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## On the triple points of singular maps

Tobias Ekholm and András Szűcs

**Abstract.** The number of triple points (mod 2) of a self-transverse immersion of a closed  $2n$ -manifold  $M$  into  $3n$ -space are known to equal one of the Stiefel–Whitney numbers of  $M$ . This result is generalized to the case of generic (i.e. stable) maps with singularities. Besides triple points and Stiefel–Whitney numbers, a certain linking number of the manifold of singular values with the rest of the image is involved in the generalized equation which corrects an erroneous formula in [9].

If  $n$  is even and the closed manifold is oriented then the equations mentioned above make sense over the integers. Together, the integer- and mod 2 generalized equations imply that a certain Stiefel–Whitney number of closed oriented  $4k$ -manifolds vanishes. This Stiefel–Whitney number is in fact the first in a family which vanish on such manifolds.

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**Keywords.** Stable map, linking number, triple point, Stiefel–Whitney number, orientable  $4k$ -manifold.

### 1. Introduction

In his classical paper [10] of 1946, Whitney showed that the number of double points of a self-transverse immersion of an  $n$ -manifold into  $2n$ -space is related to the Euler number of its normal bundle. Since then many results of a similar nature have been found. This paper deals with a generalization of one of these results, the Herbert–Ronga formula [5] which expresses the number of triple points of a self-transverse immersion of a closed  $2n$ -manifold into  $3n$ -space in terms of one of its characteristic numbers. More precisely, the Herbert–Ronga formula is extended to singular generic (i.e. stable) maps of  $2n$ -manifolds into  $3n$ -space. (In this paper all manifolds and maps are assumed to be  $C^\infty$ -smooth, unless otherwise explicitly stated.) To state the formula, some notation is needed:

Let  $M$  be a closed  $2n$ -manifold and let  $f: M \rightarrow \mathbb{R}^{3n}$  be a generic map. If  $\Delta(f) \subset \mathbb{R}^{3n}$  denotes the set of double points of  $f$  then  $\Delta(f)$  is an immersed  $n$ -dimensional submanifold with boundary. The self-intersection points of  $\Delta(f)$  are the triple points of  $f$ . The boundary of  $\Delta(f)$  is  $\Sigma(f)$ , the set of singular values of  $f$ .

Define  $t_2(f) \in \mathbb{Z}_2$  as the mod 2-number of triple points of  $f$ . Let  $\Sigma'(f)$  denote the  $(n-1)$ -dimensional submanifold of  $\mathbb{R}^{3n}$  which is obtained by shifting  $\Sigma(f)$  slightly along its outward normal vector field in  $\Delta(f)$ . Then  $\Sigma'(f) \cap f(M) = \emptyset$ . Define  $l_2(f) \in \mathbb{Z}_2$  as the mod 2-linking number of the cycles  $f(M)$  and  $\Sigma'(f)$  in  $\mathbb{R}^{3n}$ . If  $i_1 + \dots + i_m = 2n$  then let  $\bar{w}_{i_1} \dots \bar{w}_{i_m}[M] \in \mathbb{Z}_2$  denote the product of the normal Stiefel–Whitney classes of  $M$  in dimensions  $i_1, \dots, i_m$  evaluated on the fundamental homology class of  $M$ .

**Theorem 1.** *Let  $M$  be a closed manifold of dimension  $2n$  and let  $f: M \rightarrow \mathbb{R}^{3n}$  be a generic map. Then*

$$t_2(f) + l_2(f) = \bar{w}_n^2[M] + \bar{w}_{n+1}\bar{w}_{n-1}[M] \quad (1)$$

Theorem 1 is proved in Section 2. It corrects the erroneous theorem on the second page of [9], in which the second term in the right hand side of Equation (1) is missing.

