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Remarks on approximate harmonic maps

YUMNEI CHEN* AND FANG HUA LIN**

§1. Introduction

The analytical difficulties in the study of harmonic maps come from the fact that the maps take their values in a curved, compact Riemannian manifold N. One natural way to tackle such a problem is to use a so called penalty approximation, that is, to relax this nonlinear, nonconvex constraint. Roughly speaking, one studies, instead of the standard Dirichlet integrals, the following variational integral

$$\int_{M} \left[|\nabla U|^2 + \frac{1}{\varepsilon^2} d^2(U, N) \right] dx, \tag{1.1}$$

where M is a compact, Riemann manifold with (or without boundary) ∂M , and $U: M \to \mathbb{R}^k$. Here we view, via Nash's isometric embedding, N as a compact submanifold of R^k , and d(U, N) denotes the distance from U to N.

The above approach has been employed successfully by Chen and Struwe [CS] in establishing the global existence of weak solutions to the heat flow of harmonic maps. Moreover, to study such approximate energy functional (1.1) may also be natural in the Ginzburg-Laudau's approach to various physical problems, see, e.g., [BBH] and references therein.

The present note is bought out by our previous work [CL] on the evolution of harmonic maps with Dirichlet boundary conditions. We shall establish here first the Schoen-Uhlenbeck's Theorem, or "small energy regularity theorem", for energy minimizing maps. The problem is essentially reduces to obtain an a priori estimate for a family of smooth approximate solutions with small energy. As an application of our method we shall also prove one of the main results of [BBH2] concerning asymptotic limits for the Ginzburg-Landau model of scalar fields.

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Next we shall consider the uniqueness of suitable weak solutions to the heat flow of harmonic maps. As a first step we show that the weak solutions obtained in [CS] and [CL] coincide with the classical solution whenever the latter exist. This result is rather similar to the uniqueness theorem of J. Serrin [S] for the Navier-Stokes equations.

Uniqueness of weak solutions to the heat flow of harmonic maps fails in general as was shown by J. M. Coron et al. [C]. It remains an interesting open question whether certain suitable weak solutions (such as whose constructed in [CS] and [CL]) will be unique.

§2. Regularity of energy minimizing maps

Let N be a compact smooth Riemannian submanifold of R^k . As we shall discuss only the regularity of energy minimizing maps into N, the metric on the domain manifold does not play an important role (as long as one assumes a certain minimal smoothness). To simplify the presentation we therefore assume that the domain of our maps is the unit ball $B_1^m = \{x \in \mathbb{R}^m = |x| < 1\}$ in \mathbb{R}^m , $m \ge 2$, with the standard Euclidean metric.

The well-known Schoen-Uhlenbeck Theorem states that

THEOREM. [SU]. Let $U: B_1^m \to N$ be an energy minimizing map. Then there is an $\varepsilon_0 = \varepsilon_0(m, N) > 0$ such that, if $\int_{B_1} |\nabla U|^2 dx = \varepsilon \le \varepsilon_0$, then

$$\sup_{B_{1/2}(0)} |\nabla U|^2 \le C(m, N)\varepsilon. \tag{2.1}$$

Here we shall give an alternative proof of the above statement. To do so we consider a family of approximate solutions U_{δ} , $\delta \in (0, 1)$ where $U_{\delta} = B_1^m \to \mathbb{R}^K$ be defined as follows

$$U_{\delta}(x) = \begin{cases} U(x) & \text{if } x \in B_{1/2}^{m}(0) \\ V_{\delta}(x) & \text{if } \frac{1}{2} \le |x| \le 1 \end{cases}$$
 (2.2)

where V_{δ} minimizes

$$I_{\delta}(V) = \int_{B_1 \setminus B_{1/2}} \left\{ |\nabla V|^2 + \frac{1}{\delta^2} d^2(V, N) \right\} dx$$
 (2.3)

subject to the Dirichlet boundary conditions V = U on $\partial(B_1 \setminus B_{1/2})$.

