

**Zeitschrift:** Commentarii Mathematici Helvetici  
**Herausgeber:** Schweizerische Mathematische Gesellschaft  
**Band:** 74 (1999)

**Artikel:** Isotopy and invariants of Albert algebras  
**Autor:** Thakur, Maneesh L.  
**DOI:** <https://doi.org/10.5169/seals-55787>

#### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 02.02.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## Isotopy and invariants of Albert algebras

Maneesh L. Thakur

**Abstract.** Let  $k$  be a field with characteristic different from 2 and 3. Let  $B$  be a central simple algebra of degree 3 over a quadratic extension  $K/k$ , which admits involutions of second kind. In this paper, we prove that if the Albert algebras  $J(B, \sigma, u, \mu)$  and  $J(B, \tau, v, \nu)$  have same  $f_3$  and  $g_3$  invariants, then they are isotopic. We prove that for a given Albert algebra  $J$ , there exists an Albert algebra  $J'$  with  $f_3(J') = 0$ ,  $f_5(J') = 0$  and  $g_3(J') = g_3(J)$ . We conclude with a construction of Albert division algebras, which are pure second Tits' constructions.

**Mathematics Subject Classification (1991).** Primary 17C40.

**Keywords.** Jordan algebras, isotopy, Albert algebras, invariants.

### Introduction

Let  $k$  be a field with characteristic different from 2 and 3. The exceptional central simple Jordan algebras over  $k$  are now called *Albert algebras*. There are rational constructions of Albert algebras due to Tits. These are referred to as the *first Tits' construction* and the *second Tits' construction*. In the first construction, one associates to a pair  $(A, \mu)$ , where  $A$  is a degree 3 central simple algebra over  $k$  and  $\mu \in k^*$ , an Albert algebra  $J(A, \mu)$  over  $k$ . For the second construction, one starts with a quadratic extension  $K/k$  and a degree 3 central simple algebra  $B$  over  $K$  with an involution  $\sigma$  of second kind. To any unit  $u \in B$  with  $\sigma(u) = u$  and  $N(u) = \mu\bar{\mu}$  for some  $\mu \in K$ , one associates an Albert algebra  $J(B, \sigma, u, \mu)$  over  $k$  (cf. [P-R 1]). There are cohomological invariants attached to these algebras. Let  $J$  be an Albert algebra over  $k$ . One assigns two mod 2 invariants to  $J$ ,  $f_3(J) \in H^3(k, \mathbb{Z}/2)$ ,  $f_5(J) \in H^5(k, \mathbb{Z}/2)$  and a mod 3 invariant  $g_3(J) \in H^3(k, \mathbb{Z}/3)$  (cf. [P-R 1]). Serre asked whether these invariants determine the isomorphism class of  $J$ . The question is known to have affirmative answer for the *reduced* Albert algebras (cf. [P-R 1]). It was proved in ([P-S-T]) that if the Albert algebras  $J(B, \sigma, u, \mu)$  and  $J(B, \sigma, u', \mu')$  have the same invariants  $f_3$  and  $g_3$ , then they are isomorphic. In this direction, Petersson and Racine had asked a weaker question ([P-R 1]), namely, if two Albert algebras have same  $f_3$  and  $g_3$ , are they isotopic? In this paper, we answer this question for the Albert algebras  $J(B, \sigma, u, \mu)$  and

$J(B, \tau, v, \nu)$ , in the affirmative (§2). The same authors had asked in another paper ([P-R 2]), whether for a given Albert algebra  $J$ , there exists an Albert algebra  $J'$  with  $f_3(J') = 0$ ,  $f_5(J') = 0$  and  $g_3(J') = g_3(J)$ . We answer this question in the affirmative in (§3). We end with a construction of pure second Tits' construction Albert division algebras over a field  $k$  (§4).

## 1. Preliminaries

### 1.1. Isotopes of Jordan algebras

Let  $J$  be a Jordan algebra over  $k$  with 1. Let  $a \in J$  be an invertible element. One defines a new multiplication on  $J$  by

$$x_a y = \{xay\},$$

where  $\{xyz\}$  is the Jordan triple product in  $J$ , given by

$$\{xyz\} = U_{x,z}(y),$$

$$U_{x,z} = R_x R_z + R_z R_x - R_{xz},$$

$R_x$  denoting the homothety on  $J$  given by  $x$ . The algebra  $J$ , with this new multiplication, is a Jordan algebra (cf. [J]), called the  $a$ -isotope of  $J$ . It is denoted by  $J^{(a)}$ . Two Jordan algebras  $J_1$  and  $J_2$  are *isotopic* if  $J_1^{(a)}$  is isomorphic to  $J_2$  for some invertible  $a \in J_1$ . Isotopy is an equivalence relation on the class of Jordan algebras (cf. [J]).