For closed oriented  $4k$ -manifolds Equation (1) can be lifted to an integer equation: If  $n = 2k$  is even and  $M$  is oriented then there is an induced orientation on  $\Delta(f)$  as well as on the triple points of  $f$ . Define  $t(f) \in \mathbb{Z}$  as the algebraic number of triple points of  $f$ . The orientation of  $\Delta(f)$  induces an orientation of its boundary  $\Sigma(f)$  which in turn induces an orientation of  $\Sigma'(f)$ . Define  $l(f) \in \mathbb{Z}$  as the linking number of the oriented cycles  $f(M)$  and  $\Sigma'(f)$  in  $\mathbb{R}^{6k}$ . Let  $\bar{p}_k[M^{4k}]$  denote the  $k^{\text{th}}$  normal Pontryagin number of  $M$ . The following theorem is Lemma 4 in [1].

**Theorem 2.** *Let  $M$  be a closed oriented manifold of dimension  $4k$  and let  $f: M \rightarrow \mathbb{R}^{6k}$  be a generic map. Then*

$$3t(f) - 3l(f) = \bar{p}_k[M]. \quad (2)$$

Equation (2) turned out to be very useful: It is used in the derivation of a geometric formula for Smale invariants of immersions of spheres, see [1] and [2], and in the study of geometric features of the regular homotopy classification of immersions of 3-manifolds in 5-space, see [7].

If  $M$  is a closed oriented  $4k$ -manifold then the mod 2-reduction of  $\bar{p}_k[M]$  equals  $\bar{w}_{2k}^2[M]$ . Hence Theorems 1 and 2 together imply that

$$\bar{w}_{2k+1}\bar{w}_{2k-1}[M] = 0 \quad (3)$$

for any closed oriented  $4k$ -manifold  $M$ . In fact,  $\bar{w}_{2k+1}\bar{w}_{2k-1}[M]$  is the first in a sequence of Stiefel–Whitney numbers which vanish on closed oriented  $4k$ -manifolds. More precisely,

**Theorem 3.** (Stong). *If  $M$  is an oriented  $4k$ -manifold and  $(2k_1 + 1) + \dots + (2k_r + 1) = 4k$  then*

$$\bar{w}_{2k_1+1} \dots \bar{w}_{2k_r+1}[M] = 0.$$

This theorem was communicated by R. Stong to the second author together with a proof of the first case (3). A proof of Theorem 3 is presented in Section 3.

### 2. Proof of Theorem 1

Fix a generic map  $f: M \rightarrow \mathbb{R}^{3n}$  of a closed  $2n$ -manifold. Let  $\tilde{\Sigma} \subset M$  denote the  $(n - 1)$ -dimensional submanifold of singular points of  $f$  and let  $\Sigma = f(\tilde{\Sigma})$ . Then  $f$  maps  $\tilde{\Sigma}$  diffeomorphically to  $\Sigma$ .

Let  $\tilde{\Delta} \subset M$  denote the closure of the preimages of multiple points of  $f$ . Then  $\tilde{\Delta}$  is an immersed closed  $n$ -dimensional manifold with transverse double points at the preimages of triple points of  $f$ . Let  $\tilde{\Delta}_{\text{res}}$  denote the resolution of  $\tilde{\Delta}$  and let  $\tilde{\iota}: \tilde{\Delta}_{\text{res}} \rightarrow M$  denote the natural immersion with image  $\tilde{\Delta} \subset M$ .

There is a natural involution  $T: \tilde{\Delta}_{\text{res}} \rightarrow \tilde{\Delta}_{\text{res}}$  such that  $f \circ \tilde{\iota} \circ T = f \circ \tilde{\iota}$ . Since no triple point of  $f$  is singular we have a natural embedding  $\tilde{\Sigma} \subset \tilde{\Delta}_{\text{res}}$  and  $\tilde{\Sigma}$  is the fix point set of  $T$ .

