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# The rigidity of Clifford torus $S^1\left(\sqrt{\frac{1}{n}}\right) \times S^{n-1}\left(\sqrt{\frac{n-1}{n}}\right)$

QING-MING CHENG

*Abstract.* In this paper, we prove that if  $M$  is an  $n$ -dimensional closed minimal hypersurface with two distinct principal curvatures of a unit sphere  $S^{n+1}(1)$ , then  $S = n$  and  $M$  is a Clifford torus if  $n \leq S \leq n + [2n^2(n + 4)/3(n(n + 4) + 4)]$ , where  $S$  is the squared norm of the second fundamental form of  $M$ .

## 1. Introduction

Let  $M$  be an  $n$ -dimensional closed hypersurface in a unit sphere  $S^{n+1}(1)$  of dimension  $n + 1$ . Let  $S$  denote the squared norm of the second fundamental form of  $M$ . It is well-known that Chern, do Carmo and Kobayashi [2] and Lawson [3] obtained independently that Clifford tori are the only closed minimal hypersurfaces of the unit sphere with  $S = n$ . When the scalar curvature of  $M$  is constant, there are very nice results on the rigidity of the Clifford torus (see [5] and [6]). On the other hand, Otsuki[4] studied the converse problem for minimal hypersurfaces in  $S^{n+1}(1)$ . He proved that if  $M$  is a closed minimal hypersurface in  $S^{n+1}(1)$  with two distinct principal curvatures and the multiplicities of them are at least two, then  $M$  is  $S^m(\sqrt{m/n}) \times S^{n-m}(\sqrt{(n-m)/n})$  ( $1 < m < n - 1$ ). But for the case in which one of the two principal curvatures is simple, he constructed infinitely many minimal hypersurfaces other than  $S^1(\sqrt{1/n}) \times S^{n-1}(\sqrt{(n-1)/n})$  which are not congruent to each other in  $S^{n+1}(1)$ . When professor K. Shiohama visited China in 1993, he proposed the following interesting problem:

**PROBLEM.** *Let  $M$  be a closed minimal hypersurface in  $S^{n+1}(1)$  with two distinct principal curvatures  $\lambda_1$  and  $\lambda_2$  and one of them be simple (we assume  $\lambda_1$ ). Is there a constant  $\epsilon = \epsilon(n)$  such that if  $|\lambda_1 - \lambda_{10}| < \epsilon$  and  $|\lambda_2 - \lambda_{20}| < \epsilon/(n - 1)$  then  $M$  is  $S^1(\sqrt{1/n}) \times S^{n-1}(\sqrt{(n-1)/n})$ , where  $\lambda_{i0}$  are the corresponding principal curvatures of  $S^1(\sqrt{1/n}) \times S^{n-1}(\sqrt{(n-1)/n})$ .*

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This problem is equivalent to whether there is constant  $\delta = \delta(n) > 0$  such that if  $n - \delta \leq S \leq n + \delta$ , then  $M$  is  $S^1(\sqrt{1/n}) \times S^{n-1}(\sqrt{(n-1)/n})$ .

In this paper, we consider the problem and give a partial answer.

**THEOREM.** *Let  $M$  be an  $n$ -dimensional closed minimal hypersurface of a unit sphere  $S^{n+1}(1)$  with two distinct principal curvatures and one of them be simple. If*

$$n \leq S \leq n + \frac{2n^2(n+4)}{3[n(n+4)+4]}$$

*then  $S = n$  and  $M$  is  $S^1(\sqrt{1/n}) \times S^{n-1}(\sqrt{(n-1)/n})$ .*

**COROLLARY.** *Let  $M$  be an  $n$ -dimensional closed minimal hypersurface of a unit sphere  $S^{n+1}(1)$  with two distinct principal curvatures. If*

$$n \leq S \leq n + \frac{2n^2(n+4)}{3[n(n+4)+4]}$$

*then  $S = n$  and  $M$  is a Clifford torus.*

*Proof of Corollary.* This is obvious from the result due to Otsuki and Theorem.

