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Remarks on approximate harmonic maps

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§1. Introduction

The analytical difficulties in the study of harmonic maps come from the fact that the maps take their values in a curved, compact Riemannian manifold N. One natural way to tackle such a problem is to use a so called penalty approximation, that is, to relax this nonlinear, nonconvex constraint. Roughly speaking, one studies, instead of the standard Dirichlet integrals, the following variational integral

$$\int_M \left[|\nabla U|^2 + \frac{1}{\varepsilon^2} d^2(U, N) \right] dx, \tag{1.1}$$

where M is a compact, Riemann manifold with (or without boundary) ∂M, and $U : M \to \mathbb{R}^k$. Here we view, via Nash's isometric embedding, N as a compact submanifold of R^k, and $d(U, N)$ denotes the distance from U to N.

The above approach has been employed successfully by Chen and Struwe [CS] in establishing the global existence of weak solutions to the heat flow of harmonic maps. Moreover, to study such approximate energy functional (1.1) may also be natural in the Ginzburg–Laudau's approach to various physical problems, see, e.g., [BBH] and references therein.

The present note is bought out by our previous work [CL] on the evolution of harmonic maps with Dirichlet boundary conditions. We shall establish here first the Schoen–Uhlenbeck's Theorem, or "small energy regularity theorem", for energy minimizing maps. The problem is essentially reduces to obtain an a priori estimate for a family of smooth approximate solutions with small energy. As an application of our method we shall also prove one of the main results of [BBH2] concerning asymptotic limits for the Ginzburg–Landau model of scalar fields.

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Next we shall consider the uniqueness of suitable weak solutions to the heat flow of harmonic maps. As a first step we show that the weak solutions obtained in [CS] and [CL] coincide with the classical solution whenever the latter exist. This result is rather similar to the uniqueness theorem of J. Serrin [S] for the Navier–Stokes equations.

Uniqueness of weak solutions to the heat flow of harmonic maps fails in general as was shown by J. M. Coron et al. [C]. It remains an interesting open question whether certain suitable weak solutions (such as whose constructed in [CS] and [CL]) will be unique.

§2. Regularity of energy minimizing maps

Let N be a compact smooth Riemannian submanifold of R^k. As we shall discuss only the regularity of energy minimizing maps into N, the metric on the domain manifold does not play an important role (as long as one assumes a certain minimal smoothness). To simplify the presentation we therefore assume that the domain of our maps is the unit ball $B_1^m = \{x \in \mathbb{R}^m = |x| < 1\}$ in $\mathbb{R}^m$, $m \geq 2$, with the standard Euclidean metric.

The well-known Schoen-Uhlenbeck Theorem states that

THEOREM. [*SU*]. *Let* $U : B_1^m \to N$ *be an energy minimizing map. Then there is an* $\varepsilon_0 = \varepsilon_0(m, N) > 0$ *such that, if* $\int_{B_1} |\nabla U|^2 \, dx = \varepsilon \leq \varepsilon_0$, *then*

$$\sup_{B_{1/2}(0)} |\nabla U|^2 \leq C(m, N)\varepsilon. \tag{2.1}$$

Here we shall give an alternative proof of the above statement. To do so we consider a family of approximate solutions U_δ, $\delta \in (0, 1)$ where $U_\delta = B_1^m \to \mathbb{R}^K$ be defined as follows

$$U_\delta(x) = \begin{cases} U(x) & \text{if } x \in B_{1/2}^m(0) \\ V_\delta(x) & \text{if } \frac{1}{2} \leq |x| \leq 1 \end{cases} \tag{2.2}$$

where V_δ minimizes

$$I_\delta(V) = \int_{B_1 \setminus B_{1/2}} \left\{ |\nabla V|^2 + \frac{1}{\delta^2} d^2(V, N) \right] dx \tag{2.3}$$

subject to the Dirichlet boundary conditions $V = U$ on $\partial(B_1 \setminus B_{1/2})$.

