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number of values of p and q, if either U_2 or U_3 is irreducible, for which $P(x, y)$ is reducible.

Next

$$h^2 - 4ab = (p^2 h_2 - 2pq h_1 - q^2 h_3)^2 - 4(p^2 a_2 - 2pq a_1 - q^2 a_3)(p^2 b_2 - pq b_1 - q^2 b_3)$$

If $h_2^2 - 4 a_2 b_2 > 0$ and is not a perfect square, this holds for $h^2 - 4ab$ if p is large compared with q, and for an infinity of p. This proves Theorem (1).

There are many special cases not included in the theorem. We need only mention

Theorem 2

The equation

$$z^2 = k^2 + x^2 (a x^2 + b y^2), \quad a b k \neq 0,$$

has an infinity of integer solutions if k, a, b are integers and either $b > 0$, or $b < 0$, $4 a k^2 > b^2$.

We have

$$z + k = \frac{q}{p} (a x^2 + b y^2), \quad z - k = \frac{p}{q} x^2,$$

where p, q are integers and $(p, q) = 1$. Then

$$(a q^2 - p^2) x^2 + b q^2 y^2 = 2 k p q .$$

This will have the solution $x = 0$, $y = t$, where t is an arbitrary integer, if $b q t^2 = 2 k p$, and so if $\delta = (b, 2k)$, we can take

$$\lambda p = \frac{b}{\delta} t^2, \quad \lambda q = \frac{2k}{\delta}, \quad \lambda = \left(t^2, \frac{2k}{\delta}\right)$$

Hence there will be an infinity of integer solutions for x, y if $b(p^2 - a q^2) > 0$ and is not a perfect square, i.e. $b(b^2 t^4 - 4 a k^2) > 0$ and is not a perfect square. This is possible if $b > 0$ for an infinity of values of t, and also if $b < 0$, $4 a k^2 > b^2$ for $t = 1$.

The case $4 a k^2 < b^2$ seems difficult. Of course if $a < 0$, $b < 0$, there are only a finite number of solutions.

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Ungelöste Probleme

Bemerkung zu Nr. 14 (El. Math. *11*, 134–135 (1956)). A. a. O. wurde gezeigt, dass die Gleichung $x_1 + x_2 + \cdots + x_s = x_1 x_2 \ldots x_s$ für jedes natürliche s mindestens eine Lösung in natürlichen Zahlen besitzt. Nach einer Mitteilung von Herrn A. SCHINZEL (Warschau) hat M. MISIUREWICZ vor kurzem bewiesen, dass $s = 2, 3, 4, 6, 24, 144, 174, 444$ die einzigen $s \leqq 1000$ sind, für die genau eine Lösung existiert.