### 1.2. Constructions of Albert algebras

In the following, we give a brief review of the Tits' constructions and the Freudenthal's construction of Albert algebras.

**Tits' first construction:** Let  $A$  be a central simple  $k$ -algebra of degree 3. Let  $\mu \in k^*$ . On the  $k$ -vector space

$$J(A, \mu) = A_0 \oplus A_1 \oplus A_2, \text{ where } A_i = A \text{ for } i = 0, 1, 2,$$

we define a multiplication by

$$\begin{aligned} (a_0, a_1, a_2)(a'_0, a'_1, a'_2) \\ = (a_0 a'_0 + \widetilde{a_1 a'_2} + \widetilde{a'_1 a_2}, \widetilde{a_0 a'_1} + \widetilde{a'_0 a_1} + \mu^{-1} a_2 \times a'_2, a_2 \widetilde{a'_0} + a'_2 \widetilde{a_0} + \mu a_1 \times a'_1). \end{aligned}$$

Here, for  $a, b \in A$ ,

$$a.b = \frac{1}{2}(ab + ba), \quad a \times b = a.b - \frac{1}{2}t(a)b - \frac{1}{2}t(b)a + \frac{1}{2}(t(a)t(b) - t(a.b))$$

and  $\tilde{a} = \frac{1}{2}(t(a) - a)$ ,  $t$  being the reduced trace on  $A$ . It is known that  $J(A, \mu)$  is an Albert algebra. Further, it is a division algebra if and only if  $A$  is a division algebra and  $\mu$  is not a reduced norm from  $A$ .

**Tits' second construction:** Let  $K$  be a quadratic extension of  $k$  and let  $(B, \sigma)$  be a central simple  $K$ -algebra of degree 3 with an involution  $\sigma$  of the second kind over  $k$ . Let  $u \in B^*$  be such that  $\sigma(u) = u$  and  $N(u) = \mu\bar{\mu}$  for some  $\mu \in K^*$ , bar denoting the nontrivial  $k$ -automorphism of  $K$ . Let  $(B, \sigma)_+$  denote the  $k$ -vector space of  $\sigma$ -symmetric elements in  $B$ . Let  $J(B, \sigma, u, \mu) = (B, \sigma)_+ \oplus B$ . We define a multiplication on  $J(B, \sigma, u, \mu)$  by

$$(b_0, b)(b'_0, b') = (b_0.b'_0 + b\widetilde{u\sigma}(b') + b'\widetilde{u\sigma}(b), \tilde{b}_0b' + \tilde{b}'b + \bar{\mu}(\sigma(b) \times \sigma(b'))u^{-1}).$$

Then  $J(B, \sigma, u, \mu)$  is known to be an Albert algebra and

$$J(B, \sigma, u, \mu) \otimes_k K \simeq J(B, \mu)$$

over  $K$  (cf. [MC]). Further,  $J(B, \sigma, u, \mu)$  is a division algebra if and only if  $B$  is a division algebra and  $\mu$  is not a reduced norm from  $B$ .

**Freudenthal's construction:** Let  $C$  be a Cayley algebra over  $k$  and let  $\Gamma = \langle \gamma_1, \gamma_2, \gamma_3 \rangle$  be a diagonal invertible matrix with  $\gamma_i \in k$ . Let  $M_3(C)$  denote the algebra of  $3 \times 3$  matrices with entries in  $C$ . The map  $X \mapsto \Gamma^{-1}\bar{X}^t\Gamma$  stabilizes  $M_3(C)$ , where  $\bar{X}$  is the matrix obtained by applying the involution bar on  $C$  to the entries of  $X$ . Let

$$\mathcal{H}_3(C, \Gamma) = \{X \in M_3(C) \mid \Gamma^{-1}\bar{X}^t\Gamma = X\}.$$

This is closed under the multiplication  $X \cdot Y = \frac{1}{2}(XY + YX)$  and is known to be an Albert algebra (cf. [J]). These are the so called *reduced* Albert algebras.

