Applications of Hofer's geometry to Hamiltonian dynamics

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Abstract. We prove that for every subset A of a tame symplectic manifold (W, ω) meeting a semi-positivity condition, the π_1-sensitive Hofer–Zehnder capacity of A is not greater than four times the stable displacement energy of A,

$$c_{\mathrm{HZ}}^{\circ}(A, W) \leq 4e(A \times S^1, W \times T^* S^1).$$

This estimate yields almost existence of periodic orbits near stably displaceable energy levels of time-independent Hamiltonian systems. Our main applications are:

- The Weinstein conjecture holds true for every stably displaceable hypersurface of contact type in (W, ω).

- The flow describing the motion of a charge on a closed Riemannian manifold subject to a non-vanishing magnetic field and a conservative force field has contractible periodic orbits at almost all sufficiently small energies.

The proof of the above energy-capacity inequality combines a curve shortening procedure in Hofer geometry with the following detection mechanism for periodic orbits: If the ray $\{\varphi_F^t\}$, $t \geq 0$, of Hamiltonian diffeomorphisms generated by a compactly supported time-independent Hamiltonian stops to be a minimal geodesic in its homotopy class, then a non-constant contractible periodic orbit must appear.

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1. Introduction and results

On their search for periodic orbits of autonomous Hamiltonian systems, Hofer and Zehnder ([27], [28]) associated to every open subset A of a symplectic manifold (V, ω) a number, the Hofer–Zehnder capacity $c_{\mathrm{HZ}}(A) \in [0, \infty]$, in such a way that $c_{\mathrm{HZ}}(A) < \infty$ implies almost existence of periodic orbits near any compact regular

energy level of an autonomous Hamiltonian system on A. Showing that $c_{HZ}(A)$ is finite is, however, often a difficult problem. Our main result is that if a subset A of a tame symplectic manifold meeting a suitable semi-positivity condition can be displaced from itself by a Hamiltonian isotopy in a stabilized sense, then the Hofer–Zehnder capacity of A is indeed finite.

In order to set notations, we abbreviate $I = [0, 1]$ and consider an arbitrary smooth symplectic manifold (V, ω) without boundary. Denote by $\mathcal{H}_c(I \times V)$ the set of smooth functions $I \times V \to \mathbb{R}$ with compact support. The Hamiltonian vector field of $H \in \mathcal{H}_c(I \times V)$, defined by

$$\omega(X_{H_t}, \cdot) = -dH_t(\cdot),$$

generates a flow h_t. The time-1-maps h form the group

$$\mathrm{Ham}_c(V, \omega) := \{h \mid H \in \mathcal{H}_c(I \times V)\}$$

of compactly supported Hamiltonian diffeomorphisms of (V, ω). The set of functions in $\mathcal{H}_c(I \times V)$ which do not depend on $t \in I$ is denoted by $\mathcal{H}_c(V)$. We shall denote functions in $\mathcal{H}_c(I \times V)$ by H or K and functions in $\mathcal{H}_c(V)$ by F or G, and their flows by h_t or k_t and f_t or g_t.

The Hofer–Zehnder capacity we shall study is defined as follows. We say that $F \in \mathcal{H}_c(V)$ is *slow* if all non-constant contractible periodic orbits of f_t have period greater than 1. Following [27], [28] and [38], [53], [17] we define for each subset A of (V, ω) the π_1-sensitive Hofer–Zehnder capacity

$$c_{HZ}^{\circ}(A, V, \omega) = \sup\{\max F - \min F \mid F \in \mathcal{H}_c(\mathrm{Int}(A)) \text{ is slow}\}. \tag{1}$$

We shall often suppress ω from the notation, and we shall write $c_{HZ}^{\circ}(V)$ instead of $c_{HZ}^{\circ}(V, V)$. The Hofer–Zehnder capacity $c_{HZ}(A)$ mentioned above is obtained by taking the supremum over the smaller class of functions $F \in \mathcal{H}_c(\mathrm{Int}(A))$ for which *all* non-constant periodic orbits of f_t have period > 1. Therefore, $c_{HZ}(A) \le c_{HZ}^{\circ}(A, V)$.

We shall compare the Hofer–Zehnder capacity $c_{HZ}^{\circ}(A, V)$ with the displacement energy defined in [21], [32]. The norm $\|H\|$ of $H \in \mathcal{H}_c(I \times V)$ is defined as

$$\|H\| = \int_0^1 \Big(\max_{x \in V} H(t, x) - \min_{x \in V} H(t, x)\Big)dt,$$

and the displacement energy $e(A, V) = e(A, V, \omega) \in [0, \infty]$ of a subset A of V is defined as

$$e(A, V) = \inf\{\|H\| \mid H \in \mathcal{H}_c(I \times V), \ h(A) \cap A = \emptyset\}$$

if A is compact and as

$$e(A, V) = \sup\{e(K, V) \mid K \subset A \text{ is compact}\}$$

for a general subset A of V. In fact, we shall compare $c^\circ_{\mathrm{HZ}}(A, V)$ with the *stable displacement energy* defined as

$$e_S(A, V) := e(A \times S^1, V \times T^*S^1, \omega \oplus \omega_0)$$

where $\omega_0 = dp \wedge dq$ is the standard symplectic form on T^*S^1. We are able to do this for the following class of symplectic manifolds.