Since $I_{\delta}(V_{\delta}) \leq I_{\delta}(U) = \int_{B_1 \setminus B_{1/2}} |\nabla U|^2 dx$, we may choose a sequence of $\delta_i \downarrow 0$ such that $U_i = U_{\delta_i}$ converges to some U_* weakly in $H^1(B_1, N)$ and converges strongly to U_* in $L^2(B_1, N)$. Moreover $\int_{B_1} |\nabla U_*|^2 dx \leq \int_{B_1} |\nabla U|^2 dx$ by lower semicontinuity of the energy.

Since U is an energy minimizing map, we see U_* must also be energy minimizing. Moreover, $U_i \to U_*$ in $H^1(B_1, N)$ by Fatou's Lemma. Finally, for all i sufficiently large, $I_{\delta_i}(V_{\delta_i}) \le \varepsilon \le \varepsilon_0$.

Next, we let $e_{\delta}(V) = |\nabla V|^2 + 1/\delta^2 d^2(V, N)$, $0 < \delta < 1$. Then from [CS] (see also [CL]) we have the monotonicity inequality for V_{δ} :

$$\Phi(r,x) \le \Phi(\rho,x) \tag{2.4}$$

for all $x \in B_{15/16} \setminus B_{9/16}$, $0 < r < \rho < dist(x, \partial(B_1 \setminus B_{1/2}))$. Here $\Phi(r, x) = r^{2-n} \int_{B_r(x)} e_{\delta}(V_{\delta}) dx \cdot \exp(cr)$. Also we have the Bochner-type inequality:

$$\Delta e \ge -ce(1+e) \quad \text{in } B_1 \backslash B_{1/2} \tag{2.5}$$

where $e = e_{\delta}(V_{\delta})$.

By (2.4) and (2.5) and since $\int_{B_1 \setminus B_{1/2}} e_{\delta_i}(V_{\delta_i}) dx \le \varepsilon$, we have the following estimate as in [CS]:

$$\sup_{\frac{5}{8} \le |x| \le \frac{5}{6}} e_{\delta_i}(V_{\delta_i})(x) \le C(m, N)\varepsilon$$
(2.6)

provided $\varepsilon \leq \varepsilon_0(m, N)$.

Let $i \to \infty$. we obtain, in particular, that

$$\underset{\partial B_{3/4}}{OSC} U_* \le C\sqrt{\varepsilon_0} \tag{2.7}$$

Since U_* is an energy minimizing map, $U_*(B_{3/4})$ is contained in ball $B_{2c\sqrt{\epsilon}}(p)$ for some $p \in N$.

To see this, we let $p_0 = U_*(x_0)$, for some $x_0 \in \partial B_{3/4}$. Let $B_{2c\sqrt{\varepsilon}}(p_0)$ be the ball of radius $2c\sqrt{\varepsilon}$ in \mathbb{R}^K so that $U_*(\partial B_{3/4}) \subset B_{c\sqrt{\varepsilon}}(p_0)$. Let $\pi : \mathbb{R}^K \to B_{2c\sqrt{\varepsilon}}(p_0)$ be the retraction map, i.e., $\pi(x) = x$ if $x \in B_{2c\sqrt{\varepsilon}}(p_0)$, and $\pi(x) = 2c\sqrt{\varepsilon}(x-p_0)/|x-p_0|$ if $x \notin B_{2c\sqrt{\varepsilon}}(p_0)$. Since $\sqrt{\varepsilon} \le \sqrt{\varepsilon_0}$ is very small, and N is a smooth submanifold, $p_0 \in N$, we see the nearest point projection map $\partial B_{2c\sqrt{\varepsilon}}(p_0) \xrightarrow{\pi_N} N \cap B_{\partial c\sqrt{\varepsilon}}(p_0)$ is well-defined and is distance decreasing map from $\partial B_{2c\sqrt{\varepsilon}}(p_0)$ to $N \cap B_{2c\sqrt{\varepsilon}}(p_0)$.

Now, if U_* is energy minimizing with $U_*(\partial B_{3/4}) \subset B_{c\sqrt{\epsilon}}(p_0)$, then $\tilde{U} = \pi_N \circ \pi \circ U_* : B_{3/4} \to N$ is also energy minimizing with $\tilde{U} = U_*$ on $\partial B_{3/4}$. In fact, $\int_{B_{3/4}} |\nabla \tilde{U}|^2 dx \leq \int_{B_{3/4}} |\nabla U_*|^2 dx$ and the equality is valid if and only if $U_*(B_{3/4}) \subset B_{2c\sqrt{\epsilon}}(p_0) \cap N$.

