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Erratum: Erratum to: On Pi-hyperbolic knots and branched coverings: (Comment. math. Helv. 74 (1999), 467-475)

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Erratum to

On π-hyperbolic knots and branched coverings

(Comment. Math. Helv. 74 (1999), 467–475)

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The aim of this note is to discuss to which extent an error found in the proof of Lemma [3, p. 473] affects the other results of the same paper and some results of a subsequent paper [4], which are based on [3].

In the proof of this lemma it is assumed that the quotient of a minimal genus equivariant Seifert surface for a knot via the action of a finite cyclic group of positive diffeomorphisms preserving the orientation of the knot is a minimal genus Seifert surface for the quotient knot. Unfortunately this is not always the case. Note that an incompressible minimum genus Seifert surface for the quotient knot lifts to an equivariant incompressible Seifert surface for the lift of the knot, not necessarily of minimum genus. Remark that incompressibility is a consequence of the equivariant loop theorem–Dehn lemma.

With the notation of [3, Lemma], let D respectively D' be a minimum genus Seifert surface (i.e. a disk) for $p_h(K)$ respectively $p_{h'}(K)$ which is equivariant by the cyclic action induced by h' and h respectively, where h and h' are periodic symmetries of K. The existence of such surfaces is proved in [7, Theorem 6]. Let F respectively F' be the equivariant Seifert surfaces for K obtained as lifts of D respectively D'. The proof of the Lemma applies if F and F' coincide and have minimum genus. This is indeed the case under the following extra assumption:

The knot K has a unique incompressible Seifert surface up to isotopy.

Under this hypothesis, one uses the fact, which was pointed out to the author by M. Boileau, that two isotopic equivariant incompressible Seifert surfaces are equivariantly isotopic. This fact is a consequence of a result of Waldhausen [8, Proposition 5.4] and its proof follows the lines of [1, Proposition 4.5] where the case of $\mathbb{Z}_2$-actions is considered. A complete proof under the hypothesis of [3, Lemma] (namely actions of finite groups of positive diffeomorphisms acting orientation preservingly on the knot) will be given in [6] where, for any pair of fixed coprime integers $n > m \geq 2$, two non equivalent π-hyperbolic knots with the same n-fold and m-fold cyclic branched covers will also be constructed. For $m = 2$ and $n \geq 3$ odd this shows that Theorem 2 is indeed false as stated in [3].

In conclusion, Theorem 2 must then read:

Theorem 2′. *Let K and K' be two π-hyperbolic and $2\pi/n$-hyperbolic knots, $n \geq 3$, and assume that K admits a unique Seifert surface up to isotopy. If K and K' have the same 2-fold and n-fold cyclic branched coverings then K and K' are equivalent, i.e. the pairs (S^3, K) and (S^3, K') are homeomorphic.*

Remark. Theorem 2 is true if one assumes that K is a fibred knot. This fact can be proved directly thanks to the existence of an equivariant fibration [2, Theorem 5.2] which projects to a fibration for the quotient knot. Since a fibred knot admits a unique fibration up to isotopy, the fibre of the quotient fibration must be a disk and the conclusion follows.

Note that Theorem 1 is false too without the above extra hypothesis, however, the proof of Proposition, page 468, implies that given $n \geq 3$, any Conway irreducible hyperbolic knot which is not π-hyperbolic is determined by its 2-fold and n-fold cyclic branched coverings.

Remark that the Lemma shows that the genus of a hyperbolic knot admitting a unique Seifert surface and which is not determined by its n-fold cyclic branched covering, $n \geq 3$, is a multiple of $(n-1)/2$ and is precisely $(n-1)(m-1)/2$ if the knot is not determined by its n-fold and m-fold cyclic branched coverings, $n > m > 2$, where n and m are necessarily coprime. These conditions on the genus, which easily imply that a hyperbolic knot with a unique Seifert surface is determined by three of its cyclic branched coverings, are not satisfied by hyperbolic knots in general, for which only a bound can be given (this was already observed by Zimmermann in [9, Corollary 2]). However, it is still true [3, end of page 469] that a hyperbolic knot is determined by its cyclic branched coverings of orders at most 4, if it is Conway irreducible, and at most 5 otherwise. This follows from the general fact (see [5] for details) that three cyclic branched coverings suffice to determine hyperbolic knots.

In [4] examples of Conway reducible hyperbolic knots with the same 2-fold and n-fold, $n \geq 3$, cyclic branched coverings were constructed. The results of [3] were used in this paper to prove that the given construction was essentially unique. This is however not the case and examples showing a different behaviour will be illustrated in a forthcoming paper by the author. Remark also that Proposition 2 of [4] is not true as stated and must be replaced by the aforementioned result of [5] and that Claim 6 and Proposition 1 hold for knots which behave as those constructed in [4] (Case B2), but not in general.

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