

# **Erratum to "The topology at infinity of Coxeter groups and buildings"**

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## Erratum to “The topology at infinity of Coxeter groups and buildings”

Michael W. Davis and John Meier

The paper referred to in the title concerns the algebraic topology at infinity of geometric realizations of Coxeter groups and of buildings. For Coxeter groups, the arguments are correct; however, for buildings, they are not. Jan Dymara and Damian Osajda pointed out to us some serious problems with Section 5, where the results for buildings are given (see [4]). In particular, Lemmas 5.3 and 5.7 are wrong. This leads to gaps in the proofs of Theorems 5.8, 5.12 and 5.13. Nevertheless, we believe that the theorems in Section 5 remain true. (More precisely, Theorems 5.12 and 5.13 should hold as stated and a version of Theorem 5.8 should be true.)

The basic mistake in Section 5 is this. Suppose  $C$  is (the set of chambers of) a building and  $X \subset C$  is a subset which is starlike with respect to a base chamber  $c_0$ . Let  $c$  be an extreme chamber in  $X$  and put  $\check{X} := C - X$ . Implicit in both Lemmas 5.3 and 5.7 is the assumption that

$$|\check{X}| \cap |c| = |c|^{I_{\uparrow}(X, c)}. \quad (1)$$

In other words, the intersection on the left is a certain union of mirrors of  $|c|$  indexed by  $I_{\uparrow}(X, c)$ , where  $I_{\uparrow}(X, c)$  denotes the set of  $i \in I$  such that  $\check{X}$  contains a chamber  $i$ -adjacent to  $c$ . In fact, the intersection in (1) need not be a union of mirrors. For example, it can be the union of  $|c|^{I_{\uparrow}(X, c)}$  with a lower dimensional face. (This can be seen even in the case of thick, right-angled, spherical buildings of rank  $\geq 2$ .)

In the calculation of  $H_c^*(|C|)$  in Theorem 5.8, one starts by ordering  $C, c_0, c_1, \dots$ , so that  $l(\delta(c_0, c_{k+1})) \geq l(\delta(c_0, c_k))$ , where  $\delta(\cdot, \cdot)$  is the  $W$ -valued distance on  $C$  and  $l(\cdot)$  is word length on  $W$ . If  $X_m := \{c_0, c_1, \dots, c_m\}$ , then one wants formula (1) to hold with  $X = X_m$  and  $c = c_m$ . There is considerable freedom in choosing the ordering of  $C$  and not all choices work. To see this, suppose  $R$  is a spherical residue in  $C$  and  $d_R \in R$  is its chamber closest to  $c_0$ . Let  $L_R$  be the set of chambers in  $R$  which are furthest from  $d_R$ . ( $L_R$  is the set of chambers in  $R$  opposite to  $d_R$ .) Since the elements of  $L_R$  all have the same  $W$ -valued distance from  $c_0$ , when choosing the ordering of  $C$ ,  $L_R$  can be ordered arbitrarily. Most choices of orderings will not satisfy (1). In particular, if  $L_R$  is not gallery-connected, no choice will work.

When  $C$  is right-angled the situation can be remedied. For in this case,  $L_R$  is a spherical building of the same type  $(W_J, J)$  as  $R$  (at least when  $R$  is thick). Order the elements of  $L_R$  using the  $W_J$ -distance on  $L_R$ . (It may be necessary to apply this step repeatedly.) The conclusion is that for right-angled buildings there is an ordering of  $C$  satisfying (1). Hence, Theorems 5.8, 5.12 and 5.13 hold for right-angled buildings and Corollary 5.14 is true as stated. A different proof of Theorem 5.8 for right-angled buildings is given in [2]. It also uses the fact that  $L_R$  is a building of type  $(W_J, J)$ . In the right-angled case, Theorems 5.12 and 5.13, as well as, Corollary 5.11 also follow from the results in [1].

## References

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