Definition ([20], [56], [2]). A symplectic manifold (W, ω) is *tame* if W admits an almost complex structure J and a complete Riemannian metric g such that

- J is uniformly tame, i.e., there are positive constant C_1 and C_2 such that

$$\omega(X, JX) \geq C_1 \|X\|^2 \quad \text{and} \quad |\omega(X, Y)| \leq C_2 \|X\| \|Y\|$$

  for all $X, Y \in TW$.

- The sectional curvature of (W, g) is bounded from above and the injectivity radius of (W, g) is bounded away from zero.

Examples of tame symplectic manifolds are closed symplectic manifolds, standard cotangent bundles (T^*M, ω_0) as well as twisted cotangent bundles (T^*M, ω_σ) over a closed base M, and symplectic manifolds which at infinity are isomorphic to the symplectization of a closed contact manifold. The class of tame symplectic manifolds is closed under taking products or coverings.

For technical reasons we also impose a semi-positivity condition on (W, ω). The first Chern class $c_1 \in H^2(W; \mathbb{Z})$ is defined as the first Chern class of the complex vector bundle (TW, J), where J is any almost complex structure such that $\omega(\cdot, J\cdot)$ is a Riemannian metric. Recall from [43], [23], [54], [44] that a $2n$-dimensional symplectic manifold (W, ω) is *strongly semi-positive* if for all $A \in \pi_2(W)$,

$$\omega(A) > 0, \quad c_1(A) \geq 2 - n \quad \Longrightarrow \quad c_1(A) \geq 0.$$

Definition. A $2n$-dimensional symplectic manifold (W, ω) is *stably strongly semi-positive* if for all $A \in \pi_2(W)$,

$$\omega(A) > 0, \quad c_1(A) \geq 1 - n \quad \Longrightarrow \quad c_1(A) \geq 0.$$

Equivalently, (W, ω) satisfies one of the following conditions.

(i) $\omega(A) = \lambda c_1(A)$ for every $A \in \pi_2(W)$ and some $\lambda \geq 0$;

(ii) $c_1(A) = 0$ for every $A \in \pi_2(W)$;

(iii) The minimal Chern number $N \geq 0$ defined by $c_1(\pi_2(W)) = N\mathbb{Z}$ is at least n.

Since (T^*S^1, ω_0) is exact and has vanishing first Chern class, (W, ω) is stably strongly semi-positive if and only if $(W \times T^*S^1, \omega \oplus \omega_0)$ is strongly semi-positive. This assumption guarantees that the evaluation map used in the definition of the Gromov–Witten invariants relevant for our arguments is a pseudo-cycle. If one is willing to use Liu–Tian's construction of the S^1-invariant virtual moduli cycle, this assumption can be dropped throughout the paper.

Our main result is the following energy-capacity inequality.

Theorem 1.1. *Assume that A is a subset of a tame and stably strongly semi-positive symplectic manifold (W, ω). Then*

$$c^{\circ}_{\mathrm{HZ}} (A, W) \leq 4e_S(A, W).$$

We shall derive Theorem 1.1 from the following result by capitalizing on the fact that the definition of c°_{HZ} involves *only contractible* periodic orbits and by using a stabilization trick found in Macarini's work [41].

Theorem 1.2. *Assume that A is a subset of a tame and strongly semi-positive symplectic manifold (W, ω). Then*

$$c^{\circ}_{\mathrm{HZ}} (A, W) \leq 4e(A, W).$$

Up to its slightly more restrictive hypothesis, Theorem 1.1 is stronger than Theorem 1.2. Indeed, it is elementary to see that $e_S(A, V) \leq e(A, V)$ in general, and in the dynamically relevant Example 1.5 below we have $e_S(A, V) < e(A, V) = \infty$.

The energy-capacity inequality

$$c^{\circ}_{\mathrm{HZ}} (A, V) \leq e(A, V) \tag{2}$$

is known for every subset A of a weakly exact symplectic manifold (V, ω) which is closed or convex ([22], [53], [12], [16], [11]). For the open ball $B^{2n}(r)$ of radius r in $(\mathbb{R}^{2n}, \omega_0)$ it holds that

$$c^{\circ}_{\mathrm{HZ}} \left(B^{2n}(r), \mathbb{R}^{2n}\right) = e \left(B^{2n}(r), \mathbb{R}^{2n}\right) = \pi r^2,$$

see [28], and so (2) is sharp. It is conceivable that the factor 4 in Theorems 1.1 and 1.2 can be omitted.

Following Polterovich [50] we shall obtain Theorem 1.2 by combining an elementary curve shortening technique in Hofer's geometry with the following detection mechanism for periodic orbits.