Let  $\nu(\tilde{\iota})$  denote the normal bundle of the immersion  $\tilde{\iota}$  and let  $\nu$  denote its restriction to  $\tilde{\Sigma}$ . Since  $\nu$  is an  $n$ -dimensional vector bundle over an  $(n - 1)$ -manifold there exists a non-zero section. Let  $\tilde{s}$  be such a section.

A standard transversality argument allows us to extend  $\tilde{s}$  to a section  $\tilde{S}$  of  $\nu(\tilde{\iota})$  which is transverse to the 0-section and which satisfies the following two conditions:

- If  $x$  is a double point of  $\tilde{\iota}$  then  $\tilde{S}(x) \neq 0$ .
- If  $\tilde{S}(x) = 0$  then  $\tilde{S}(T(x)) \neq 0$ .

Let  $\Delta \subset \mathbb{R}^{3n}$  denote the closure of the double points of  $f$ . Then  $\Delta$  is an immersed submanifold with boundary  $\Sigma$  and  $\Delta$  has triple points at the triple points of  $f$ . Let  $\Delta_{\text{res}}$  denote the resolution of  $\Delta$  and let  $\iota: \Delta_{\text{res}} \rightarrow \mathbb{R}^{3n}$  denote the natural immersion with image  $\Delta$ . Let  $\nu(\iota)$  denote the normal bundle of the immersion  $\iota$ . Note that there is a natural map  $\Pi: \tilde{\Delta}_{\text{res}} \rightarrow \Delta_{\text{res}}$  which is a double cover of  $\Delta_{\text{res}} - \Sigma$  when restricted to  $\tilde{\Delta}_{\text{res}} - \tilde{\Sigma}$ , and which maps  $\tilde{\Sigma}$  diffeomorphically onto  $\Sigma$ .

Define the section  $S$  of  $\nu(\iota)$  as follows:

$$S(y) = \begin{cases} df(\tilde{S}(y_1)) + df(\tilde{S}(y_2)) & \text{if } y \in \Delta_{\text{res}} - \Sigma, \text{ where } y_1 \neq y_2, \Pi(y_1) = \Pi(y_2) = y, \\ 2df(\tilde{S}(y_1)) & \text{if } y \in \Sigma, \text{ where } \Pi(y_1) = y. \end{cases}$$

Let  $C(\Sigma) \subset \Delta_{\text{res}}$  be a small open collar on the boundary  $\Sigma$  of  $\Delta_{\text{res}}$ . Let  $\Delta''$  denote the image of the immersion  $y \mapsto \iota(y) + \epsilon S(y)$ ,  $y \in \Delta_{\text{res}} - C(\Sigma)$  for some small  $\epsilon > 0$ . Then, if  $\epsilon$  and the collar  $C(\Sigma)$  are small enough,  $\Delta''$  is a chain with boundary  $\partial\Delta'' = \Sigma''$  satisfying  $\Sigma'' \cap f(M) = \emptyset$ . If  $\text{lk}_2$  denotes the mod 2-linking number,  $\bullet$  denotes the mod 2-intersection number, and  $\sharp(F)$  denotes the mod 2-number of elements in the finite set  $F$ , then

$$\text{lk}_2(\Sigma'', f(M)) = \Delta'' \bullet f(M) = \sharp(\tilde{S}^{-1}(0)) + t_2(f), \tag{4}$$

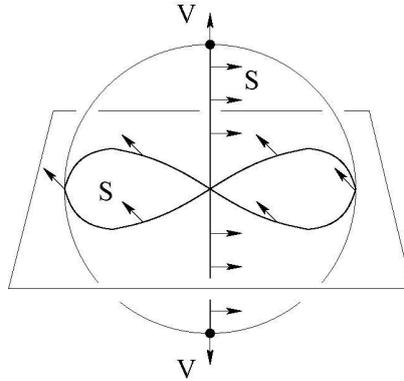


Figure 1. A piece of  $f(M)$  (represented by a 2-sphere and a piece of a plane) with the double point set  $\Delta$  (fat lines), its normal field  $S$ , and singularity set  $\Sigma$  (dots) with its outward normal field  $V$  in  $\Delta$ .

since near each zero  $z$  of  $\tilde{S}$  there is a unique intersection point of  $\Delta''$  and  $f(M)$  near  $f(z)$ , and near each triple point of  $f$  there are exactly three such intersection points.