## 2. Local formulae

Let  $M$  be an  $n$ -dimensional closed minimal hypersurface in a unit sphere  $S^{n+1}(1)$ . We choose a local orthonormal frame field  $\{e_1, \dots, e_{n+1}\}$  in  $S^{n+1}(1)$ , restricted to  $M$ , so that  $e_1, \dots, e_n$  are tangent to  $M$ . Let  $\omega_1, \dots, \omega_n$  denote the dual coframe field on  $M$ . The connection form  $\omega_{ij}$  are characterized by the structure equations

$$\begin{aligned} d\omega_i + \sum_j \omega_{ij} \wedge \omega_j &= 0, & \omega_{ij} + \omega_{ji} &= 0, \\ d\omega_{ij} + \sum_k \omega_{ik} \wedge \omega_{kj} &= \Omega_{ij}, \\ \Omega_{ij} &= \frac{1}{2} \sum_{k,l} R_{ijkl} \omega_k \wedge \omega_l, \end{aligned} \tag{2.1}$$

where  $\Omega_{ij}$  (resp.  $R_{ijkl}$ ) denotes the curvature form (resp. the components of the curvature tensor) of  $M$ . The second fundamental form  $\alpha$  of  $M$  is given by

$$\alpha = \sum_{i,j} h_{ij} \omega_i \omega_j e_{n+1} \quad \text{and} \quad \sum_i h_{ii} = 0. \quad (2.2)$$

Since  $\alpha$  is a symmetric tensor,  $h_{ij} = h_{ji}$ . The Gauss equation, Codazzi equation and Ricci formulas for the second fundamental form and its covariant derivatives are given by

$$R_{ijkl} = (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) + (h_{ik} h_{jl} - h_{il} h_{jk}), \quad (2.3)$$

$$h_{ijk} = h_{ikj} = h_{jik}, \quad (2.4)$$

$$h_{ijkl} - h_{ikjl} = \sum_m h_{im} R_{mjkl} + \sum_m h_{mj} R_{mikl}, \quad (2.5)$$

$$h_{ijklm} - h_{ijkml} = \sum_r h_{rjk} R_{rilm} + \sum_r h_{irk} R_{rjlm} + \sum_r h_{ijr} R_{rklm}, \quad (2.6)$$

where  $h_{ijk}$ ,  $h_{ijkl}$  and  $h_{ijklm}$  are the coefficients of the first, the second and the third covariant derivatives of the second fundamental form of  $M$ , respectively. The components of the Ricci curvature and the scalar curvature are given by

$$R_{ij} = (n-1)\delta_{ij} - \sum_k h_{ik} h_{jk}, \quad (2.7)$$

$$R = n(n-1) - \sum_{ij} h_{ij}^2. \quad (2.8)$$

Now we compute some local formulae. For any fixed point  $p$  in  $M$ , we can choose a local frame field  $e_1, \dots, e_n$  such that

$$h_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ \lambda_i & \text{if } i = j. \end{cases} \quad (2.9)$$

The following formulas can be found in [1]. Let

$$S := \sum_{ij} h_{ij}^2 = \sum_i \lambda_i^2.$$

$$\frac{1}{2} \Delta S = \sum_{i,j,k} h_{ijk}^2 - S(S - n). \quad (2.10)$$

$$\frac{1}{2} \Delta \sum_{i,j,k} h_{ijk}^2 = \sum_{i,j,k,l} h_{ijkl}^2 + (2n + 3 - S) \sum_{i,j,k} h_{ijk}^2 + 3(2B - A) - \frac{3}{2} |\nabla S|^2, \quad (2.11)$$

where  $A = \sum_{i,j,k} \lambda_i^2 h_{ijk}^2$  and  $B = \sum_{i,j,k} \lambda_i \lambda_j h_{ijk}^2$ .

$$\frac{1}{3} \Delta f_3 = (n - S) f_3 + 2 \sum_{i,j,k} \lambda_i h_{ijk}^2, \quad (2.12)$$

where  $f_3 = \sum_i \lambda_i^3$ .

### 3. Proofs of theorems

At first we give an algebra Lemma which will play a crucial role in the proof of our theorems.

**LEMMA.** *Let  $a_{ij}$  and  $b_i$  ( $i, j = 1, \dots, n$ ) be real numbers satisfying  $\sum_i b_i = 0$  and  $\sum_i b_i^2 = b > 0$ ,  $\sum_{i,j} b_i a_{ij} = b(n - b)$  and  $\sum_{i,j} b_j a_{ij} = 0$ . Then*

$$\begin{aligned} & \sum_i a_{ii}^2 + 3 \sum_{i \neq j} a_{ij}^2 + 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} - 3b(n - b) \\ & \geq \frac{3b(n - b)^2}{2(n + 4)} - \frac{3}{2} \left[ b \sum_i b_i^4 - \left( \sum_i b_i^3 \right)^2 - 2b^2 + nb \right]. \end{aligned}$$

