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## Erratum

**Autor:** [s.n.]

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## Errata

P. Sankaran and V. Uma

In the article “Cohomology of toric bundles” by P. Sankaran and V. Uma published in Volume 78/3 (2003), pp. 540–554 in the journal *Commentarii Mathematici Helvetici* were errors. The corrections are as follows:

We correct here the errors in our paper [6] which we found recently much to our embarrassment. The notations of [6] will be in force unless otherwise stated.

**1.** In Lemma 2.2(i) it was asserted that the elements  $z_u \in \mathcal{I} := \mathcal{I}_S$  for every  $u \in M$  the proof of which was left out as an “easy exercise”. Upon re-examining our proof we realized that it is not valid without further hypotheses! We circumvent the problem by modifying the definition of  $\mathcal{I}$  as follows so that Lemma 2.2(i) is redundant:

Assume that  $r_i, 1 \leq i \leq n$ , are invertible elements in the centre of  $S$ . Let  $\mathcal{I}$  be the (two-sided) ideal of the polynomial algebra  $S[x_1, \dots, x_d]$  generated by the following two types of elements:

$$x_{j_1} \cdots x_{j_k}, \quad 1 \leq j_p \leq d, \quad (i)$$

whenever  $v_{j_1}, \dots, v_{j_k}$  do *not* span a cone of  $\Delta$ ; for each  $u := \sum_{1 \leq i \leq n} a_i u_i \in M$ , the element

$$z_u := \prod_{j, \langle u, v_j \rangle > 0} (1 - x_j)^{\langle u, v_j \rangle} - r_u \prod_{j, \langle u, v_j \rangle < 0} (1 - x_j)^{-\langle u, v_j \rangle} \quad (ii)'$$

where  $r_u = \prod_{1 \leq i \leq n} r_i^{a_i}$ . Define  $\mathcal{R}(S, \Delta) := S[x_1, \dots, x_d]/\mathcal{I}$ .

With this definition of  $\mathcal{I}$ , Lemma 2.2(i) is a tautology. Remaining parts of Lemma 2.2 (the proofs of which used part (i)) are now valid as given in [6].

Lemma 2.2 was used in Proposition 4.3(iii). But with the corrected definition of  $\mathcal{R}$ , it continues to hold because in 4.3(ii), we established the stronger condition  $\prod [L_j]^{\langle u, v_j \rangle} = 1$ . This (together with 4.3(i)) ensures that  $x_j \mapsto (1 - [L_j]^\vee)$ ,  $1 \leq j \leq d$ , does yield a well-defined ring homomorphism  $\mathcal{R} \rightarrow K(X)$  (where  $r_i = 1$ ,  $1 \leq i \leq n$ ).

Thanks to equations (7) and (8), p. 552 of [6], the proof of Theorem 1.2(iv) is valid verbatim with this modified definition of  $\mathcal{R}$ .

**2.** In Theorem 1.2(ii), we need, besides the new definition of  $\mathcal{R}$ , that  $B$  be Hausdorff.

*Proof of Theorem 1.2(ii).* We now give a proof that  $K^*(E(X))$  is a free  $K^*(B)$ -module of rank  $m$ , the number of  $n$ -dimensional cones in  $\Delta$  where  $X = X(\Delta)$ . With notations as in §4, [6] the restriction of  $[\mathcal{L}(\tau_i)]$ ,  $1 \leq i \leq m$ , to the fibre  $X$  forms a  $\mathbb{Z}$ -basis for  $K^*(X)$ . Since  $B$  is compact Hausdorff, it is locally compact and normal. Therefore  $B$  can be covered by finitely many *compact* subsets  $W_1, \dots, W_k$  such that the bundle  $\pi|_{W_r}$  is trivial for  $1 \leq r \leq k$ . Let  $Y$  be a closed subspace of  $W_r$ . Now using the Künneth theorem for  $K$ -theory, which is also valid for general compact spaces (cf. [2]), we see that  $K^*(\pi^{-1}(Y))$  is a free  $K^*(Y)$ -module with basis  $[\mathcal{L}(\tau_i)|_{\pi^{-1}(Y)}]$ ,  $1 \leq i \leq m$ . Applying Theorem 1.3, Ch. IV, [4], we conclude that  $K^*(E(X))$  is a free  $K(B)$ -module with basis  $[\mathcal{L}(\tau_i)]$ ,  $1 \leq i \leq m$ . In view of equations (7) and (8), p. 552, [6], setting  $r_i = \pi^*(\xi_i^\vee)$ , one has a well-defined homomorphism  $\mathcal{R}(K(B), \Delta) \rightarrow K(E(X))$  of  $K(B)$  algebras defined by  $x_j \mapsto (1 - \mathcal{L}_j)$ . Rest of the proof is exactly as given in p. 552, [6].  $\square$

**3.** It was asserted after the proof of Lemma 4.2, [6], that flag varieties  $G/B$  where  $G$  is semi simple and  $B$  a Borel subgroup and smooth Schubert varieties in  $G/B$  satisfy the hypotheses of Lemma 4.2. In fact it turns out that  $H^*(G/B; \mathbb{Z})$  is not generated by  $H^2(G/B; \mathbb{Z})$  in general. This is related to the presence of torsion in the integral cohomology of the classifying space  $BG$ . (See §4 of [3].) However  $H^*(SL(n, \mathbb{C})/B; \mathbb{Z})$  is generated as an algebra by  $H^2(SL(n, \mathbb{C})/B; \mathbb{Z})$ . More importantly, the *conclusion* of Lemma 4.2 is valid for any  $G/B$ . This follows from the surjectivity of the “ $\alpha$ -construction” established by Atiyah–Hirzebruch (Theorem 5.8, [1]) and Pittie [5].

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P. Sankaran and V. Uma  
 Institute of Mathematical Sciences  
 CIT Campus  
 Chennai 600 113  
 India  
 e-mail: sankaran@imsc.res.in  
 uma@imsc.res.in