The homology class of the cycle  $\tilde{\Delta}$  in  $M$  is Poincaré dual to  $n^{\text{th}}$  normal Stiefel-Whitney class  $\bar{w}_n$  of  $M$ , see [6]. Thus

$$\bar{w}_n^2[M] = \tilde{\Delta} \bullet \tilde{\Delta} = \sharp(\tilde{S}^{-1}(0)), \tag{5}$$

since the image of a slight shift of the immersion  $\tilde{t}$  along  $\tilde{S}$  intersects  $\tilde{\Delta}$  near each zero of  $\tilde{S}$  and in *two* points near each double point of  $\tilde{t}$ .

Equations (4) and (5) imply

$$\text{lk}_2(\Sigma'', f(M)) = \bar{w}_n^2[M] + t_2(f). \tag{6}$$

Recall that  $\Sigma' \subset \mathbb{R}^{3n}$  is the submanifold which results when  $\Sigma$  is shifted slightly along its unit outward normal vector field  $V$  in  $\Delta$ , and that  $\Sigma' \cap f(M) = \emptyset$ . We compare the linking numbers  $\text{lk}_2(\Sigma'', f(M))$  and  $\text{lk}_2(\Sigma', f(M))$ :

Let  $\tilde{\Sigma}_0 \subset M$  be the submanifold which results when  $\tilde{\Sigma}$  is shifted a small distance along  $\tilde{S}$ . Let  $\Sigma_0 = f(\tilde{\Sigma}_0)$  and for  $p \in \Sigma$ , let  $p_0 = f(\tilde{p}_0)$  where  $\tilde{p}_0$  is the point in  $\tilde{\Sigma}_0$  corresponding to  $\tilde{p} \in \tilde{\Sigma}$  with  $f(\tilde{p}) = p$ .

For small  $\epsilon > 0$  and  $p \in \Sigma$  let  $l_p(\epsilon)$  be the segment of the straight line through  $p + \epsilon V(p)$  and  $p_0$  of length  $2\epsilon$  and centered at  $p_0$ . For  $\epsilon > 0$  and the shifting of  $\tilde{\Sigma}$  in  $M$  small enough,

$$\Gamma = \bigcup_{p \in \Sigma} l_p(\epsilon)$$

is a submanifold of  $\mathbb{R}^{3n}$ . If the collar  $C(\Sigma)$  is chosen small enough and if the

shifting distance along  $S$  is small enough then the boundary  $\partial\Gamma$  of  $\Gamma$  is isotopic to  $\Sigma' \cup \Sigma''$  in  $\mathbb{R}^{3n} - f(M)$ . Thus

$$\text{lk}_2(\Sigma', f(M)) = \text{lk}_2(\Sigma_0, f(M)) + \Gamma \bullet f(M) = \text{lk}_2(\Sigma'', f(M)) + \Gamma \bullet f(M). \quad (7)$$

We compute  $\Gamma \bullet f(M)$ : The intersection  $\Gamma \cap f(M)$  is a clean intersection. That is,  $\Gamma \cap f(M) = \Sigma_0$  is a manifold and the tangent bundle

$$T\Sigma_0 = T f(M) \cap T\Gamma \subset T\mathbb{R}^{3n}, \quad (8)$$

where all bundles in the left hand side are restricted to  $\Sigma_0$ .