Theorem 1.3. *Assume that (W, ω) is a tame and strongly semi-positive symplectic manifold, and that the autonomous Hamiltonian $F \in \mathcal{H}_c(W)$ is slow. Then the path f_t, $t \in [0, 1]$, is length minimizing in its homotopy class.*

Here, the length of f_t is defined as $\|F\|$. This result was discovered by Hofer [22] for $(\mathbb{R}^{2n}, \omega_0)$ and has been proved in [34] for weakly exact tame symplectic manifolds; it removes an additional assumption on F in [9], [44] and verifies Conjecture 1.2 in [44] for tame strongly semi-positive symplectic manifolds.

Theorems 1.1 and 1.2 show that if $e_S(A, W)$ or $e(A, W)$ is finite, then so is $c_{HZ}^{\circ}(A, W)$, and the finiteness of $c_{HZ}^{\circ}(A, W)$ implies existence of contractible periodic orbits on almost every compact regular energy level of an autonomous Hamiltonian system on A. We thus want to understand which compact subsets of a symplectic manifold V have finite (stable) displacement energy. *Every* compact subset of a symplectic manifold of the form $(V \times \mathbb{R}^2, \omega \oplus \omega_0)$ has finite displacement energy. Less obvious sufficient assumptions on A alone are collected in the following proposition essentially due to Laudenbach [35] and to Polterovich [49] and Laudenbach–Sikorav [36]. Recall that a middle-dimensional submanifold L of a symplectic manifold (V, ω) is called *Lagrangian* if ω vanishes on L.

Proposition 1.4. *Let A be a compact subset of a $2n$-dimensional symplectic manifold (V, ω).*

(i) *If A is contained in an embedded finite CW-complex X of dimension $< n$, then $e_S(A, V) < \infty$.*

(ii) *If A is contained in an n-dimensional closed submanifold M which is not Lagrangian, then $e_S(A, V) = 0$.*

(iii) *If A is strictly contained in a closed Lagrangian submanifold L, then $e_S(A, V) = 0$.*

The example $S^1 \subset (T^*S^1, \omega_0)$ shows that neither the dimension assumption in (i) nor the assumption $\omega|_M \neq 0$ in (ii) nor the assumption $A \subsetneq L$ in (iii) can be omitted. The following example will play an important role in our applications.

Example 1.5. Let σ be a non-vanishing closed 2-form on a closed manifold M and let $\omega_\sigma = \omega_0 + \pi^*\sigma$ be the twisted symplectic form on its cotangent bundle $\pi : T^*M \to M$. Then $e_S(M, T^*M, \omega_\sigma) = 0$ by Proposition 1.4 (ii). Note that if the Euler characteristic $\chi(M)$ does not vanish, then $e(M, T^*M, \omega_\sigma) = \infty$.

Theorems 1.1 and 1.2 and Proposition 1.4, which are proved in the next section, have various applications to the existence problem of periodic orbits of time-independent Hamiltonian systems. Some of them are given in Section 3 below. Further such applications as well as an application to Lagrangian intersections can be found in [52].

magnetic field in [50] with the approach in [12]. I cordially thank him for sharing his insight with me. I also thank Urs Frauenfelder and Viktor Ginzburg for their generous help, and Ely Kerman and Jean-Claude Sikorav for valuable discussions. Much of this work has been written during my stay at Tel Aviv University in April 2003, and it was finished at FIM of ETH Zürich and at Leipzig University. I wish to thank these institutions for their support, and I thank Hari and Harald and Matthias Schwarz for their warm hospitality.

2. Proofs

2.1. Proof of Theorem 1.2. We follow Polterovich's beautiful argument in [50, Section 9.A]. The proof consists of two steps.

Step 1. Curve shortening in Hofer's geometry

Curve shortening in Hofer's geometry was invented by Sikorav in [55] and further developed in [33, Proposition 2.2]. Here, we closely follow the proof of Theorem 8.3.A in [51], see also Theorem 3.3.A in [3].

We consider an arbitrary symplectic manifold (V, ω). Two Hamiltonians $H, K \in \mathcal{H}_c(I \times V)$ are *equivalent*, $H \sim K$, if $h = k$ and the paths $\{h_t\}, \{k_t\}$, $t \in [0, 1]$, are homotopic in $\mathrm{Ham}_c(V, \omega)$ with fixed end points. In other words, there exists a smooth family $\{H^s\}, s \in [0, 1]$, in $\mathcal{H}_c(I \times V)$ such that $h_t^0 = h_t$ and $h_t^1 = k_t$ for all t and $h^s = h = k$ for all s. The group of equivalence classes $\mathcal{H}_c(I \times V)/\sim$ form the universal cover $\widetilde{\mathrm{Ham}}_c(V, \omega)$ of $\mathrm{Ham}_c(V, \omega)$. We denote the lift of the Hofer norm to $\widetilde{\mathrm{Ham}}_c(V, \omega)$ by

$$\rho\,[h_t] \equiv \rho[H] := \inf\{\|K\| \mid K \sim H\}.$$

Proposition 2.1. *Consider a compact subset A of an arbitrary symplectic manifold (V, ω) such that $e(A, V) < \infty$. If $F\colon V \to \mathbb{R}$ is supported in A and $\|F\| > 4e(A, V)$, then $\rho\,[F] < \|F\|$.*