Having seen $U_*(B_{3/4}) \subset B_{2c\sqrt{\epsilon}}(p_0) \cap N$, the regularity of U_* on $B_{1/2}$ follows from the standard elliptic theory (See e.g. [J]). By our definition of U_* , $U_* = U$ on $B_{1/2}$, we, in particular, obtain that

$$\sup_{B_{1/2}} |\nabla U|^2 \le C \int_{B_1} |\nabla U|^2 = c\varepsilon. \tag{2.8}$$
Q.E.D.

§3. A theorem of Bethuel-Brezis-Helein

Let $\Omega \subseteq \mathbb{R}^2$ be a smooth bounded connected domain. Consider the functional

$$E_{\varepsilon}(U) = \int_{\Omega} |\nabla U|^2 + \frac{1}{2\varepsilon^2} \int_{\Omega} (|U|^2 - 1)^2$$
(3.1)

which is defined for maps $U \in H^1(\Omega, \mathbb{C})$, we let $g : \partial\Omega \to \mathbb{C}$ be a smooth map with |g(x)| = 1, $\forall x \in \partial\Omega$. We also assume that $\deg(g, \partial\Omega) = 0$ and hence there is a smooth extension of $g^* : \Omega \to S^1$ with $g^* = g$ on $\partial\Omega$.

By a theorem of C. B. Morrey, there is a map $U_0: \Omega \to S^1$ which minimizes $\int_{\Omega} |\nabla U|^2 dx$ over the set $H_g^1(\Omega, S^1) = \{U \in H^1(\Omega, \mathbb{C}) : u = g \text{ on } \partial\Omega\}$. Moreover, U_0 is smooth. When Ω is simply connected, a simple lifting argument shows that $U_0 = e^{i\phi_0}$. Here ϕ_0 is the harmonic extension of ϕ , $e^{i\phi} = g$ on $\partial\Omega$.

Here we want to show a uniform estimate for the minimizers U_{ε} of (3.1) for $0 < \varepsilon < 1$ under the hypothesis that $deg(g, \partial \Omega) = 0$.

To do so, we note first that

$$E_{\varepsilon}(U_{\varepsilon}) \le \int_{\Omega} |\nabla U_0|^2 dx$$
, for all $0 < \varepsilon < 1$. (3.2)

For any sequence $\varepsilon_i \to 0$, there is a subsequence of U_{ε_i} which converges weakly in H^1 and strongly in L^2 to some $U_* \in H^1(\Omega, S^1)$. Moreover, $U_* = g$ on $\partial \Omega$, and $\int_{\Omega} |\nabla U_*|^2 dx \le \int_{\Omega} |\nabla U_0|^2 dx$. By the minimizing property of U_0 , we see U_* again is a minimizer of $\int_{\Omega} |\nabla U|^2 dx$ over $H_g^1(\Omega, S^1)$. In particular, U_* is smooth. Moreover, it follows from (3.2) that U_{e_i} converges strongly to U_* . We therefore obtain the following

LEMMA. For any $\varepsilon_0 > 0$, there is an $r_0 > 0$ depending only on $\partial \Omega$ and g such that if U_{ε} is a minimizer of (3.1) then

$$\int_{\Omega \cap B(x, r_0)} \left[|\nabla U_{\varepsilon}|^2 + \frac{1}{2\varepsilon^2} (|U_{\varepsilon}|^2 - 1)^2 \right] dx \le \varepsilon_0$$
 (3.3)

for all $x \in \overline{\Omega}$ provided $0 < \varepsilon \le \varepsilon_*(r_0, \varepsilon_0)$.

Proof. Let F denote the set of energy minimizing maps over the set $H_g^1(\Omega, S^1)$. Then it is easy to see that F is compact in $H^1(\Omega, S^1)$. Moreover, by Morrey's theorem, one has for any $\varepsilon_0 > 0$, $U_* \in F$

$$\int_{\Omega \cap B(x,r)} |\nabla U_*|^2 dx \le \varepsilon_0/2 \tag{3.4}$$

for all $x \in \overline{\Omega}$ and $0 < r \le r_0$ provided that r_0 is chosen to be suitably small.