Since $I_\delta(V_\delta) \le I_\delta(U) = \int_{B_1 \setminus B_{1/2}} |\nabla U|^2\, dx$, we may choose a sequence of $\delta_i \downarrow 0$ such that $U_i = U_{\delta_i}$ converges to some U_* weakly in $H^1(B_1, N)$ and converges strongly to U_* in $L^2(B_1, N)$. Moreover $\int_{B_1} |\nabla U_*|^2\, dx \le \int_{B_1} |\nabla U|^2\, dx$ by lower semicontinuity of the energy.

Since U is an energy minimizing map, we see U_* must also be energy minimizing. Moreover, $U_i \to U_*$ in $H^1(B_1, N)$ by Fatou's Lemma. Finally, for all i sufficiently large, $I_{\delta_i}(V_{\delta_i}) \le \varepsilon \le \varepsilon_0$.

Next, we let $e_\delta(V) = |\nabla V|^2 + 1/\delta^2\, d^2(V, N)$, $0 < \delta < 1$. Then from [CS] (see also [CL]) we have the monotonicity inequality for V_δ:

$$\Phi(r, x) \le \Phi(\rho, x) \tag{2.4}$$

for all $x \in B_{15/16} \setminus B_{9/16}$, $0 < r < \rho < dist(x, \partial(B_1 \setminus B_{1/2}))$. Here $\Phi(r, x) = r^{2-n} \int_{B_r(x)} e_\delta(V_\delta)\, dx \cdot \exp(cr)$. Also we have the Bochner-type inequality:

$$\Delta e \ge -ce(1+e) \quad \text{in } B_1 \setminus B_{1/2} \tag{2.5}$$

where $e = e_\delta(V_\delta)$.

By (2.4) and (2.5) and since $\int_{B_1 \setminus B_{1/2}} e_{\delta_i}(V_{\delta_i})\, dx \le \varepsilon$, we have the following estimate as in [CS]:

$$\sup_{\frac{5}{8} \le |x| \le \frac{5}{6}} e_{\delta_i}(V_{\delta_i})(x) \le C(m, N)\varepsilon \tag{2.6}$$

provided $\varepsilon \le \varepsilon_0(m, N)$.

Let $i \to \infty$. we obtain, in particular, that

$$\underset{\partial B_{3/4}}{OSC}\, U_* \le C\sqrt{\varepsilon} \le C\sqrt{\varepsilon_0} \tag{2.7}$$

Since U_* is an energy minimizing map, $U_*(B_{3/4})$ is contained in ball $B_{2c\sqrt{\varepsilon}}(p)$ for some $p \in N$.

To see this, we let $p_0 = U_*(x_0)$, for some $x_0 \in \partial B_{3/4}$. Let $B_{2c\sqrt{\varepsilon}}(p_0)$ be the ball of radius $2c\sqrt{\varepsilon}$ in $\mathbb{R}^K$ so that $U_*(\partial B_{3/4}) \subset B_{c\sqrt{\varepsilon}}(p_0)$. Let $\pi : \mathbb{R}^K \to B_{2c\sqrt{\varepsilon}}(p_0)$ be the retraction map, i.e., $\pi(x) = x$ if $x \in B_{2c\sqrt{\varepsilon}}(p_0)$, and $\pi(x) = 2c\sqrt{\varepsilon}\,(x - p_0)/|x - p_0|$ if $x \notin B_{2c\sqrt{\varepsilon}}(p_0)$. Since $\sqrt{\varepsilon} \le \sqrt{\varepsilon_0}$ is very small, and N is a smooth submanifold, $p_0 \in N$, we see the nearest point projection map $\partial B_{2c\sqrt{\varepsilon}}(p_0) \xrightarrow{\pi_N} N \cap B_{\partial c\sqrt{\varepsilon}}(p_0)$ is well-defined and is distance decreasing map from $\partial B_{2c\sqrt{\varepsilon}}(p_0)$ to $N \cap B_{2c\sqrt{\varepsilon}}(p_0)$.

