Erratum to "The topology at infinity of Coxeter groups and buildings"

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The paper referred to in the title concerns the algebraic topology at infinity of geometric realizations of Coxeter groups and of buildings. For Coxeter groups, the arguments are correct; however, for buildings, they are not. Jan Dymara and Damian Osajda pointed out to us some serious problems with Section 5, where the results for buildings are given (see [4]). In particular, Lemmas 5.3 and 5.7 are wrong. This leads to gaps in the proofs of Theorems 5.8, 5.12 and 5.13. Nevertheless, we believe that the theorems in Section 5 remain true. (More precisely, Theorems 5.12 and 5.13 should hold as stated and a version of Theorem 5.8 should be true.)

The basic mistake in Section 5 is this. Suppose C is (the set of chambers of) a building and $X \subset C$ is a subset which is starlike with respect to a base chamber c_0. Let c be an extreme chamber in X and put $\check{X} := C - X$. Implicit in both Lemmas 5.3 and 5.7 is the assumption that

$$|\check{X}| \cap |c| = |c|^{I_{\uparrow}(X,c)}. \tag{1}$$

In other words, the intersection on the left is a certain union of mirrors of $|c|$ indexed by $I_{\uparrow}(X,c)$, where $I_{\uparrow}(X,c)$ denotes the set of $i \in I$ such that $\check{X}$ contains a chamber i-adjacent to c. In fact, the intersection in (1) need not be a union of mirrors. For example, it can be the union of $|c|^{I_{\uparrow}(X,c)}$ with a lower dimensional face. (This can be seen even in the case of thick, right-angled, spherical buildings of rank ≥ 2.)

In the calculation of $H^*_c(|C|)$ in Theorem 5.8, one starts by ordering C, $c_0, c_1, \dots,$ so that $l(\delta(c_0, c_{k+1})) \geq l(\delta(c_0, c_k))$, where $\delta(\ ,\ )$ is the W-valued distance on C and $l(\ )$ is word length on W. If $X_m := \{c_0, c_1, \dots, c_m\}$, then one wants formula (1) to hold with $X = X_m$ and $c = c_m$. There is considerable freedom in choosing the ordering of C and not all choices work. To see this, suppose R is a spherical residue in C and $d_R \in R$ is its chamber closest to c_0. Let L_R be the set of chambers in R which are furthest from d_R. (L_R is the set of chambers in R opposite to d_R.) Since the elements of L_R all have the same W-valued distance from c_0, when choosing the ordering of C, L_R can be ordered arbitrarily. Most choices of orderings will not satisfy (1). In particular, if L_R is not gallery-connected, no choice will work.

When C is right-angled the situation can be remedied. For in this case, L_R is a spherical building of the same type (W_J, J) as R (at least when R is thick). Order the elements of L_R using the W_J-distance on L_R. (It may be necessary to apply this step repeatedly.) The conclusion is that for right-angled buildings there is an ordering of C satisfying (1). Hence, Theorems 5.8, 5.12 and 5.13 hold for right-angled buildings and Corollary 5.14 is true as stated. A different proof of Theorem 5.8 for right-angled buildings is given in [2]. It also uses the fact that L_R is a building of type (W_J, J). In the right-angled case, Theorems 5.12 and 5.13, as well as, Corollary 5.11 also follow from the results in [1].

References

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