Proof. Choose a path $\{h_t\}$, $t \in [0, 1]$, in $\mathrm{Ham}_c(V, \omega)$ such that $h(A) \cap A = \emptyset$ and

$$\rho\,[h_t] \;<\; \tfrac{1}{4}\,\|F\|. \tag{3}$$

For $t \in [0, 1]$ we decompose the path f_t as

$$f_t = \left(f_{t/2} \circ h_t \circ f_{t/2} \circ h_t^{-1}\right) \circ \left(h_t \circ f_{t/2}^{-1} \circ h_t^{-1} \circ f_{t/2}\right) \equiv b_t \circ a_t.$$

As we shall see below,

$$\rho\,[a_t] < \tfrac{1}{2}\,\|F\| \quad \text{and} \quad \rho\,[b_t] \le \tfrac{1}{2}\,\|F\|. \tag{4}$$

Since $\{b_t \circ a_t\}$ is equivalent to the juxtaposition of $\{a_t\}$ and $\{b_t \circ a_1\}$ and since ρ satisfies the triangle inequality, the estimates (4) imply Proposition 2.1. In order to prove the first estimate in (4), notice that the paths $\left\{ f_{t/2}^{-1} \circ h_t^{-1} \circ f_{t/2} \right\}$ and $\left\{ f_{1/2}^{-1} \circ h_t^{-1} \circ f_{1/2} \right\}$ are equivalent and that

$$\rho\left[f_{1/2}^{-1} \circ h_t^{-1} \circ f_{1/2} \right] = \rho\left[h_t^{-1} \right] = \rho\left[h_t \right].$$

Together with the triangle inequality and the estimate (3) we can estimate

$$\begin{aligned}
\rho\left[a_t \right] &= \rho\left[h_t \circ f_{t/2}^{-1} \circ h_t^{-1} \circ f_{t/2} \right] \\
&\leq \rho[h_t] + \rho\left[f_{t/2}^{-1} \circ h_t^{-1} \circ f_{t/2} \right] \\
&= 2\rho[h_t] \\
&< \tfrac{1}{2}\|F\|.
\end{aligned}$$

To prove the second estimate in (4), notice that the path $\{b_t\} = \left\{ f_{t/2} \circ h_t \circ f_{t/2} \circ h_t^{-1} \right\}$ is equivalent to the path $\left\{ f_{t/2} \circ h \circ f_{t/2} \circ h^{-1} \right\}$ generated by the Hamiltonian

$$K(t,x) = \tfrac{1}{2}F(x) + \tfrac{1}{2}F\left(h^{-1} f_{t/2}^{-1} x \right), \quad t \in [0,1].$$

Since F is autonomous, $F = F \circ f_{t/2}$, and since h displaces $\operatorname{supp} F \subset A$, so does h^{-1}. Therefore,

$$\begin{aligned}
\|K_t\| &= \tfrac{1}{2}\left\| F + F \circ h^{-1} \circ f_{t/2}^{-1} \right\| \\
&= \tfrac{1}{2}\left\| F \circ f_{t/2} + F \circ h^{-1} \right\| \\
&= \tfrac{1}{2}\left\| F + F \circ h^{-1} \right\| \\
&= \tfrac{1}{2}\|F\|,
\end{aligned}$$

and so $\rho[b_t] \leq \tfrac{1}{2}\|F\|$. The proof of Proposition 2.1 is complete.   □

Step 2. The cut point has a non-constant contractible periodic orbit

Consider an arbitrary symplectic manifold (V, ω). We recall from the introduction that $F \in \mathcal{H}_c(V)$ is *slow* if all non-constant contractible periodic orbits of f_t have period > 1. We say that $F \in \mathcal{H}_c(V)$ is *flat* if all non-constant periodic orbits of the linearized flow of F at its critical points have period > 1.

Lemma 2.2. *Assume that (W, ω) is a tame strongly semi-positive symplectic manifold, and that the autonomous Hamiltonian $F \in \mathcal{H}_c(W)$ is slow and flat. Then the path f_t, $t \in [0, 1]$, is length minimizing in its homotopy class.*

Proof. If W is closed, this result is proved in [9], [44], see also [34]. If (W, ω) is not closed but tame, then the compactness theorems in [20], [56] hold, and so the arguments in [44] establishing compactness of the relevant Floer moduli space go through.   □

Following a suggestion by Viktor Ginzburg, we derive Theorem 1.3 from Lemma 2.2 by elementary means:

Proof of Theorem 1.3. Let $F \in \mathcal{H}_c(W)$ be slow. Arguing by contradiction, we assume that $\rho[F] < \|F\|$. Choose $\varepsilon > 0$ so small that

$$\rho[F] + 2\varepsilon < \|F\|.$$

Since F is smooth and compactly supported and by Sard's theorem, the set C of critical values of F is compact and has zero Lebesgue measure. If $F(W) = [a, b]$, we thus find finitely many intervals $[a_i, b_i] \subset [a, b] \setminus C$ such that $\sum_i (b_i - a_i) \geq (b - a) - \varepsilon$. Choose a smooth function $r \colon [a, b] \to \mathbb{R}$ such that $r(a) = a$ and such that $0 \leq r'(t) \leq 1$ for all t and

$$r'(t) = 1 \text{ if } t \in \bigcup_i [a_i, b_i] \quad \text{and} \quad r'(t) = 0 \text{ if } t \in C.$$

The function $G = r \circ F$ belongs to $\mathcal{H}_c(W)$ and is both slow and flat. Moreover,

$$\max G = r(b) \geq r(a) + (b - a) - \varepsilon = \max F - \varepsilon.$$

Since the path $\{g_t \circ f_t^{-1}\}$ is generated by $G - F = r \circ F - F$ and since $\|r \circ F - F\| = \max F - \max G \leq \varepsilon$, we have $\rho[g_t \circ f_t^{-1}] \leq \varepsilon$. Therefore,

$$
\begin{aligned}
\rho[G] &= \rho\left[g_t \circ f_t^{-1} \circ f_t\right] \\
&\leq \rho\left[g_t \circ f_t^{-1}\right] + \rho[F] \\
&\leq \varepsilon + \rho[F] \\
&< \|F\| - \varepsilon \\
&\leq \|G\|.
\end{aligned}
$$

We have constructed a slow and flat $G \in \mathcal{H}_c(W)$ with $\rho[G] < \|G\|$, in contradiction to Lemma 2.2. □

We would like to point out that the proof of Lemma 2.2 is the only place were we use a semi-positivity assumption on (W, ω). As explained in [44] the S^1-invariant virtual moduli cycle can be used to establish Lemma 2.2 for arbitrary tame symplectic manifolds. The above argument then yields Theorem 1.3 and hence Conjecture 1.2 in [44] for all tame symplectic manifolds.

End of the proof of Theorem 1.2. We can assume that $e(A, W) < \infty$, and in view of the definitions of the capacity c_{HZ}° and the displacement energy e we can assume that A is compact. Let $F \in \mathcal{H}_c(\mathrm{Int}\, A)$ be such that $\max F - \min F = \|F\| > 4e(A, W)$. According to Proposition 2.1 we have $\rho[F] < \|F\|$, and so Theorem 1.3 shows that F is not slow. Therefore, $c_{\mathrm{HZ}}^\circ(A, W) \leq 4e(A, W)$. □

2.2. Proof of Theorem 1.1. We shall derive Theorem 1.1 from Theorem 1.2 by a stabilization argument. Let $G(q, p) = \frac{1}{2}p^2$ be the Hamiltonian generating the geodesic flow on T^*S^1, and abbreviate $G^\varepsilon = \{(q, p) \mid G(q, p) \leq \varepsilon\}$.

Lemma 2.3. *For any subset A of a symplectic manifold (V, ω) and any $\varepsilon > 0$,*

$$c^\circ_{HZ}(A, V) \leq c^\circ_{HZ}\left(A \times G^\varepsilon, V \times T^*S^1\right).$$

Proof. We can assume that $\operatorname{Int} A \neq \emptyset$. Let $F \in \mathcal{H}_c(\operatorname{Int} A)$ be slow. We choose a smooth function $a \colon \mathbb{R} \to [0, 1]$ such that

$$a(t) = 1 \ \text{if} \ t \leq \tfrac{1}{3}\varepsilon \quad \text{and} \quad a(t) = 0 \ \text{if} \ t \geq \tfrac{2}{3}\varepsilon.$$

The function $F_S \colon V \times T^*S^1 \to \mathbb{R}$ given by $(v, w) \mapsto F(v)a(G(w))$ belongs to $\mathcal{H}_c(\operatorname{Int}(A \times G^\varepsilon))$. In order to see that F_S is slow, assume that $x(t)$ is a contractible periodic orbit of its Hamiltonian flow. Then $x(t) = (x_1(t), x_2(t)) \subset V \times T^*S^1$, where both $x_1(t)$ and $x_2(t)$ are contractible periodic orbits. Denoting the Hamiltonian vector fields of F and G by X_F and X_G, we find

$$\dot{x}_1(t) = a\big(G(x_2(t))\big)X_F\left(x_1(t)\right),$$
$$\dot{x}_2(t) = F(x_1(t))a'\big(G(x_2(t))\big)X_G\left(x_2(t)\right).$$