### 1.3. Cohomological invariants of Albert algebras

Let  $k$  be as before. Let  $J$  be an Albert algebra over  $k$ . It is a fact that  $J$  carries a linear trace form  $T$  defined on it (cf. [J]) and this gives rise to a quadratic form  $Q_J$  on  $J$  given by

$$Q_J(x) = \frac{1}{2}T(x^2).$$

There exists a 3-fold Pfister form  $\phi_3$  and a 5-fold Pfister form  $\phi_5$  over  $k$  such that

$$Q_J \perp \phi_3 \simeq \langle 2, 2, 2 \rangle \perp \phi_5$$

over  $k$  (cf. [S]). Further, this property characterizes  $\phi_3$  and  $\phi_5$  upto isometry. For an  $n$ -fold Pfister form  $\phi_n = \langle \langle a_1, a_2, \dots, a_n \rangle \rangle = \langle 1, -a_1 \rangle \otimes \dots \otimes \langle 1, -a_n \rangle$ , one has the Arason invariant  $A(\phi_n) \in H^n(k, \mathbb{Z}/2)$  given by

$$A(\phi_n) = (-a_1) \cup (-a_2) \cup \dots \cup (-a_n),$$

where, for  $a \in k^*$ ,  $(a)$  denotes the class of  $a$  in  $H^1(k, \mathbb{Z}/2)$ . The mod 2 invariants for  $J$  are defined as

$$f_3(J) = A(\phi_3), \quad f_5(J) = A(\phi_5).$$

If  $J = \mathcal{H}_3(C, \Gamma)$  then  $f_3(J) = A(n_C)$  and  $f_5(J) = A(<1, \gamma_1^{-1}\gamma_2 > \otimes <1, \gamma_2^{-1}\gamma_3 > \otimes n_C)$ , where  $n_C$  is the norm on the Cayley algebra  $C$ , which is known to be a 3-fold Pfister form. Rost ([R]) attached an invariant mod 3 to  $J$ , denoted by  $g_3(J)$ , which is defined as follows. If  $J = J(B, \sigma, u, \mu)$  for some central simple algebra  $B$  of degree 3 over a quadratic field extension  $K$  of  $k$ , with an involution of second kind, then define

$$g_3(J) = -Cor_{K/k}([B] \cup [\mu]) \in H^3(k, \mathbb{Z}/3),$$

and if  $J = J(A, \nu)$  for a central simple algebra  $A$  of degree 3 over  $k$ , then define

$$g_3(J) = ([A] \cup [\nu]) \in H^3(k, \mathbb{Z}/3).$$

These are independent of the expression of  $J$  as a first or a second Tits' construction (cf. [R], [P-R 3]). Rost showed ([R]) that  $J$  is a division algebra if and only if  $g_3(J) \neq 0$ . Further,  $g_3$  is compatible with base change.

## 2. Classification of Albert algebras upto isotopy

The question, whether the invariants  $f_3$  and  $g_3$  classify a given Albert algebra upto isotopy, itself is an important question. We answer this question in a particular case of second Tits' construction. Namely,

**Theorem 2.1.** *Let  $K$  be a quadratic extension of  $k$  and let  $B$  denote a central simple algebra of degree 3 over  $K$ , which admits involutions of second kind over  $k$ . Let  $J = J(B, \sigma, u, \mu)$  and  $J' = J(B, \sigma', u', \mu')$  be second Tits' construction Albert algebras. Assume that  $f_3(J) = f_3(J')$  and  $g_3(J) = g_3(J')$ . Then  $J$  and  $J'$  are isotopic.*

We need the following result. We supply a proof for the sake of completeness.

**Theorem 2.2.** ([P-R 1]). *Let  $J_1$  and  $J_2$  be two Albert algebras over  $k$  which are isotopic. Then  $f_3(J_1) = f_3(J_2)$  and  $g_3(J_1) = g_3(J_2)$ .*

*Proof.* Since isotopic first Tits' constructions Albert algebras are isomorphic ([P-R 4], 4.9), we may assume that  $J_1$  and  $J_2$  are both second Tits' constructions. Let  $J_1 = J(B, \sigma, u, \mu)$  and let  $K$  be the centre of  $B$ . There is a cubic extension  $L$  of  $k$  such that  $J_1 \otimes_k L$  is reduced. Since  $J_1$  is isotopic to  $J_2$ , for some invertible  $v \in J_1$  we have,

$$J_1^{(v)} \simeq J_2.$$

Thus

$$(J_1 \otimes_k K)^{(v)} \simeq J_2 \otimes_k K.$$

But  $J_1 \otimes_k K \simeq J(B, \mu)$  over  $K$  (cf. [MC]), and using the fact that isotopic first Tits' Albert algebras are isomorphic ([P-R 4], 4.9), we get

$$J_1 \otimes_k K \simeq J_2 \otimes_k K$$

over  $K$ . This proves  $g_3(J_1) = g_3(J_2)$ .