Now, if U_* is energy minimizing with $U_*(\partial B_{3/4}) \subset B_{c\sqrt{\varepsilon}}(p_0)$, then $\tilde{U} = \pi_N \circ \pi \circ U_* : B_{3/4} \to N$ is also energy minimizing with $\tilde{U} = U_*$ on $\partial B_{3/4}$. In fact, $\int_{B_{3/4}} |\nabla \tilde{U}|^2\, dx \le \int_{B_{3/4}} |\nabla U_*|^2\, dx$ and the equality is valid if and only if $U_*(B_{3/4}) \subset B_{2c\sqrt{\varepsilon}}(p_0) \cap N$.

Having seen $U_*(B_{3/4}) \subset B_{2c\sqrt{\varepsilon}}(p_0) \cap N$, the regularity of U_* on $B_{1/2}$ follows from the standard elliptic theory (See e.g. [J]). By our definition of U_*, $U_* = U$ on $B_{1/2}$, we, in particular, obtain that

$$\sup_{B_{1/2}} |\nabla U|^2 \le C \int_{B_1} |\nabla U|^2 = c\varepsilon. \tag{2.8}$$

Q.E.D.

§3. A theorem of Bethuel–Brezis–Helein

Let $\Omega \subseteq \mathbb{R}^2$ be a smooth bounded connected domain. Consider the functional

$$E_\varepsilon(U) = \int_\Omega |\nabla U|^2 + \frac{1}{2\varepsilon^2} \int_\Omega (|U|^2 - 1)^2 \tag{3.1}$$

which is defined for maps $U \in H^1(\Omega, \mathbb{C})$, we let $g : \partial\Omega \to \mathbb{C}$ be a smooth map with $|g(x)| = 1, \forall x \in \partial\Omega$. We also assume that $\deg(g, \partial\Omega) = 0$ and hence there is a smooth extension of $g^* : \Omega \to S^1$ with $g^* = g$ on $\partial\Omega$.

By a theorem of C. B. Morrey, there is a map $U_0 : \Omega \to S^1$ which minimizes $\int_\Omega |\nabla U|^2 \, dx$ over the set $H^1_g(\Omega, S^1) = \{U \in H^1(\Omega, \mathbb{C}) : u = g \text{ on } \partial\Omega\}$. Moreover, U_0 is smooth. When Ω is simply connected, a simple lifting argument shows that $U_0 = e^{i\phi_0}$. Here ϕ_0 is the harmonic extension of ϕ, $e^{i\phi} = g$ on $\partial\Omega$.

Here we want to show a uniform estimate for the minimizers U_ε of (3.1) for $0 < \varepsilon < 1$ under the hypothesis that *deg*$(g, \partial\Omega) = 0$.

To do so, we note first that

$$E_\varepsilon(U_\varepsilon) \le \int_\Omega |\nabla U_0|^2 \, dx, \qquad \text{for all } 0 < \varepsilon < 1. \tag{3.2}$$

For any sequence $\varepsilon_i \to 0$, there is a subsequence of U_{ε_i} which converges weakly in H^1 and strongly in L^2 to some $U_* \in H^1(\Omega, S^1)$. Moreover, $U_* = g$ on $\partial\Omega$, and $\int_\Omega |\nabla U_*|^2 \, dx \le \int_\Omega |\nabla U_0|^2 \, dx$. By the minimizing property of U_0, we see U_* again is a minimizer of $\int_\Omega |\nabla U|^2 \, dx$ over $H^1_g(\Omega, S^1)$. In particular, U_* is smooth. Moreover, it follows from (3.2) that U_{e_i} converges strongly to U_*. We therefore obtain the following

LEMMA. *For any $\varepsilon_0 > 0$, there is an $r_0 > 0$ depending only on $\partial\Omega$ and g such that if U_ε is a minimizer of* (3.1) *then*

$$\int_{\Omega \cap B(x, r_0)} \left[|\nabla U_\varepsilon|^2 + \frac{1}{2\varepsilon^2}(|U_\varepsilon|^2 - 1)^2 \right] dx \le \varepsilon_0 \tag{3.3}$$

for all $x \in \bar{\Omega}$ provided $0 < \varepsilon \le \varepsilon_*(r_0, \varepsilon_0)$.