Therefore, the orbits $x_1(t)$ and $x_2(t)$ are, up to reparametrization, orbits of X_F and X_G. Since F and G are autonomous, we conclude that the functions $a\big(G(x_2(t))\big)$ and $F(x_1(t))a'\big(G(x_2(t))\big)$ are constant. Since $|a(G(x_2))| \in [0, 1]$ and F is slow, the orbit $x_1(t)$ is constant or has period > 1, and since all contractible periodic orbits of the flow of G are constant, the orbit $x_2(t)$ is constant. We have constructed for every slow $F \in \mathcal{H}_c(\operatorname{Int} A)$ a slow $F_S \in \mathcal{H}_c(\operatorname{Int}(A \times G^\varepsilon))$ with $\max F = \max F_S$. Lemma 2.3 thus follows. $\square$

In order to prove Theorem 1.1 we need to show that for every compact subset A of W,

$$c^\circ_{HZ}(A, W) \leq 4e\left(A \times S^1, W \times T^*S^1\right). \tag{5}$$

We can assume that $e(A \times S^1, W \times T^*S^1)$ is finite. Fix $\delta > 0$, and choose $H \in \mathcal{H}_c(I \times W \times T^*S^1)$ such that h displaces $A \times S^1$ and

$$\|H\| \leq e\left(A \times S^1, W \times T^*S^1\right) + \delta.$$

We then find $\varepsilon > 0$ such that h displaces $A \times G^\varepsilon$. It follows that

$$e\left(A \times G^\varepsilon, W \times T^*S^1\right) \leq \|H\| \leq e\left(A \times S^1, W \times T^*S^1\right) + \delta.$$

Since both (W, ω) and (T^*S^1, ω_0) are tame, so is their product, and since (W, ω) is stably strongly semi-positive, $(W \times T^*S^1, \omega \oplus \omega_0)$ is strongly semi-positive. Together with Lemma 2.3 and Theorem 1.2 we can thus estimate

$$\begin{aligned} c_{\mathrm{HZ}}^{\circ}(A, W) &\leq c_{\mathrm{HZ}}^{\circ}\left(A \times G^{\varepsilon}, W \times T^*S^1\right) \\ &\leq 4e\left(A \times G^{\varepsilon}, W \times T^*S^1\right) \\ &\leq 4e\left(A \times S^1, W \times T^*S^1\right) + 4\delta. \end{aligned}$$

Since $\delta > 0$ was arbitrary, inequality (5) follows, and so Theorem 1.1 is proved. $\square$

2.3. Proof of Proposition 1.4. (i) By assumption, the set $A \times S^1$ is contained in the finite CW-complex $X \times S^1$ of dimension $< n + 1$ in the $(2n + 2)$-dimensional symplectic manifold $(V \times T^*S^1, \omega \oplus \omega_0)$. Since $X \times S^1$ can be displaced from itself in $V \times T^*S^1$ by a smooth isotopy, a result of Laudenbach [35] implies that $X \times S^1$ can be displaced from itself in $(V \times T^*S^1, \omega \oplus \omega_0)$ by a Hamiltonian isotopy. It follows that $e_S(A, V) \leq e_S(X, V) < \infty$.

(ii) Consider the closed submanifold $M \times S^1$ of $V \times T^*S^1$. Since $\omega|_M \neq 0$ we have $\omega \oplus \omega_0|_{M \times S^1} \neq 0$. Moreover, the Euler characteristic of $M \times S^1$ vanishes. A result of Polterovich [49] and Laudenbach–Sikorav [36] thus implies that $e(M \times S^1, V \times T^*S^1) = 0$, and so $e_S(A, V) = 0$.

(iii) The proof of the case $n = 1$ is elementary and omitted. So assume that $n \geq 2$. Since A is compact, $L \setminus A$ is open. Using the Lagrangian Neighbourhood Theorem we easily find a closed submanifold L' of V which is not Lagrangian and such that $A \subset L'$. By assertion (ii) we have $e_S(L', V) = 0$, and so $e_S(A, V) = 0$. $\square$

3. Applications

Throughout this section, (V, ω) denotes an arbitrary symplectic manifold, while (W, ω) denotes a tame and stably strongly semi-positive symplectic manifold. We say that a compact subset A of (V, ω) is *displaceable* if there exists $h \in \mathrm{Ham}_c(V, \omega)$ such that $h(A) \cap A = \emptyset$, and we say that A is *stably displaceable* if $A \times S^1$ is displaceable in $(V \times T^*S^1, \omega \oplus \omega_0)$. Thus $A \subset V$ is (stably) displaceable if and only if $e(A, V) < \infty$ (resp. $e_S(A, V) < \infty$). Note that if A is (stably) displaceable, then an entire neighbourhood of A is (stably) displaceable.