To compare the  $f_3$  invariant, we appeal to the following

**Theorem 2.3.** (cf. [F], 1.9) *Let  $J$  be an Albert algebra isotopic to  $\mathcal{H}_3(C, \Gamma)$ ,  $C$  a Cayley algebra over  $k$  and  $\Gamma \in GL_3(k)$  a diagonal matrix. Then there is an isomorphism of  $J$  onto  $\mathcal{H}_3(C, \Gamma')$  for some  $\Gamma' \in GL_3(k)$ , a diagonal matrix.*

Now we base change to  $L$  to reduce  $J_1$ , so that over  $L$  we have,

$$(J_1 \otimes_k L)^{(v)} \simeq J_2 \otimes_k L.$$

Let  $J_1 \otimes_k L \simeq \mathcal{H}_3(C, \Gamma)$  for some Cayley algebra  $C$  over  $L$  and  $\Gamma$  defined over  $L$ . By the above theorem,  $J_2 \otimes_k L \simeq \mathcal{H}_3(C, \Gamma')$  for some  $\Gamma'$ . By a theorem of Serre and Rost (cf. [P-R 5], 1.8), there exists a Cayley algebra  $\mathcal{O}$  over  $k$  such that  $\mathcal{O} \otimes_k L \simeq C$ . Now, by the definition of  $f_3$ , it follows that  $f_3(J_1) = f_3(J_2)$ .

For the proof of Theorem 2.1, we need the following

**Theorem 2.4.** ([P-R 5], 1.5). *Let  $(B, \sigma)$  be as above. Let  $v \in B^*$  be such that  $\sigma(v) = v$ . Let  $\sigma' = Int(v)\sigma$ . Then the map*

$$(a_o, a) \mapsto (v^{-1}a_0, a)$$

*is an isomorphism from  $J(B, \sigma', uv^\#, N(v)\mu)$  onto  $J(B, \sigma, u, \mu)^{(v)}$ .*

**Theorem 2.5.** ([P-S-T], 2.8) *Let  $J = J(B, \sigma, u, \mu)$  and  $J' = J(B, \sigma', u', \mu')$  be Albert algebras arising from Tits' second construction. Assume  $f_3(J) = f_3(J')$  and  $g_3(J) = g_3(J')$ . Then  $J$  is isomorphic to  $J'$ .*

*Proof of Theorem 2.1.* We have  $J = J(B, \sigma, u, \mu)$  and  $J' = J(B, \sigma', u', \mu')$ . Let  $v \in (B, \sigma)_+$  be such that  $Int(v)\sigma = \sigma'$ . Then, by Theorem 2.2,

$$f_3(J) = f_3(J^{(v)}), \quad g_3(J) = g_3(J^{(v)}).$$

Now, invoking Theorem 2.4, we have,

$$J^{(v)} \simeq J(B, \sigma', uv^\#, N(v)\mu).$$

Thus

$$f_3(J) = f_3(J(B, \sigma', uv^\#, N(v)\mu)), \quad g_3(J) = g_3(J(B, \sigma', uv^\#, N(v)\mu)).$$

Therefore, by Theorem 2.5, we get

$$J(B, \sigma', u', \mu') \simeq J(B, \sigma', uv^\#, N(v)\mu) \simeq J(B, \sigma, u, \mu)^{(v)}.$$

Hence  $J$  and  $J'$  are isotopic.

### 3. Albert algebras with trivial mod 2 invariants

In this section, we construct, for a given Albert algebra  $J$ , an Albert algebra  $J'$  with  $f_3(J') = 0$ ,  $f_5(J') = 0$  and  $g_3(J') = g_3(J)$ . We begin with reviewing some results on involutions of second kind. Let  $K/k$  be a quadratic extension. Let  $(B, \sigma)$  be a central simple algebra of degree 3 over  $K$  with an involution  $\sigma$  of second kind over  $k$ . The restriction  $Q_\sigma$  of the trace quadratic form  $x \mapsto T(x^2)$  to  $(B, \sigma)_+$ , the  $k$ -space of  $\sigma$ -symmetric elements in  $B$ , is a quadratic form with values in  $k$ . It is shown in ([H-K-R-T]) that  $Q_\sigma$  is an invariant of  $\sigma$ . The decomposition of  $Q_\sigma$  is given by the following