Now we apply the convergence argument above to conclude that (3.3) is valid for all minimizers U_{ε} whenever $0 < \varepsilon \le \varepsilon_*$. Note that $1/\varepsilon^2 \int_{\Omega} (|U_{\varepsilon}|^2 - 1)^2 dx \to 0$ is $\varepsilon \to 0^+$.

THEOREM. Let U_{ε} be a minimizer of (3.1) over the set $H_g^1(\Omega, \mathbb{C})$ with $deg(g, \partial\Omega) = 0$. Then

$$\sup_{\Omega} \left[|\nabla U_{\varepsilon}|^2 + \frac{1}{2\varepsilon^2} (|u_{\varepsilon}|^2 - 1)^2 \right] \le C(g), \tag{3.5}$$

for all $0 < \varepsilon < 1$.

Proof. It is obvious, by the maximum-principle, that $|U_{\varepsilon}| \le 1$ on Ω . Thus $|U_{\varepsilon}|(1-|U_{\varepsilon}|^2)1/\varepsilon^2 \le 1/\varepsilon^2 \le 1/\varepsilon^2_*$ whenever $\varepsilon \ge \varepsilon_* = \varepsilon_*(g) > 0$. It follows that (3.5) is true whenever $\varepsilon \ge \varepsilon_*$.

For $0 < \varepsilon < \varepsilon_*$, we use the estimate (3.3). It follows from the identical arguments as in the previous section, that one obtians the interior estimate

$$\sup_{\Omega'} \left[|\nabla U_{\varepsilon}|^2 + \frac{1}{2\varepsilon^2} (1 - |U_{\varepsilon}|^2)^2 \right] \le C(g, \Omega')$$
(3.6)

for all $0 < \varepsilon \le \varepsilon_*$.

For the estimate near the boundary of Ω , we refer to [CL]. We should point out that the monotonicity inequality in the present situation is automatically valid. Combining (3.6) with the boundary estimate one concludes

$$\sup_{\Omega} \left[|\nabla U_{\varepsilon}|^2 + \frac{1}{2\varepsilon^2} (1 - |U_{\varepsilon}|^2)^2 \right] \le C(\varepsilon_0, r_0)$$
(3.7)

for all $0 < \varepsilon \le \varepsilon_*$.

Remark. When Ω is, in addition, simply connected, the energy-minimizer over $H_g^1(\Omega, S^1)$ is also unique. In this case, one can show that $U_{\varepsilon} \to U_*$ energy-minim-

izer uniformly and strongly in $H^1(\Omega, \mathbb{C})$. Moreover, as in [BBH2], one has, for $\psi_{\varepsilon} = 1/\varepsilon^2 (1 - |U_{\varepsilon}|^2) \ge 0$,

$$\begin{cases} 2\varepsilon^2 \Delta \psi_{\varepsilon} - \psi_{\varepsilon} \ge -4 |\nabla U_{\varepsilon}|^2 \ge -C_1, & \text{for all } 0 < \varepsilon \le \varepsilon_*, \\ \psi_{\varepsilon}|_{\partial \Omega} = 0 \end{cases}$$
 (3.8)

Let $x_0 \in \Omega$ be such that $\psi_{\varepsilon}(x_0) = \max_{x \in \bar{\Omega}} \psi_{\varepsilon}(x) > 0$, then $\Delta \psi_{\varepsilon}(x_0) \le 0$, and thus

$$\psi_{\varepsilon}(x_0) \le C_1. \tag{3.9}$$

From (3.9) we obtain $\|\Delta U_{\varepsilon}\|_{L^{\infty}(\Omega)} \leq C_1$, and $U_{\varepsilon} \to U_0$ in $C^{1,\alpha}(\Omega)$, $\forall \alpha < 1$, follows.