*Proof.* We consider  $F = \sum_i a_{ii}^2 + 3 \sum_{i \neq j} a_{ij}^2 + 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} - 3b(n - b)$  as a function of  $a_{ij}$ . Solve the following problem for the conditional extremum:

$$\begin{aligned} f = & \sum_i a_{ii}^2 + 3 \sum_{i \neq j} a_{ij}^2 + 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} \\ & - 3b(n - b) + \lambda \left( \sum_{i,j} b_i a_{ij} - b(n - b) \right) + \mu \sum_{i,j} b_j a_{ij}, \end{aligned} \quad (3.1)$$

where  $\lambda$  and  $\mu$  are the Lagrange multipliers. It is obvious that the critical point of  $f$  is the minimum point of  $f$ . Taking derivatives of  $f$  with respect to  $a_{ij}$ , we get

$$f_{a_{ij}} = 6a_{ij} + 3(b_j^2b_i - b_i^2b_j) + \lambda b_i + \mu b_j = 0, \quad \text{for } i \neq j, \quad (3.2)$$

$$f_{a_{ii}} = 2a_{ii} + \lambda b_i + \mu b_i = 0, \quad \text{for } i = j. \quad (3.3)$$

Hence

$$\sum_i a_{ii} f_{a_{ii}} = 2 \sum_i a_{ii}^2 + \lambda \sum_i b_i a_{ii} + \mu \sum_i b_i a_{ii} = 0, \quad (3.4)$$

$$\sum_{i \neq j} a_{ij} f_{a_{ij}} = 6 \sum_{i \neq j} a_{ij}^2 + 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} + \lambda \sum_{i \neq j} b_i a_{ij} + \mu \sum_{i \neq j} b_j a_{ij} = 0, \quad (3.5)$$

$$\sum_i b_i f_{a_{ii}} = 2 \sum_i a_{ii} b_i + \lambda \sum_i b_i^2 + \mu \sum_i b_i^2 = 0, \quad (3.6)$$

$$\sum_{i \neq j} b_i f_{a_{ij}} = 6 \sum_{i \neq j} b_i a_{ij} + 3b^2 + \lambda \sum_{i \neq j} b_i^2 + \mu \sum_{i \neq j} b_i b_j = 0, \quad (3.7)$$

$$\sum_{i \neq j} b_j f_{a_{ij}} = 6 \sum_{i \neq j} b_j a_{ij} - 3b^2 + \lambda \sum_{i \neq j} b_i b_j + \mu \sum_{i \neq j} b_j^2 = 0. \quad (3.8)$$

(3.4) + (3.5) implies

$$\begin{aligned} & 2 \left( \sum_i a_{ii}^2 + 3 \sum_{i \neq j} a_{ij}^2 + 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} - 3b(n-b) \right) \\ & - 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} + \lambda \sum_{i,j} b_i a_{ij} + \mu \sum_{i,j} b_j a_{ij} = -6b(n-b). \end{aligned}$$

Thus

$$\begin{aligned} 2f_{\min} &= 2 \left( \sum_i a_{ii}^2 + 3 \sum_{i \neq j} a_{ij}^2 + 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} - 3b(n-b) \right) \\ &= 3 \sum_{i,j} (b_j^2 b_i - b_i^2 b_j) a_{ij} - 6b(n-b) - \lambda b(n-b). \end{aligned} \quad (3.9)$$

According to (3.6), we get

$$2 \sum_i a_{ii} b_i + (\lambda + \mu) b = 0. \quad (3.10)$$

(3.6) + (3.7) + (3.8) yield

$$-4 \sum_i a_{ii} b_i + 3b^2 + 6b(n - b) + nb\lambda = 0, \quad (3.11)$$

$$-4 \sum_i a_{ii} b_i - 3b^2 + nb\mu = 0. \quad (3.12)$$

Solving the system of the linear equations (3.10), (3.11) and (3.12) with unknown  $\lambda$ ,  $\mu$  and  $\sum_i a_{ii} b_i$ , we obtain

$$\lambda - \mu = -6$$

and

$$\begin{aligned} \lambda + \mu &= -\frac{6(n - b)}{(n + 4)}. \\ -\lambda &= 3 + \frac{3(n - b)}{(n + 4)}. \end{aligned} \quad (3.13)$$