Proof. Let F denote the set of energy minimizing maps over the set $H_g^1(\Omega, S^1)$. Then it is easy to see that F is compact in $H^1(\Omega, S^1)$. Moreover, by Morrey's theorem, one has for any $\varepsilon_0 > 0$, $U_* \in F$

$$\int_{\Omega \cap B(x,r)} |\nabla U_*|^2 \, dx \leq \varepsilon_0/2 \tag{3.4}$$

for all $x \in \bar{\Omega}$ and $0 < r \leq r_0$ provided that r_0 is chosen to be suitably small.

Now we apply the convergence argument above to conclude that (3.3) is valid for all minimizers U_ε whenever $0 < \varepsilon \leq \varepsilon_*$. Note that $1/\varepsilon^2 \int_\Omega (|U_\varepsilon|^2 - 1)^2 \, dx \to 0$ is $\varepsilon \to 0^+$.

THEOREM. *Let* U_ε *be a minimizer of* (3.1) *over the set* $H_g^1(\Omega, \mathbb{C})$ *with* $deg(g, \partial\Omega) = 0$. *Then*

$$\sup_\Omega \left[|\nabla U_\varepsilon|^2 + \frac{1}{2\varepsilon^2} (|u_\varepsilon|^2 - 1)^2 \right] \leq C(g), \tag{3.5}$$

for all $0 < \varepsilon < 1$.

Proof. It is obvious, by the maximum-principle, that $|U_\varepsilon| \leq 1$ on Ω. Thus $|U_\varepsilon|(1 - |U_\varepsilon|^2) 1/\varepsilon^2 \leq 1/\varepsilon^2 \leq 1/\varepsilon_*^2$ whenever $\varepsilon \geq \varepsilon_* = \varepsilon_*(g) > 0$. It follows that (3.5) is true whenever $\varepsilon \geq \varepsilon_*$.

For $0 < \varepsilon < \varepsilon_*$, we use the estimate (3.3). It follows from the identical arguments as in the previous section, that one obtians the interior estimate

$$\sup_{\Omega'} \left[|\nabla U_\varepsilon|^2 + \frac{1}{2\varepsilon^2} (1 - |U_\varepsilon|^2)^2 \right] \leq C(g, \Omega') \tag{3.6}$$

for all $0 < \varepsilon \leq \varepsilon_*$.

For the estimate near the boundary of Ω, we refer to [CL]. We should point out that the monotonicity inequality in the present situation is automatically valid. Combining (3.6) with the boundary estimate one concludes

$$\sup_\Omega \left[|\nabla U_\varepsilon|^2 + \frac{1}{2\varepsilon^2} (1 - |U_\varepsilon|^2)^2 \right] \leq C(\varepsilon_0, r_0) \tag{3.7}$$

for all $0 < \varepsilon \leq \varepsilon_*$.

Remark. When Ω is, in addition, simply connected, the energy-minimizer over $H_g^1(\Omega, S^1)$ is also unique. In this case, one can show that $U_\varepsilon \to U_*$ energy-minimizer uniformly and strongly in $H^1(\Omega, \mathbb{C})$. Moreover, as in [BBH2], one has, for $\psi_\varepsilon = 1/\varepsilon^2(1 - |U_\varepsilon|^2) \ge 0$,

$$\begin{cases} 2\varepsilon^2 \Delta\psi_\varepsilon - \psi_\varepsilon \ge -4|\nabla U_\varepsilon|^2 \ge -C_1, & \text{for all } 0 < \varepsilon \le \varepsilon_*, \\ \psi_\varepsilon|_{\partial\Omega} = 0 \end{cases} \tag{3.8}$$