3.1. Almost existence of closed characteristics and the Weinstein conjecture. A *hypersurface* S in a symplectic manifold (V, ω) is a smooth compact connected orientable codimension 1 submanifold of V without boundary. A closed characteristic on S is an embedded circle in S all of whose tangent lines belong to the distinguished

line bundle

$$\mathcal{L}_S = \{(x, \xi) \in TS \mid \omega(\xi, \eta) = 0 \text{ for all } \eta \in T_x S\}.$$

Examples show that $\mathcal{L}_S$ might not carry any closed characteristic, see [15], [17]. We therefore follow [26] and consider parametrized neighbourhoods of S. Since S is orientable, there exists an open neighbourhood I of 0 and a smooth diffeomorphism

$$\vartheta : S \times I \to U \subset V$$

such that $\vartheta(x, 0) = x$ for $x \in S$. We call ϑ a *thickening of* S, and we abbreviate $S_\varepsilon = \vartheta(S \times \{\varepsilon\})$. Denote by $\mathcal{P}^\circ(S_\varepsilon)$ the set of closed characteristics on S_ε which are contractible in V. The refinement of the Hofer–Zehnder argument [28, Sections 4.1 and 4.2] in [42] shows

Proposition 3.1. *For any thickening* $\vartheta : S \times I \to U \subset V$ *of a hypersurface* S *in* (V, ω) *with* $c^\circ_{\mathrm{HZ}}(U, V) < \infty$ *it holds that* $\mathcal{P}^\circ(S_\varepsilon) \neq \emptyset$ *for almost all* $\varepsilon \in I$.

Together with Theorem 1.2 we obtain

Corollary 3.2. *Assume that* S *is a stably displaceable hypersurface in* (W, ω). *Then for any stably displaceable thickening* $\vartheta : S \times I \to U \subset W$ *it holds that* $\mathcal{P}^\circ(S_\varepsilon) \neq \emptyset$ *for almost all* $\varepsilon \in I$.

In [61], Zehnder constructed a symplectic form on the 4-torus $T^4 = (\mathbb{R}/\mathbb{Z})^4$ such that none of the hypersurfaces $\{x_4 = \mathrm{const}\}$ carries a closed characteristic. The assumption in Corollary 3.2 that S is stably displaceable thus cannot be omitted.

A hypersurface S in a symplectic manifold (V, ω) is called *of contact type* if there exists a Liouville vector field X (i.e., $\mathcal{L}_X \omega = d\iota_X \omega = \omega$) which is defined in a neighbourhood of S and is everywhere transverse to S. Weinstein conjectured in [60] that every hypersurface S of contact type with $H^1(S; \mathbb{R}) = 0$ carries a closed characteristic.

Corollary 3.3. *Assume that* S *is a stably displaceable hypersurface of contact type in* (W, ω). *Then* $\mathcal{P}^\circ(S) \neq \emptyset$. *In particular, the Weinstein conjecture holds true for* S.

The Weinstein conjecture has been proved for various classes of hypersurfaces of contact type in various classes of symplectic manifolds ([57], [26], [24], [10], [25], [29], [40], [58], [38], [59], [4], [37], [39], [46]). Corollary 3.3 generalizes or complements the results in [57], [26], [10], [59], [37], where the ambient symplectic manifold is of the form $(V \times \mathbb{R}^2, \omega \oplus \omega_0)$. Under the additional assumption that (W, ω) is weakly exact and convex, Corollary 3.3 has been proved in [12].

3.2. Periodic orbits of autonomous Hamiltonian systems. We consider a smooth proper Hamiltonian F on (V, ω) which attains its minimum at 0. We abbreviate the sublevel set $F^{-1}([0, r])$ by F^r, and define $d_1(F) \in [0, \infty]$ by

$$d_1(F) = \sup\{r \in \mathbb{R} \mid F^r \text{ is stably displaceable}\}.$$

Thus $d_1(F) > 0$ if and only if $F^{-1}(0)$ is stably displaceable. Denote by $\mathscr{P}^\circ(F^{-1}(r))$ the set of non-constant periodic orbits on $F^{-1}(r)$ which are contractible in V. Since the set of critical values of F is closed and, by Sard's theorem, of Lebesgue measure zero, Corollary 3.2 yields

Corollary 3.4. *Consider a proper Hamiltonian F on (W, ω) with minimum 0, and assume that $d_1(F) > 0$. Then $\mathscr{P}^\circ(F^{-1}(r)) \neq \emptyset$ for almost all $r \in]0, d_1(F)]$.*

Discussion. 1. Recall that Corollary 3.4 becomes relevant in conjunction with Proposition 1.4 applied to $A = F^{-1}(0)$.

2. According to [17], every symplectic manifold (V, ω) of dimension $2n \geq 4$ admits a proper C^2-smooth Hamiltonian F with minimum 0 and $d_1(F) > 0$ such that for a sequence $r_k \to 0$ of regular values the levels $F^{-1}(r_k)$ carry no periodic orbit, and if $2n \geq 6$, then F can be chosen C^∞-smooth.