From (3.2) we have

$$-6a_{ij} = 3(b_j^2 b_i - b_i^2 b_j) + \lambda b_i + \mu b_j.$$

Hence

$$\begin{aligned} & \sum_{ij} 3(b_j^2 b_i - b_i^2 b_j) a_{ij} \\ &= -\frac{3}{2} \sum_{ij} (b_j^2 b_i - b_i^2 b_j)^2 - \frac{1}{2} \sum_{ij} (b_j^2 b_i - b_i^2 b_j) (\lambda b_i + \mu b_j) \\ &= -3 \left[ b \sum_i b_i^4 - \left( \sum_i b_i^3 \right)^2 \right] - \frac{1}{2} b^2 (\lambda - \mu) \\ &= -3 \left[ b \sum_i b_i^4 - \left( \sum_i b_i^3 \right)^2 \right] + 3b^2. \end{aligned}$$

From (3.9) and (3.13) and the above equality, we conclude

$$f_{\min} = \frac{3b(n-b)^2}{2(n+4)} - \frac{3}{2} \left[ b \sum_i b_i^4 - \left( \sum_i b_i^3 \right)^2 - 2b^2 + nb \right]. \quad (3.14)$$

Thus we complete the proof of Lemma.

*Proof of theorem.*

$$\begin{aligned} \frac{1}{3} \sum_{ij} h_{ij}(f_3)_{ij} &= \frac{1}{3} \sum_i \lambda_i (f_3)_{ii} \\ &= \sum_i \lambda_i \left( \sum_j \lambda_j^2 h_{jji} + 2 \sum_{j,k} \lambda_k h_{jki}^2 \right) \\ &= \sum_{ij} \lambda_i \lambda_j^2 h_{jji} + 2 \sum_{i,j,k} \lambda_i \lambda_j h_{ijk}^2 \\ &= \sum_{ij} \lambda_i \lambda_j^2 (h_{ijj} + (\lambda_j - \lambda_i)(1 + \lambda_i \lambda_j)) + 2 \sum_{i,j,k} \lambda_i \lambda_j h_{ijk}^2 \\ &= \sum_i \frac{\lambda_i^2 S_{ii}}{2} + \sum_{ij} \lambda_i \lambda_j^2 (\lambda_j - \lambda_i)(1 + \lambda_i \lambda_j) + 2B - A \\ &= \frac{1}{2} \sum_{i,j,k} h_{ik} h_{kj} S_{ij} + \left[ S \sum_i \lambda_i^4 - S^2 - \left( \sum_i \lambda_i^3 \right)^2 \right] + 2B - A. \end{aligned}$$

Hence

$$\begin{aligned} 0 &= \frac{1}{3} \int_{\mathcal{M}} \sum_{ij} h_{jji}(f_3)_i dM = \frac{1}{3} \int_{\mathcal{M}} \sum_{ij} h_{ij}(f_3)_{ij} dM \\ &= \int_{\mathcal{M}} \left[ \frac{1}{3} \sum_{i,j,k} h_{ik} h_{kj} S_{ij} + (2B - A) + S f_4 - f_3^2 - S^2 \right] dM, \end{aligned}$$

where  $f_4$  is defined by  $f_4 = \sum_i \lambda_i^4$ , i.e.,

$$\int_{\mathcal{M}} (A - 2B) dM = \int_{\mathcal{M}} \left[ S f_4 - S^2 - f_3^2 - \frac{1}{4} |\nabla S|^2 \right] dM. \quad (3.15)$$

$$\begin{aligned}
\sum_{i,j,k,l} h_{ijkl}^2 &\geq \sum_i h_{iii}^2 + 3 \sum_{i \neq j} h_{ijij}^2 \\
&= \sum_i h_{iii}^2 + \frac{3}{4} \sum_{i \neq j} (h_{ijij} + h_{jjji})^2 + \frac{3}{4} \sum_{i \neq j} (h_{ijij} - h_{jjji})^2 \\
&= \sum_i h_{iii}^2 + 3 \sum_{i \neq j} h_{iijj} h_{jjii} + \frac{3}{2} \sum_{i \neq j} (h_{ijij} - h_{jjji})^2 \\
&= \sum_i h_{iii}^2 + 3 \sum_{i \neq j} h_{iijj} h_{jjii} + 3 \left[ nS - 2S^2 + S \sum_i \lambda_i^4 - \left( \sum_i \lambda_i^3 \right)^2 \right]. \tag{3.16}
\end{aligned}$$

Since

$$\sum_p h_{ijpp} = -(S - n)h_{ij},$$

$\sum_i \lambda_i = 0$  and  $\sum_i \lambda_i^2 = S > 0$ , we have

$$\sum_{i,j} h_{iijj} \lambda_i = S(n - S) \quad \text{and} \quad \sum_{i,j} h_{iijj} \lambda_j = 0. \tag{3.17}$$