Let $x_0 \in \Omega$ be such that $\psi_\varepsilon(x_0) = \max_{x \in \bar{\Omega}} \psi_\varepsilon(x) > 0$, then $\Delta\psi_\varepsilon(x_0) \le 0$, and thus

$$\psi_\varepsilon(x_0) \le C_1. \tag{3.9}$$

From (3.9) we obtain $\|\Delta U_\varepsilon\|_{L^\infty(\Omega)} \le C_1$, and $U_\varepsilon \to U_0$ in $C^{1,\alpha}(\Omega), \forall\alpha < 1$, follows.

§4. Uniqueness problems

Let M be a compact Riemannian manifold with possible nonempty boundary ∂M, and let N be as before. The equation for harmonic maps $U: M \to N$ can be written as

$$\Delta U + A(U)(\nabla U, \nabla U) = 0 \tag{4.1}$$

where Δ is the Beltrami-operator on M, and $A(U)$ is the second fundamental form of N at U. Thus the corresponding equations for the heat flow are

$$\frac{\partial U}{\partial t} - \Delta U = A(U)(\nabla U, \nabla U), \qquad (x, t) \in M \times (0, \infty). \tag{4.2}$$

Given initial data $U_0: M \to N$, one then is interested in solving (4.2) with

$$U(x, t) = U_0(x) \qquad \text{for } (x, t) \in (M \times \{0\}) \cup (\partial M \times (0, \infty)). \tag{4.3}$$

Suppose U_0, M, ∂M and N are smooth. It is well-known (see, e.g. [J]) that the problem (4.2) (4.3) has a unique smooth solution $U(x, t)$, $(x, t) \in M \times [0, T]$, for some $T > 0$ which may depend on the various data mentioned. On the other hand, it was shown in [CS] (for the case $\partial M = \phi$) and [CL] that (4.2), (4.3) has a global weak-solution which is smooth off a relatively small closed subset of $M \times (0, \infty)$. One of the natural question is that whether such suitable weak solutions obtained in [CS] and [CL] are unique.

Here we want to show that the weak solution obtained in [CS] and [CL] must coincide with the classical solution on the time interval $[0, T^*]$, here $0 < T^* \le \infty$ is the first time of blow-up for the classical solution. The latter means that there is a smooth solution $\tilde{U}$ of (4.2), (4.3) on the time interval $[0, T^*)$ and that $\lim_{t \to T^*_-} \|\nabla \tilde{U}\|_{L^\infty(M)}(t) = +\infty$.

For this purpose, we adopt the same notations as that in [CS] and [CL]. Consider a sequence of approximate solutions U^k such that

$$\frac{\partial}{\partial t} U^k - \Delta U^k + k\chi'(d^2(U^k, N)) \frac{d}{dU}\left(\frac{d^2(U^k, N)}{2}\right) = 0 \tag{4.4}$$

in $M \times (0, \infty)$,

$$U^k(x, t) = (U_0(x), \qquad \text{on } (M \times \{0\}) \cup (\partial M \times (0, \infty)). \tag{4.5}$$

We claim there are positive constant C_0, T_0 depending only on $U_0, \partial M, M, N$ such that

$$\sup_{M \times [0, T_0]} e(U^k) \leq C_0, \qquad \text{for } k = 1, 2, \ldots \tag{4.6}$$

where $e(U^k) = |\nabla U^k|^2 + (k/2)\chi(d^2(U^k, N))$.