**Theorem 3.1.** ([H-K-R-T], 4) *Let  $K = k(\sqrt{\alpha})$ . Then there exist  $b, c \in k^*$  such that*

$$Q_\sigma \simeq \langle 1, 1, 1 \rangle \perp \langle 2 \rangle \ll \alpha \gg \ll -b, -c, bc \gg.$$

In the same paper, it is shown that the Arason invariant of the 3-fold Pfister form  $\ll \alpha, b, c \gg$  determines the isomorphism class of  $\sigma$ . More precisely,

**Proposition 3.2.** ([H-K-R-T], 15) *The following statements are equivalent for  $(B, \sigma)$ , with  $B$  as above.*

- (1)  $\sigma \simeq \sigma'$ .
- (2)  $A(\ll \alpha, b, c \gg) = A(\ll \alpha, b', c' \gg)$ . Where  $b', c'$  are the elements of  $k^*$ , corresponding to the decomposition of  $Q_{\sigma'}$ .

The invariant  $A(\ll \alpha, b, c \gg) \in H^3(k, \mathbb{Z}/2)$  is also denoted by  $f_3(B, \sigma)$ . An involution  $\sigma$  on  $B$  is called *distinguished* if  $f_3(B, \sigma) = 0$ . The following result from ([H-K-R-T]) will be needed.

**Proposition 3.3.** ([H-K-R-T], 17) *On any central simple algebra  $B$  of degree 3 over  $K$ , with an involution of second kind, there exists a distinguished involution.*

We now come to the promised construction. We note first that we need only consider the case of second Tits' construction Albert algebras.

**Theorem 3.4.** *Let  $J = J(B, \sigma, u, \mu)$  be an Albert algebra,  $B$  a degree 3 central simple algebra over a quadratic extension  $K/k$  and with an involution of second kind. There exists an Albert algebra  $J'$  with  $f_3(J') = 0$ ,  $f_5(J') = 0$  and  $g_3(J') = g_3(J)$ .*

*Proof.* By ([K-M-R-T], 39.2), we may assume that  $N(u) = 1 = \mu\bar{\mu}$ , bar denoting the nontrivial  $k$ -automorphism of  $K$ . Let  $\sigma'$  be a distinguished involution on  $B$  (3.3). Set  $J' = J(B, \sigma', 1, \mu)$ . Then by ([K-M-R-T], 40.2), we have  $f_3(J') = f_3(B, \sigma') = 0$  and  $f_5(J')$ , being a multiple of  $f_3(J')$ , is zero as well. Further,  $g_3(J) = -\text{Cor}_{K/k}([B] \cup [\mu]) = g_3(J')$ . This completes the proof.

#### 4. Pure second Tits' construction Albert algebras.

In this brief section, we exhibit how one can construct pure second Tits' construction Albert division algebras. We recall (cf. [P-R 6]) that an Albert algebra  $J$  over  $k$  is called a *pure second Tits' construction* if it does not arise from Tits' first construction.

Let  $B$  be a central division algebra of degree 3 over a quadratic extension  $K/k$ . Assume that  $B$  admits involutions of second kind. Assume further that  $\sigma$  is an involution on  $B$  which is not distinguished. In the terms of the invariant mod 3 associated to  $(B, \sigma)$ , this means that  $f_3(B, \sigma) \neq 0$ . Let  $\mu \in K^*$  be such that  $\mu\bar{\mu} = 1$  and  $\mu$  is not a reduced norm from  $B$ . Set  $J = J(B, \sigma, 1, \mu)$ . Then  $J$  is an Albert division algebra over  $k$ . Further, by ([K-M-R-T], 40.2),  $f_3(J) = f_3(B, \sigma) \neq 0$ . Thus by ([K-M-R-T], 40.5),  $J$  is a pure second Tits' construction. We record this as

**Theorem 4.1.** *Let  $(B, \sigma)$  be a central division algebra of degree 3 over  $K$ , with an involution  $\sigma$  of second kind. Assume that  $f_3(B, \sigma) \neq 0$ . Let  $\mu \in K^*$  be such that  $\mu$  is not a reduced norm from  $B$  and  $\mu\bar{\mu} = 1$ . Then the Albert algebra  $J(B, \sigma, 1, \mu)$  is a pure second Tits' construction division algebra.*

