evaluation of the Poisson Integral

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2} \tag{1}$$

does not appear to have been considered in the literature. On the contrary (1) is invariably borrowed from real variable theory to derive Fresnel's integrals by integrating e^{iz^2} (or e^{-z^2}) along a sector. The Fresnel integrals, however, may be obtained independently by the procedure for evaluating Gauss's sums [2] suggesting that a suitable modification of this procedure should also yield (1). In fact, we integrate

$$f(z) = \frac{e^{\frac{iz^2}{2}}}{e^{-\sqrt{\pi}z} - 1} \tag{2}$$

along the parallelogram $A\overset{\frown}{BC}DEFGA$ as shown in the diagram.



We have

$$\left|\int_{DE} f(z)\,dz\right| \leqq \frac{e^{-R^2}}{1 - e^{-\sqrt{\pi}R}} \int_0^{\sqrt{\pi}} e^{-Rt}\,dt = \frac{e^{-R^2}}{R} \to 0 \quad (R \to \infty)\,. \tag{3}$$

Similarly, $\int_{GA} f(z)\,dz$ also vanishes as $R \to \infty$. Now,

$$\int_{CD} f(z)\,dz = (1+i)\int_r^R \frac{e^{-x^2}}{e^{-(1+i)\sqrt{\pi}\,x}-1}\,dx$$

and

$$\int_{AB} f(z)\,dz = (1+i)\int_{-R}^{-r} \frac{e^{-x^2}}{e^{-(1+i)\sqrt{\pi}\,x}-1}\,dx = (1+i)\int_r^R \frac{e^{-x^2}}{e^{(1+i)\sqrt{\pi}\,x}-1}\,dx$$

giving

$$\int_{AB} f(z)\,dz + \int_{CD} f(z)\,dz = -(1+i)\int_r^R e^{-x^2}\,dx\,. \tag{4}$$

Likewise, we have

$$\int_{GF} f(z)\,dz + \int_{FE} f(z)\,dz = i\,(1+i)\int_0^R e^{-x^2}\,dx\,. \tag{5}$$

Equating the integrals along AD and GE in the limit as $r \to 0$ and $R \to \infty$ and taking into account the contribution $i\sqrt{\pi}$ from the indentation at the origin, we obtain, from (4) and (5),

$$-(1+i)\int_0^\infty e^{-x^2}\,dx + i\sqrt{\pi} = i\,(1+i)\int_0^\infty e^{-x^2}\,dx$$

yielding (1).

More generally, by integrating

$$f(z) = \frac{e^{i z^2 \cos^2\alpha}}{e^{-\sqrt{2\pi}\,z\cos\alpha}-1} \tag{6}$$

along a parallelogram inclined at α to the real axis, $0 \leqq \alpha < \pi/2$ so that

$$D \equiv R + i\,R\tan\alpha\,, \quad E \equiv R + i\,R\tan\alpha + i\sqrt{\frac{\pi}{2}}\sec\alpha\,, \quad F \equiv i\sqrt{\frac{\pi}{2}}\sec\alpha$$

we obtain, in exactly the same manner and with no more effort,

$$\int_0^\infty e^{-x^2\sin 2\alpha + i x^2\cos 2\alpha}\,dx = \frac{1+i}{2}\sqrt{\frac{\pi}{2}}\,e^{-i\alpha}\,, \quad 0 \leqq \alpha < \pi/2\,, \tag{7}$$

which evidently includes the integrals of Fresnel ($\alpha = 0$) and Poisson ($\alpha = \pi/4$) as special cases. Here again, the usual proof [1] of the generalisation (7) depends on an appeal to (1).

Changing the variable from x to $\varrho\, x$ where $\varrho > 0$ is arbitrary and setting $a = \varrho^2\, e^{2i\alpha}$ (7) assumes the more compact form

$$\int_0^\infty e^{iax^2}\, dx = \frac{1+i}{2} \sqrt{\frac{\pi}{2a}}\,, \quad a \neq 0\,,\ \operatorname{Im} a \geqq 0\,, \tag{8}$$

the principal value of $\sqrt{a}$ being taken on the right.

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Aufgaben

Aufgabe 647. Für eine streng monoton wachsende Folge (a_i) natürlicher Zahlen seien $A(n) = \sum\limits_{a_i < n} 1$ $(n = 1, 2, \ldots)$, $\limsup A(n)/n$ $[n \to \infty]$ die *obere Dichte* und – im Falle der Existenz – $\lim A(n)/n$ $[n \to \infty]$ die *Dichte*. Man beweise:

a) Jede streng monoton wachsende Folge natürlicher Zahlen mit oberer Dichte 1 besitzt eine unendliche Teilfolge, welche aus paarweise teilerfremden Zahlen besteht.

b) Zu jedem $\varepsilon > 0$ gibt es stets eine streng monoton wachsende Folge natürlicher Zahlen mit Dichte $> 1 - \varepsilon$ derart, dass für keine ihrer unendlichen Teilfolgen die Glieder paarweise denselben grössten gemeinsamen Teiler haben.

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Für die Lösung zu Teil a) vgl. diesen Band, p. 65.

Lösung zu Teil b): Zu jedem $\varepsilon > 0$ wollen wir eine streng monoton wachsende Folge (a_i) natürlicher Zahlen mit Dichte $> 1 - \varepsilon$ konstruieren derart, dass für jede natürliche Zahl d nur endlich viele verschiedene a_i paarweise den grössten gemeinsamen Teiler d haben.

Es sei $n_0 = n_0(\varepsilon)$ genügend gross. Eine natürliche Zahl a gehöre nun genau dann zur Folge (a_i), wenn gilt:

1) Der kleinste Primfaktor von a ist $\leq n_0$, und
2) ist $p_k | a$ und p_k nicht der grösste Primfaktor von a, dann hat a im Intervall $(p_k, e^{n_0 p_k^2})$ einen Primfaktor.

Wäre $a_{i_1} < a_{i_2} < \ldots.$ eine unendliche Teilfolge mit $(a_{i_{r_1}}, a_{i_{r_2}}) = d$ für $r_1 \neq r_2$, so würde gelten $a_{i_r} = d s_r$, $(s_{r_1}, s_{r_2}) = 1$. Also hätten die a_{i_r} beliebig grosse Primfaktoren, daher hätten unendlich viele unter ihnen einen Primfaktor im Intervall