From Ricci formulas, we have

$$h_{iijj} - h_{jjii} = (\lambda_i - \lambda_j)(1 + \lambda_i \lambda_j).$$

Note that in view of (3.17)  $h_{iijj}$  and  $\lambda_i$  satisfy the conditions of Lemma. Hence we have

$$\begin{aligned}
&\sum_i h_{iii}^2 + 3 \sum_{i \neq j} h_{iijj} h_{jjii} \\
&= \sum_i h_{iii}^2 + 3 \sum_{i \neq j} h_{iijj}^2 + 3 \sum_{i,j} (\lambda_j^2 \lambda_i - \lambda_i^2 \lambda_j) h_{iijj} - 3S(n - S) \\
&\geq \frac{3S(S - n)^2}{2(n + 4)} - \frac{3}{2} (Sf_4 - f_3^2 - 2S^2 + nS). \tag{3.18}
\end{aligned}$$

Since  $M$  has only two distinct principal curvatures  $\lambda_1$  and  $\lambda_2$  and  $\lambda_1$  is simple, we have  $\lambda_1 = -(n - 1)\lambda_2$  and

$$\frac{3}{2} (Sf_4 - f_3^2 - 2S^2 + nS) = \frac{3}{2n} S(S - n)^2.$$

Thus we get, from (3.16) and the above inequality,

$$\sum_{i,j,k,l} h_{ijkl}^2 \geq 3(Sf_4 - f_3^2 - 2S^2 + nS) - \frac{6S(n-S)^2}{n(n+4)}. \quad (3.19)$$

According to (2.10) and (2.11), we obtain

$$\int_M \sum_{i,j,k} h_{ijk}^2 dM = \int_M [S(S-n)] dM, \quad (3.20)$$

$$\int_M \sum_{i,j,k,l} h_{ijkl}^2 dM = \int_M \left[ -(2n+3-S) \sum_{i,j,k} h_{ijk}^2 - 3(2B-A) + \frac{3}{2} |\nabla S|^2 \right] dM. \quad (3.21)$$

From (3.15), (3.19), (3.20) and (3.21), we infer

$$\int_M \left\{ (2n-S) \sum_{i,j,k} h_{ijk}^2 - \frac{3}{4} |\nabla S|^2 - \frac{6S(n-S)^2}{n(n+4)} \right\} dM \leq 0. \quad (3.22)$$

From (2.10), we get

$$-\int_M \frac{1}{2} |\nabla S|^2 = \int_M \left[ S \sum_{i,j,k} h_{ijk}^2 + (n-S)S^2 \right] dM. \quad (3.23)$$

(3.22) and (3.23) yield

$$\int_M \left\{ \left( 2n + \frac{S}{2} \right) \sum_{i,j,k} h_{ijk}^2 - \frac{3}{2} S^2(S-n) - \frac{6}{n(n+4)} S(S-n)^2 \right\} dM \leq 0.$$

Using again (3.20) and the inequality

$$n \leq S \leq n + \frac{2n^2(n+4)}{3[(n(n+4)+4)],}$$

we have

$$\begin{aligned}
 0 &\geq \int_M \left\{ \left( 2n + \frac{n}{2} \right) \sum_{i,j,k} h_{ijk}^2 + \frac{1}{2} (S - n) \sum_{i,j,k} h_{ijk}^2 \right. \\
 &\quad \left. - \frac{3}{2} S^2 (S - n) - \frac{6}{n(n+4)} S (S - n)^2 \right\} dM \\
 &= \int_M \left\{ \frac{1}{2} (S - n) \sum_{i,j,k} h_{ijk}^2 + \left( \frac{5}{2} n - \frac{3}{2} S - \frac{6(S-n)}{n(n+4)} \right) S (S - n) \right\} dM \geq 0.
 \end{aligned}$$

Hence

$$\int_M \frac{1}{2} (S - n) \sum_{i,j,k} h_{ijk}^2 dM = 0.$$

Since  $S$  and  $\sum_{i,j,k} h_{ijk}^2$  are continuous functions, we have  $S = n$ . Thus from the assumption of Theorem,  $M$  is  $S^1(\sqrt{1/n}) \times S^{n-1}(\sqrt{(n-1)/n})$  according to a result due to Chern, do Carmo and Kobayashi [2] or Lawson [3]. We complete the proof of Theorem.

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