

**Zeitschrift:** Commentarii Mathematici Helvetici  
**Herausgeber:** Schweizerische Mathematische Gesellschaft  
**Band:** 81 (2006)

**Erratum:** Erratum to "A gap theorem for hypersurfaces with constant scalar curvature one"  
**Autor:** Alencar, H./Carmo, do M./Santos, W.

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 08.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

**Erratum to “A gap theorem for hypersurfaces with constant scalar curvature one”**

H. Alencar, M. do Carmo and W. Santos

**Proof of Lemma 4.1**

In our paper [AdCS], the following lemma is stated.

**Lemma 4.1.** *Let  $M^n \rightarrow \mathbb{S}^{n+1}$  be a closed orientable hypersurface with scalar curvature  $R = 1$  (i.e.  $S_2 = 0$ ). Then the index of the quadratic form*

$$I(f, f) = - \int_M \{fL_1f + ((n - 1)S_1 - 3S_3)f^2\} dM$$

*is greater than one.*

Here  $S_r, r = 1, \dots, n$ , is the  $r^{\text{th}}$ -symmetric function of the principal curvatures of the immersion and  $L_1$  is the linearized operator corresponding to the equation of hypersurfaces of  $\mathbb{S}^{n+1}$  with  $S_2 \equiv 0$ .

The proof of Lemma 4.1 presented in [AdCS] is incorrect. We had set  $N = \sum_{i=1}^{n+2} n_i e_i$ , where  $\{e_1, \dots, e_{n+2}\}$  is an orthonormal basis of  $\mathbb{R}^{n+2}$  and  $N$  is the unit normal vector to the immersion of  $M^n \rightarrow \mathbb{S}^{n+1} \subset \mathbb{R}^{n+2}$ . By assuming the index of  $I$  is greater than one and noticing that  $I(n_i, n_i) \leq 0$ , we concluded that all  $n_i$  but one were zero. This is not true. Replace the proof of the lemma by the following.

*Proof.* To prove that  $\text{Ind}(M)$  is greater than one, we will follow Simons [S], where a similar proposition is proved for the minimal case in arbitrary codimension (Proposition 5.1.1 of [S]). The crucial point is that Simons argument in codimension one does not depend on minimality. For convenience of the reader we present here the details.

Let  $\xi$  denote the  $(n + 2)$ -dimensional space of vector fields that are tangential projections to  $\mathbb{S}^{n+1}$  of parallel vector fields in  $\mathbb{R}^{n+2} \supset \mathbb{S}^{n+1}$ . Clearly at each point  $p \in M$ ,  $\xi$  spans the entire tangent space  $T_p(\mathbb{S}^{n+1})$ . Let  $\xi^N$  be the space of all normal vector fields of the form  $\langle a, N \rangle N$ , where  $a$  is a fixed vector of  $\mathbb{R}^{n+2}$  and  $N$  is the unit normal of  $M$ .

We first show that restricted to  $\xi^N$ , the index form is negative definite. Indeed, setting  $f_a = \langle a, N \rangle$ ,  $g_a = \langle a, x \rangle$  and noticing that

$$L_1(f_a) = -(S_1 S_2 - 3S_3) f_a - 2S_2 g_a,$$

we obtain, since  $S_2 = 0$ , that  $L_1(f_a) = 3S_3 f_a$ .

Thus,

$$\begin{aligned} I(f_a, f_a) &= - \int_M (f_a L_1(f_a) + [(n-1)S_1 - 3S_3] f_a^2) dM \\ &= - \int_M (n-1)S_1 f_a^2 dM < 0, \end{aligned}$$

since  $S_1 > 0$ . It follows that the index formula restricted to  $\xi^N$  is negative definite, hence  $\text{Ind}(M) \geq 1$ .

To show that  $\text{Ind}(M) > 1$ , we need a few lemmas taken from [S]. Let  $\nabla$ ,  $\bar{\nabla}$  and  $\overline{\nabla}$  the covariant derivatives of  $M^n$ ,  $\mathbb{S}^{n+1}$  and  $\mathbb{R}^{n+2}$ , respectively.

