

**Zeitschrift:** Commentarii Mathematici Helvetici  
**Herausgeber:** Schweizerische Mathematische Gesellschaft  
**Band:** 80 (2005)

**Erratum:** Erratum to: On Pi-hyperbolic knots and branched coverings: (Comment. math. Helv. 74 (1999), 467-475)  
**Autor:** Paoluzzi, Luisa

#### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 14.01.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

**Erratum to****On  $\pi$ -hyperbolic knots and branched coverings**

(Comment. Math. Helv. 74 (1999), 467–475)

Luisa Paoluzzi

The aim of this note is to discuss to which extent an error found in the proof of Lemma [3, p. 473] affects the other results of the same paper and some results of a subsequent paper [4], which are based on [3].

In the proof of this lemma it is assumed that the quotient of a minimal genus equivariant Seifert surface for a knot via the action of a finite cyclic group of positive diffeomorphisms preserving the orientation of the knot is a minimal genus Seifert surface for the quotient knot. Unfortunately this is not always the case. Note that an incompressible minimum genus Seifert surface for the quotient knot lifts to an equivariant incompressible Seifert surface for the lift of the knot, not necessarily of minimum genus. Remark that incompressibility is a consequence of the equivariant loop theorem–Dehn lemma.

With the notation of [3, Lemma], let  $D$  respectively  $D'$  be a minimum genus Seifert surface (i.e. a disk) for  $p_h(K)$  respectively  $p_{h'}(K)$  which is equivariant by the cyclic action induced by  $h'$  and  $h$  respectively, where  $h$  and  $h'$  are periodic symmetries of  $K$ . The existence of such surfaces is proved in [7, Theorem 6]. Let  $F$  respectively  $F'$  be the equivariant Seifert surfaces for  $K$  obtained as lifts of  $D$  respectively  $D'$ . The proof of the Lemma applies if  $F$  and  $F'$  coincide and have minimum genus. This is indeed the case under the following extra assumption:

*The knot  $K$  has a unique incompressible Seifert surface up to isotopy.*

Under this hypothesis, one uses the fact, which was pointed out to the author by M. Boileau, that two isotopic equivariant incompressible Seifert surfaces are equivariantly isotopic. This fact is a consequence of a result of Waldhausen [8, Proposition 5.4] and its proof follows the lines of [1, Proposition 4.5] where the case of  $\mathbb{Z}_2$ -actions is considered. A complete proof under the hypothesis of [3, Lemma] (namely actions of finite groups of positive diffeomorphisms acting orientation preservingly on the knot) will be given in [6] where, for any pair of fixed coprime integers  $n > m \geq 2$ , two non equivalent  $\pi$ -hyperbolic knots with the same  $n$ -fold and  $m$ -fold cyclic branched covers will also be constructed. For  $m = 2$  and  $n \geq 3$  odd this shows that Theorem 2 is indeed false as stated in [3].

In conclusion, Theorem 2 must then read:

**Theorem 2'.** *Let  $K$  and  $K'$  be two  $\pi$ -hyperbolic and  $2\pi/n$ -hyperbolic knots,  $n \geq 3$ , and assume that  $K$  admits a unique Seifert surface up to isotopy. If  $K$  and  $K'$  have the same 2-fold and  $n$ -fold cyclic branched coverings then  $K$  and  $K'$  are equivalent, i.e. the pairs  $(S^3, K)$  and  $(S^3, K')$  are homeomorphic.*

**Remark.** Theorem 2 is true if one assumes that  $K$  is a fibred knot. This fact can be proved directly thanks to the existence of an equivariant fibration [2, Theorem 5.2] which projects to a fibration for the quotient knot. Since a fibred knot admits a unique fibration up to isotopy, the fibre of the quotient fibration must be a disk and the conclusion follows.

Note that Theorem 1 is false too without the above extra hypothesis, however, the proof of Proposition, page 468, implies that given  $n \geq 3$ , any Conway irreducible hyperbolic knot which is not  $\pi$ -hyperbolic is determined by its 2-fold and  $n$ -fold cyclic branched coverings.

Remark that the Lemma shows that the genus of a hyperbolic knot admitting a unique Seifert surface and which is not determined by its  $n$ -fold cyclic branched covering,  $n \geq 3$ , is a multiple of  $(n-1)/2$  and is precisely  $(n-1)(m-1)/2$  if the knot is not determined by its  $n$ -fold and  $m$ -fold cyclic branched coverings,  $n > m > 2$ , where  $n$  and  $m$  are necessarily coprime. These conditions on the genus, which easily imply that a hyperbolic knot with a unique Seifert surface is determined by three of its cyclic branched coverings, are not satisfied by hyperbolic knots in general, for which only a bound can be given (this was already observed by Zimmermann in [9, Corollary 2]). However, it is still true [3, end of page 469] that a hyperbolic knot is determined by its cyclic branched coverings of orders at most 4, if it is Conway irreducible, and at most 5 otherwise. This follows from the general fact (see [5] for details) that three cyclic branched coverings suffice to determine hyperbolic knots.

In [4] examples of Conway reducible hyperbolic knots with the same 2-fold and  $n$ -fold,  $n \geq 3$ , cyclic branched coverings were constructed. The results of [3] were used in this paper to prove that the given construction was essentially unique. This is however not the case and examples showing a different behaviour will be illustrated in a forthcoming paper by the author. Remark also that Proposition 2 of [4] is not true as stated and must be replaced by the aforementioned result of [5] and that Claim 6 and Proposition 1 hold for knots which behave as those constructed in [4] (Case B2), but not in general.

## References

- [1] M. Boileau, B. Zimmermann, The  $\pi$ -orbifold group of a link. *Math. Z.* **200** (1989), 187–208. Zbl 0663.57006 MR 0978294
- [2] A. L. Edmonds, C. Livingston, Group actions on fibered three-manifolds. *Comment. Math. Helv.* **58** (1983), 529–542. Zbl 0532.57024 MR 0728451

- [3] L. Paoluzzi, On  $\pi$ -hyperbolic knots and branched coverings. *Comment. Math. Helv.* **74** (1999), 467–475. Zbl 0942.57003 MR 1710147
- [4] L. Paoluzzi, Non-equivalent hyperbolic knots. *Topology Appl.* **124** (2002), 85–101. Zbl 1024.57006 MR 1926137
- [5] L. Paoluzzi, Three cyclic branched covers suffice to determine hyperbolic knots. To appear in *J. Knot Theory Ramifications*.
- [6] L. Paoluzzi, Conway irreducible hyperbolic knots with two common covers. Preprint.
- [7] J. L. Tollefson, Innermost disk pairs in least weight normal surfaces. *Topology Appl.* **65** (1995), 139–154. Zbl 0845.57013 MR 1355291
- [8] F. Waldhausen, On irreducible 3-manifolds which are sufficiently large. *Ann. of Math.* (2) **87** (1968), 56–88. Zbl 0157.30603 MR 0224099
- [9] B. Zimmermann, On hyperbolic knots with homeomorphic cyclic branched coverings. *Math. Ann.* **311** (1998), 665–673. Zbl 0913.57008 MR 1637960

Received May 24, 2004

Luisa Paoluzzi, IMB – UMR 5584 du CNRS, Université de Bourgogne, 9, avenue Alain Savary – BP. 47870, 21078 Dijon Cedex, France  
E-mail: paoluzzi@u-bourgogne.fr

Leere Seite  
Blank page  
Page vide