In fact, for any $x_0 \in M$, $0 < t_0 < R_M$ (R_M is the injectivity radius of M) and $r = \sqrt{t_0}$, one has that (we adopt the same notations as that in [CS] and [CL])

$$\begin{aligned}\Phi(r, U^k, (x_0, t_0)) &= \frac{t_0}{2} \int_{R^m} e(U^k) G_{(x_0, t_0)} |_{t=0}\, \phi^2(|x - x_0|)\, dx \\ &\leq \frac{t_0}{2}\left(\int_{B_{t_0^{(1-2)/2}}} + \int_{R^m \setminus B_{t_0^{(1-2)/2}}}\right) e(U^k) G_{(x_0, t_0)} |_{t=0}\, \phi^2(|x - x_0|)\, dx \\ &\leq C t_0^{1-(\varepsilon m/2)} \|U_0\|_{C^1(M)} + C t_0^{1-(m/2)} e^{-(1/t_0^{\varepsilon})} E(U_0) \leq \varepsilon_0,\end{aligned} \tag{4.7}$$

if $\varepsilon < 2/m$ and t_0 is suitably small. Then, (4.6) follows from the small energy regularity theorems in [CS] and [CL].

By the definition of $\chi(d^2)$ and (4.6) (cf. [CL]), one has $\chi(d^2(U^k, N)) = d^2(U^k, N)$ for all large k's. We claim that (4.6) implies that U^k converges to the classical solution $\tilde{U}$ in $W_p^{2,1}(M \times (0, T_0))$ as $k \to \infty$. Here

$$W_p^{2,1}(M \times (0, T_0)) = \{V : V, D_x V, D_x^2 V, V_t \in L^p(M \times (0, T_0))\}, \qquad 1 < p < \infty.$$

Suppose, for the moment, that the above claim is true. Then we want to show the weak solution obtained in [CS] and [CL] coincides with the classical solution on

$[0, T^*)$. To do so, we let $0 < T_0^* \le T^*$ be such that

$$T_0^* = \sup\{t \in [0, T) : \lim_{k \to \infty} U^k = \tilde{U}\}. \tag{4.8}$$

Here the limit is taking in $W_p^{2,1}(M \times (0, t)), (p \ge m + 1)$. If $T_0^* < T^*$, then for $\varepsilon_0 > 0$, then is an $r_0 > 0$, such that $C_m r^2 \int_{B2r(x_0)} |\nabla \tilde{U}|^2(t_0)\, dx < \varepsilon_0$, for all $0 < r \le r_0$, and $(x_0, t_0) \in \bar{M} \times [0, T_0^*]$. We let $t_1 < T_0^*$ be such that $T_0^* - t_1 \ll r_0^2$. Then since $U^k(\,\cdot\,, t_1) \to \tilde{U}(\,\cdot\,, t_1)$ in $W^{2,p}(M)$ as $k \to \infty$, we may assume, for all large $k's$, that

$$C_m r_0^{2-m} \oint_{B_{2r_0(x_0)}} |\nabla U^k|^2(t_1)\, dx < \varepsilon_0.$$

Then by small energy regularity theorem of [CS] and [CL], one has, as above, $U^k \to \tilde{U}$ in $W_p^{2,1}(t_1, t_1 + r_0)$. This contradicts the definition of T_0^*.

Finally we would like to prove the above claim.

Let $\psi_k = d(U^k, N)$, then by a simple calculation (cf. [CL], (4.18)]), one has, by (4.5)–(4.6), that

$$\begin{cases} \dfrac{\partial}{\partial t}\psi_k - \Delta\psi_k \le -k\psi_k + |\nabla U^k|^2 & \text{in } M \times [0, T_0] \\ \psi_k = 0 \quad \text{on } (M \times \{0\}) \cup (\partial M \times [0, T_0]). \end{cases} \tag{4.9}$$

In deriving (4.9), we have used the fact that $d(U^k, N) \to 0$ uniformly as $k \to \infty$. (cf. (4.6)). Again, by the maximum principle, one has

$$\max_{(x,t) \in M \times [0,T_0]} \psi_k \le \frac{1}{k} \max_{(x,t) \in M \times [0,T_0]} |\nabla U^k|^2 \le \frac{1}{k} C_0. \tag{4.10}$$

Hence from (4.4), $(\partial/\partial t)U^k - \Delta U^k \in L^\infty(M \times [0, T_0])$, and our claim follows from the standard L^p-theory for parabolic systems [LSU].

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