§4. Uniqueness problems

Let M be a compact Riemannian manifold with possible nonempty boundary ∂M , and let N be as before. The equation for harmonic maps $U: M \to N$ can be written as

$$\Delta U + A(U)(\nabla U, \nabla U) = 0 \tag{4.1}$$

where Δ is the Beltrami-operator on M, and A(U) is the second fundamental form of N at U. Thus the corresponding equations for the heat flow are

$$\frac{\partial U}{\partial t} - \Delta U = A(U)(\nabla U, \nabla U), \qquad (x, t) \in M \times (0, \infty). \tag{4.2}$$

Given initial data $U_0: M \to N$, one then is interested in solving (4.2) with

$$U(x,t) = U_0(x) \qquad \text{for } (x,t) \in (M \times \{0\}) \cup (\partial M \times (0,\infty)). \tag{4.3}$$

Suppose U_0 , M, ∂M and N are smooth. It is well-known (see, e.g. [J]) that the problem (4.2) (4.3) has a unique smooth solution U(x, t), $(x, t) \in M \times [0, T]$, for some T > 0 which may depend on the various data mentioned. On the other hand, it was shown in [CS] (for the case $\partial M = \phi$) and [CL] that (4.2), (4.3) has a global weak-solution which is smooth off a relatively small closed subset of $M \times (0, \infty)$. One of the natural question is that whether such suitable weak solutions obtained in [CS] and [CL] are unique.

Here we want to show that the weak solution obtained in [CS] and [CL] must coincide with the classical solution on the time interval $[0, T^*]$, here $0 < T^* \le \infty$ is

the first time of blow-up for the classical solution. The latter means that there is a smooth solution \tilde{U} of (4.2), (4.3) on the time interval $[0, T^*)$ and that $\lim_{t \to T^*} \|\nabla \tilde{U}\|_{L^{\infty}(M)}(t) = +\infty$.

For this purpose, we adopt the same notations as that in [CS] and [CL]. Consider a sequence of approximate solutions U^k such that

$$\frac{\partial}{\partial t} U^k - \Delta U^k + k\chi'(d^2(U^k, N)) \frac{d}{dU} \left(\frac{d^2(U^k, N)}{2} \right) = 0$$
 (4.4)

in $M \times (0, \infty)$,

$$U^{k}(x,t) = (U_{0}(x), \quad \text{on } (M \times \{0\}) \cup (\partial M \times (0,\infty)). \tag{4.5}$$

We claim there are positive constant C_0 , T_0 depending only on U_0 , ∂M , M, N such that

$$\sup_{M \times [0, T_0]} e(U^k) \le C_0, \quad \text{for } k = 1, 2, \dots$$
 (4.6)

where $e(U^k) = |\nabla U^k|^2 + (k/2)\chi(d^2(U^k, N))$.

In fact, for any $x_0 \in M$, $0 < t_0 < R_M$ (R_M is the injectivity radius of M) and $r = \sqrt{t_0}$, one has that (we adopt the same notations as that in [CS] and [CL])

$$\Phi(r, U^{k}, (x_{0}, t_{0})) = \frac{t_{0}}{2} \int_{R^{m}} e(U^{k}) G_{(x_{0}, t_{0})} \Big|_{t=0} \phi^{2}(|x - x_{0}|) dx$$

$$\leq \frac{t_{0}}{2} \left(\int_{B_{t_{0}^{(1-2)/2}}} + \int_{R^{m} \setminus B_{t_{0}^{(1-2)/2}}} e(U^{k}) G_{(x_{0}, t_{0})} \Big|_{t=0} \phi^{2}(|x - x_{0}|) dx$$

$$\leq C t_{0}^{1 - (\epsilon m/2)} \|U_{0}\|_{C^{1}(M)} + C t_{0}^{1 - (m/2)} e^{-(1/t_{0}^{\epsilon})} E(U_{0}) \leq \varepsilon_{0}, \tag{4.7}$$

if $\varepsilon < 2/m$ and t_0 is suitably small. Then, (4.6) follows from the small energy regularity theorems in [CS] and [CL].