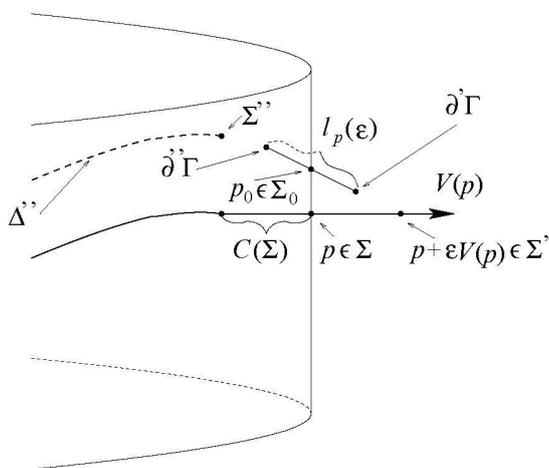


Figure 2. The normal space of  $\Sigma$  in  $\mathbb{R}^{3n}$  at  $p \in \Sigma$ . In the figure the boundary of  $\Gamma$  is the union of  $\partial'\Gamma$ , isotopic to  $\Sigma'$  in  $\mathbb{R}^{3n} - f(M)$ , and  $\partial''\Gamma$  isotopic to  $\Sigma''$  in  $\mathbb{R}^{3n} - f(M)$ .

As in [4], we find

$$\Gamma \bullet f(M) = w_{n-1}(\xi),$$

where  $\xi$  is the so called excess bundle over  $\Sigma_0$ :

$$\xi = T\mathbb{R}^{3n} / (T\Gamma + T f(M)),$$

where all bundles are restricted to  $\Sigma_0$ .

To finish the proof it remains to calculate  $w_{n-1}(\xi)$ . Note that

$$T\Gamma|_{\Sigma_0} = T\Sigma_0 \oplus \epsilon^1,$$

where  $\epsilon^1$  is the trivial line bundle directed along the intervals  $l_p(\epsilon)$ . Thus, by (8),

$$\xi \oplus T f(M)|_{\Sigma_0} \oplus \epsilon^1 = T\mathbb{R}^{3n}|_{\Sigma_0}. \quad (9)$$

The bundle  $T f(M)|_{\Sigma_0}$  is identified with  $TM|_{\tilde{\Sigma}_0}$  by the differential of  $f$ . Hence if  $i_0: \tilde{\Sigma}_0 \rightarrow M$  denotes the inclusion then  $w(\xi) = i_0^* \bar{w}(M)$ . Therefore, if  $F_V$  denotes

the fundamental homology class of the manifold  $V$  and PD denotes the Poincaré duality operator,

$$\begin{aligned} \langle w_{n-1}(\xi), F_{\Sigma_0} \rangle &= \langle i_0^* \bar{w}(M), F_{\Sigma_0} \rangle = \langle \bar{w}(M), i_{0*}(F_{\Sigma_0}) \rangle = \langle \bar{w}(M), \text{PD } \bar{w}_{n+1}(M) \rangle \\ &= \langle \bar{w}(M) \cup \bar{w}_{n+1}(M), F_M \rangle = \bar{w}_{n-1} \bar{w}_{n+1}[M]. \end{aligned} \tag{10}$$

Here, the third equality follows from the well-known formula  $\text{PD } \bar{w}_{n+1}(M) = i_* F_{\tilde{\Sigma}}$ , where  $i: \tilde{\Sigma} \rightarrow M$  denotes the inclusion, together with  $i_* F_{\tilde{\Sigma}} = i_{0*} F_{\Sigma_0}$ . Equations (6), (7), and (10) prove the theorem.  $\square$

### 3. Proof of Theorem 3

Let  $\mathfrak{N}_*$ ,  $\Omega_*$ , and  $\Omega_*^U$  denote the cobordism ring, the oriented cobordism ring, and the complex cobordism ring, respectively. Note that there are natural forgetting homomorphisms

$$\Omega_*^U \longrightarrow \Omega_* \longrightarrow \mathfrak{N}_*.$$

For a manifold  $M$ , let  $[M]$  denote its cobordism class.