3. Consider a tame symplectic manifold (W^{2n}, ω) for which $[\omega]$ and c_1 vanish on $\pi_2(W)$, and assume that the proper function $F \colon W \to \mathbb{R}$ attains its minimum 0 along a closed symplectic submanifold M^{2k} of (W, ω). It has been shown in [17, Corollary 2.16] that $\mathscr{P}^\circ(F^{-1}(r)) \neq \emptyset$ for almost all $r \in]0, b(F)]$, where

$$b(F) = \sup\{r \in \mathbb{R} \mid F^r \subset B(M, F)\} \in]0, \infty] \tag{6}$$

and $B(M, F)$ is "the F-maximal symplectic ball neighbourhood of M in (W, ω)", see [17, Section 4.1] for details. For $k \in \{0, 1, \dots, \lfloor n/2 \rfloor\}$, this result is covered by Proposition 1.4 and Corollary 3.4 with $d_1(F) > 0$ instead of $b(F)$. It would be interesting to compare these two constants.

3.3. Closed trajectories of a charge in a magnetic field and a potential. Consider a closed Riemannian manifold (M, g) of dimension at least 2, and endow the cotangent bundle T^*M with the standard symplectic form $\omega_0 = \sum_i dp_i \wedge dq_i$. We fix a closed 2-form σ on M and define the twisted symplectic form ω_σ on $\pi \colon T^*M \to M$ by $\omega_\sigma = \omega_0 + \pi^*\sigma$. We also fix a function V on M with minimum 0. The flow of the Hamiltonian system

$$F_V \colon (T^*M, \omega_\sigma) \to \mathbb{R}, \quad F_V(q, p) \mapsto \tfrac{1}{2}|p|^2 + V(q),$$

describes (for example) the motion of a unit charge on (M, g) subject to the magnetic field σ and the potential V, cf. [45], [31], [14]. As before we denote by $\mathscr{P}^\circ(F_V^{-1}(r))$ the set of periodic orbits on the level $F_V^{-1}(r)$ which are contractible in T^*M and hence project to contractible closed trajectories on M.

Corollary 3.5. *Consider a closed Riemannian manifold* (M, g) *endowed with a closed 2-form* σ *which does not vanish identically, and let* V *be a potential on* M *with minimum* 0. *Then* $d_1(F_V) > 0$ *and* $\mathcal{P}^\circ(F_V^{-1}(r)) \neq \emptyset$ *for almost all* $r \in]0, d_1(F_V)]$.

Proof. It is shown in [5] that for any closed 2-form σ on a closed manifold M the symplectic manifold (T^*M, ω_σ) is tame. Since the kernel of the differential of the projection $\pi \colon T^*M \to M$ defines a Lagrangian distribution in the tangent bundle of (T^*M, ω_σ), the first Chern class vanishes, so that (T^*M, ω_σ) is stably strongly semi-positive. Moreover, F_V is proper, has minimum 0, and $F_V^{-1}(0) \subset M$; and since σ does not vanish, M is not Lagrangian. Proposition 1.4 (ii) thus yields $d_1(F_V) > 0$, and so Corollary 3.5 follows from Corollary 3.4. $\qquad\square$

Specializing to the case $V = 0$, we set $d_1(g, \sigma) = d_1(F_0)$ and denote the sphere bundle $F_0^{-1}(r)$ by E_r.

Corollary 3.6. *Consider a closed Riemannian manifold* (M, g) *endowed with a closed 2-form* σ *which does not vanish identically. Then* $d_1(g, \sigma) > 0$ *and* $\mathcal{P}^\circ(E_r) \neq \emptyset$ *for almost all* $r \in]0, d_1(g, \sigma)]$.

Discussion. 1. There has been much recent progress in the existence problem for periodic orbits of a charge in a magnetic field ([45], [31], [1], [13], [24], [14], [38], [50], [18], [30], [7], [19], [5], [17], [41], [8], [6], [12], [47]). Corollary 3.6 solves the almost existence problem at small energies. Under additional assumptions on M, g or σ, stronger results are known. We refer to [14], [52], [47] for the state of the art.

2. If σ is exact, $d_1(g, \sigma) \leq \frac{1}{2} \max_{x \in M} |\alpha(x)|^2$ for all α with $d\alpha = \sigma$, see [12]. If σ is non-exact, $d_1(g, \sigma)$ can be infinite; examples with infinite $d_1(g, \sigma)$ are non-exact closed 2-forms σ on tori, see [18], [52].

3. One cannot expect that $\mathcal{P}^\circ(E_r) \neq \emptyset$ for almost all $r > 0$ in general. Indeed, let M be a closed oriented surface of genus 2, and let g and σ either be a Riemannian metric of constant curvature -1 and its area form or the Riemannian metric and the exact 2-form constructed in [48]. Then $\mathcal{P}^\circ(E_r) = \emptyset$ for all $r \geq \frac{1}{2}$, see [14, Example 3.7] and [48].

4. Assume that M is neither a 2-sphere nor an orientable surface of genus ≥ 2. If σ is non-exact, then none of the hypersurfaces E_r in (T^*M, ω_σ) is of contact type, see e.g. [52]. Therefore, Corollary 3.6 does not follow from existence results of closed characteristics on contact type hypersurfaces.

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