By the definition of $\chi(d^2)$ and (4.6) (cf. [CL]), one has $\chi(d^2(U^k, N)) = d^2(U^k, N)$ for all large k's. We claim that (4.6) implies that U^k converges to the classical solution \tilde{U} in $W_p^{2,1}(M \times (0, T_0))$ as $k \to \infty$. Here

$$W_p^{2,1}(M\times(0,T_0)) = \{V: V, D_x V, D_x^2 V, V_t \in L^P(M\times(0,T_0))\}, \qquad 1$$

Suppose, for the moment, that the above claim is true. Then we want to show the weak solution obtained in [CS] and [CL] coincides with the classical solution on

[0, T^*). To do so, we let $0 < T_0^* \le T^*$ be such that

$$T_0^* = \sup\{t \in [0, T) : \lim_{k \to \infty} U^k = \tilde{U}\}.$$
 (4.8)

Here the limit is taking in $W_p^{2,1}(M\times(0,t))$, $(p\geq m+1)$. If $T_0^*< T^*$, then for $\varepsilon_0>0$, then is an $r_0>0$, such that $C_m r^2 \int_{B2r(x_0)} |V\tilde{U}|^2(t_0) dx < \varepsilon_0$, for all $0< r\leq r_0$, and $(x_0,t_0)\in \bar{M}\times[0,T_0^*]$. We let $t_1< T_0^*$ be such that $T_0^*-t_1\leqslant r_0^2$. Then since $U^k(\cdot,t_1)\to \tilde{U}(\cdot,t_1)$ in $W^{2,p}(M)$ as $k\to\infty$, we may assume, for all large k's, that

$$C_m r_0^{2-m} \oint_{B_{2r_0(x_0)}} |\nabla U^k|^2(t_1) dx < \varepsilon_0.$$

Then by small energy regularity theorem of [CS] and [CL], one has, as above, $U^k \to \tilde{U}$ in $W_p^{2,1}(t_1, t_1 + r_0)$. This contradicts the definition of T_0^* .

Finally we would like to prove the above claim.

Let $\psi_k = d(U^k, N)$, then by a simple calculation (cf. [CL], (4.18)]), one has, by (4.5)-(4.6), that

$$\begin{cases} \frac{\partial}{\partial t} \psi_k - \Delta \psi_k \le -k \psi_k + |\nabla U^k|^2 & \text{in } M \times [0, T_0] \\ \psi_k = 0 & \text{on } (M \times \{0\}) \cup (\partial M \times [0, T_0]). \end{cases}$$

$$(4.9)$$

In deriving (4.9), we have used the fact that $d(U^k, N) \to 0$ uniformly as $k \to \infty$. (cf. (4.6)). Again, by the maximum principle, one has

$$\max_{(x,t) \in M \times [0,T_0]} \psi_k \le \frac{1}{k} \max_{(x,t) \in M \times [0,T_0]} |\nabla U^k|^2 \le \frac{1}{k} C_0. \tag{4.10}$$

Hence from (4.4), $(\partial/\partial t)U^k - \Delta U^k \in L^{\infty}(M \times [0, T_0])$, and our claim follows from the standard L^p -theory for parabolic systems [LSU].

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Preface – I. Conformal and Quasiconformal Mappings – II. Fixed Points and Conjugations – III. Basic Rational Iteration – IV. Classification of Periodic Components – V. Critical Points and Expanding Maps – VI. Application of Quasiconformal Mappings – Local Geometry of the Fatou Set – VII. Quadratic Polynomials – Epilogue – References – Index – Symbol Index.

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Introduction – I. Circle Diffeomorphism – 1. The Combinatorial Theory of Poincaré – 2. The Topological Theory of Denjoy – 2.a The Denjoy Inequality – 2.b Ergodicity – 3. Smooth Conjugacy Results – 4. Families of Circle Diffeomorphisms; Arnol'd Tongues – 5. Counter-Examples to Smooth Linearizability – 6. Frequency of Smooth Linearizability – 7. Some Historical Comments and Further Remarks – II. The Combinatorics of One-Dimensional Endomorphisms – 1. The Theorem of Sarkovskii – 2. Coverings Maps of the Circle as Dynamical Systems – 3. The Kneading Theory and Combinatorial Equivalence – 3.a Examples – 3.b Hofbauer's Tower Construction – 4. Full Families and Realization of Maps – 5. Families of Maps and Renormalization – 6. Piecewise Monotone Maps can be Modelled by Polynomial Maps – 7. The Topological Entropy – 8. The Piecewise Linear Model – 9. Continuity of the Topological Entropy – 10. Monotonicity of the Kneading Invariant for the Quadratic