**Lemma 1.1.** *Let  $Z \in \xi$ . Then, given  $p \in \mathbb{S}^{n+1}$ , there exists a  $\lambda$  such that, for any  $x \in T_p(\mathbb{S}^{n+1})$ ,*

$$\bar{\nabla}_x Z = \lambda x. \quad (1)$$

*Proof.* Since  $Z = W^T$ , where  $W$  is a parallel field in  $\mathbb{R}^{n+2}$ , we have, by setting  $W = W^T + W^N$ ,

$$\bar{\nabla}_x Z = (\overline{\nabla}_x W^T)^T = -(\overline{\nabla}_x W^N)^T = \lambda x,$$

because  $\mathbb{S}^{n+1}$  is umbilic.  $\square$

**Lemma 1.2.** *Let  $Z \in \xi$  and write  $Z = Z^T + Z^N$ , where  $Z^N$  is the projection into  $N(M)$  and  $Z^T$  is the projection into  $T(M)$ . Then, for any  $x \in T_p(M)$ ,*

$$\nabla_x^N Z^N = -B(x, Z^T). \quad (2)$$

*Proof.* By using (1),

$$\begin{aligned} \nabla_x^N Z^N &= (\bar{\nabla}_x Z^N)^N = (\bar{\nabla}_x Z - \bar{\nabla}_x Z^T)^N \\ &= (\lambda x - \bar{\nabla}_x Z^T)^N = -(\bar{\nabla}_x Z^T)^N = -B(x, Z^T). \quad \square \end{aligned}$$

**Lemma 1.3.** *Assume that  $\dim \xi^N = 1$ . Then given  $z \in T_p(M)$ , there exists  $Z \in \xi$  such that  $Z(p) = z$  and  $Z$  is everywhere tangent to  $M$ .*

*Proof.* Let  $\eta$  be the kernel of the homomorphism  $\xi \rightarrow \xi^N$ . If  $Z \in \eta$ , then  $Z^T = Z$  at every point of  $M$ . Now, for some  $p \in M$ , let  $\beta_p$  be the kernel of the homomorphism  $\xi \rightarrow N_p(M)$  defined by  $Z \mapsto Z^N(p)$ . Clearly  $\eta \subset \beta_p$ . On the other hand,  $\dim \beta_p = n+2-1$  and the assumption that  $\dim \xi^N = 1$  implies that  $\dim \eta = n+2-1$ . Thus  $\eta = \beta_p$ . Since  $Z \rightarrow Z^T(p)$  maps  $\beta_p$  onto  $T_p(M)$ , it also maps  $\eta$  onto  $T_p(M)$ . This implies the claim of the lemma.  $\square$

We can now conclude our proof. By using Lemma 1.3 and (2), we obtain

$$B(x, z) = B(x, Z) = B(x, Z^T) = -\nabla_x^N Z^N = 0.$$

Thus  $B \equiv 0$  and  $M$  is totally geodesic. Since  $S_1 > 0$ , this is a contradiction and shows that  $\text{Ind}(M) > 1$ .  $\square$

## References

- [AdCS] H. Alencar, M. do Carmo, W. Santos, A gap theorem for hypersurfaces with constant scalar curvature one. *Comment. Math. Helv.* **77** (2002), 549–562. Zbl 1032.53045 MR 1933789
- [S] J. Simons, Minimal varieties in riemannian manifolds. *Ann. of Math. (2)* **88** (1968) 62–105. Zbl 0181.49702 MR 0233295

Received December 27, 2004

H. Alencar, Universidade Federal de Alagoas, Departamento de Matemática, 57072-900, Maceió, AL, Brazil

E-mail: hilario@mat.ufal.br

M. do Carmo, Instituto de Matemática Pura e Aplicada (IMPA), Estrada Dona Castorina 110, 22460-320, Rio de Janeiro, RJ, Brazil

E-mail: manfredo@impa.br

W. Santos, Universidade Federal do Rio de Janeiro, Departamento de Matemática, Caixa Postal 68530, 21945-970, Rio de Janeiro, RJ, Brazil

E-mail: walcy@im.ufrj.br

Leere Seite  
Blank page  
Page vide