#### Remarks.

- (1) The construction of  $J'$  in the proof of Theorem 3.4 yields a division algebra if  $J$  is division.
- (2) The Albert algebra  $J'$  as above, must be a first Tits' construction due to the fact that Albert algebras of first Tits' type are precisely those with the  $f_3$  invariant zero ([K-M-R-T], 40.5).
- (3) As a consequence of Remark 2 and Theorem 3.4, we see that  $g_3(J)$  is always decomposable, i.e., is a product of  $H^1$ -classes, since this is the case when  $J$  is a first Tits' construction (cf. also [K-M-R-T, 40.9]).
- (4) In light of the fact that the Albert algebra  $J(B, \sigma, u, \mu)$  has the  $f_5$  invariant zero if and only if  $\sigma$  is distinguished ([K-M-R-T], 40.7), we note that the invariant

$f_5$  is sensitive to isotopy, in contrast with  $f_3$  and  $g_3$  (2.2). For example, if  $\sigma' = \text{Int}(v)\sigma$  is distinguished, then  $f_5(J(B, \sigma, u, \mu)^{(v)}) = 0$ . Whereas by (2.2), the  $f_3$  and  $g_3$  for this isotope are the same as for  $J(B, \sigma, u, \mu)$ .

(5) The Albert algebra  $J(B, \sigma, u, \mu)$  can be pure even when  $\sigma$  is distinguished, as the above remark shows.

### Acknowledgement

The author thanks the referee for suggesting Remark 3. The author has greatly benefited from many stimulating discussions with Professor M. A. Knus. He thanks him for his help and generosity. He would also like to express his gratitude to Professor R. Parimala for her encouragement and guidance. This work was done while the author was a visitor at the ETH, Zurich. He would like to thank the Forschungsinstitut fur Mathematik for its hospitality and excellent research facilities.

### References

- [F] J. R. Faulkner, Octonian planes defined by quadratic Jordan algebras, *Mem. AMS.* **104**, Providence, RI, 1970.
- [H-K-R-T] D. E. Haile, M. A. Knus, M. Rost, J. P. Tignol, Algebras of odd degree with involution, trace forms and dihedral extensions, *Israel Journal of Math.* **96** (1996), 299-340.
- [J] Nathan Jacobson, Structure and representations of Jordan algebras, *AMS Colloquium publications*, Volume **39**. AMS Providence, Rhode Island, 1968.
- [K-M-R-T] M. A. Knus, A. S. Merkurjev, M. Rost, J. P. Tignol, *The book of involutions*, AMS Coll. Publ. Vol. **44**, 1998.
- [MC] K. McCrimmon, The Freudenthal, Springer, Tits' constructions of exceptional Jordan algebras. *Trans. Amer. Math. Soc.* **139** (1969), 495-510.
- [P-R 1] H. P. Petersson, M. Racine, Albert algebras, Proceedings of a conference on Jordan algebras, Oberwolfach, 1992.
- [P-R 2] H. P. Petersson, M. Racine, *Enumeration and classification of Albert algebras: reduced models and the invariants mod 2*, Ed. S. Gonza'lez, Non-associative algebra and its applications, 334-340, 1994; Kluwer Academic Publishers, Netherlands.
- [P-R 3] H. P. Petersson, M. Racine, An elementary approach to the Serre-Rost invariant of Albert algebras, *Indag. Math. N.S.* **7** (1996), 343-365.
- [P-R 4] H. P. Petersson, M. Racine, Springer forms and the first Tits' construction of exceptional Jordan division algebras, *Manuscripta. Math.* **45** (1984), 249-272.
- [P-R 5] H. P. Petersson, M. Racine, On the invariants mod 2 of Albert algebras, *J. Alg.* **174** (1995), 1049-1072.
- [P-R 6] H. P. Petersson, M. Racine, Pure and generic first Tits' constructions of exceptional Jordan division algebras, *Alg. Grps. Geom.* **3**(1986), 386-398.
- [P-S-T] R. Parimala, R. Sridharan, Maneesh L. Thakur, A classification theorem for Albert algebras, *Trans. AMS.* **350**(3), (1998), 1277-1284.
- [R] M. Rost, A (mod 3) invariant for exceptional Jordan algebras, *C. R. Acad. Sci. Paris.* t. **313**, serie 1. (1991), 823-827.
- [S] J. P. Serre, *Cohomologie Galoisiennne: Progress et problemes*, Seminaire Bourbaki, exposé n°783. 1993/94.

Maneesh L. Thakur  
School of Mathematics  
Tata Institute of Fundamental Research  
Homi Bhabha Road  
Mumbai-5  
India  
e-mail: maneesh@math.tifr.res.in

(Received: December 9, 1997)