Using some facts from cobordism theory which can all be found in Chapter 4 of Stong's book [8], we show that it is enough to prove the theorem for oriented  $4k$ -manifolds  $M$  such that either

- (a)  $[M] \in \Omega_{4k}$  maps to a square  $[N \times N] \in \mathfrak{N}_{4k}$ , or
- (b)  $[M]$  is a torsion element of  $\Omega_{4k}$  (in fact,  $[M]$  torsion implies  $2 \cdot [M] = 0$ ):

Let  $\text{Tors}(\Omega_*)$  denote the torsion subgroup of  $\Omega_*$ . The homomorphism  $\Omega_*^U \rightarrow \Omega_*$  induces an epimorphism

$$\Omega_*^U \longrightarrow \Omega_* / \text{Tors}(\Omega_*).$$

and the image  $\Omega_*^U \rightarrow \mathfrak{N}_*$  consists of squares of elements in  $\mathfrak{N}_*$ .

Hence, if  $M$  is any oriented  $4k$ -manifold then there exists some oriented  $4k$ -manifold  $V$  such that  $[V]$  is torsion in  $\Omega_{4k}$  and  $[M] + [V] = [N \times N]$  in  $\mathfrak{N}_{4k}$ . This implies that the theorem follows once it is proved for manifolds satisfying (a) or (b) above.

First consider (a): let  $M = N \times N$ . Then  $\bar{w}(M) = \bar{w}(N) \times \bar{w}(N)$  and hence

$$\bar{w}_{2k+1}(M) = \sum_{i+j=2k+1} \bar{w}_i(N) \times \bar{w}_j(N).$$

Thus

$$\begin{aligned} &\langle \bar{w}_{2k_1+1}(M) \dots \bar{w}_{2k_r+1}(M), F_M \rangle = \\ &= \sum \langle \bar{w}_{i_1}(N) \dots \bar{w}_{i_r}(N), F_N \rangle \cdot \langle \bar{w}_{j_1}(N) \dots \bar{w}_{j_r}(N), F_N \rangle. \end{aligned} \tag{11}$$

Since  $i_s + j_s$  is odd for all  $i_s, j_s$  there is a fixed point free involution  $T$  acting on

the set of the terms in the sum in (11) such that  $i_1 + \dots + i_r = 2k = j_1 + \dots + j_r$ :

$$\begin{aligned} T: & \langle \bar{w}_{i_1}(N) \dots \bar{w}_{i_r}(N), F_N \rangle \cdot \langle \bar{w}_{j_1}(N) \dots \bar{w}_{j_r}(N), F_N \rangle \\ & \mapsto \langle \bar{w}_{j_1}(N) \dots \bar{w}_{j_r}(N), F_N \rangle \cdot \langle \bar{w}_{i_1}(N) \dots \bar{w}_{i_r}(N), F_N \rangle. \end{aligned}$$

Thus the terms in the left hand side of (11) which does not vanish for dimensional reasons cancel in pairs and hence  $\bar{w}_{2k_1+1} \dots \bar{w}_{2k_r+1}[M] = 0$ .

Next consider (b): let  $u: \mathfrak{N}_{4k} \rightarrow \mathbb{Z}_2$  denote the homomorphism induced by the product of odd-dimensional normal Stiefel–Whitney classes  $\bar{w}_{2k_1+1} \dots \bar{w}_{2k_r+1}$ ,  $\sum 2k_j + 1 = 4k$ . Odd-dimensional Stiefel–Whitney classes are mod 2-reductions of twisted integer classes, see [3], p. 140. Hence, a product of an even number of such classes is an integer class so the map

$$\Omega_{4k} \xrightarrow{\pi} \mathfrak{N}_{4k} \xrightarrow{u} \mathbb{Z}_2$$

lifts to a homomorphism

$$\Omega_{4k} \xrightarrow{U} \mathbb{Z}.$$

Thus  $U$  and therefore  $u \circ \pi$  is zero on any torsion element of  $\Omega_{4k}$ .  $\square$

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