

**Zeitschrift:** Commentarii Mathematici Helvetici  
**Herausgeber:** Schweizerische Mathematische Gesellschaft  
**Band:** 69 (1994)

**Artikel:** Polynomial invariants of representations of quivers.  
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**DOI:** <https://doi.org/10.5169/seals-52251>

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## Polynomial invariants of representations of quivers

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Let  $k$  be an algebraically closed field of arbitrary characteristic. Recall that if an affine algebraic group  $G$  over  $k$  acts on an affine variety  $Z$  then we get an induced action of  $G$  on the coordinate algebra  $k[Z]$ , given by  $(x \cdot f)(z) = f(x^{-1}z)$ , for  $x \in G$ ,  $f \in k[Z]$  and  $z \in Z$ . We consider here the space  $R(Q, \alpha)$  of all  $k$ -representations of a quiver  $Q$  with given dimension vector  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ . There is a natural action of the product of general linear groups  $GL(\alpha) = GL(\alpha_1, k) \times GL(\alpha_2, k) \times \dots \times GL(\alpha_n, k)$  on  $R(Q, \alpha)$  and the purpose of this note is to describe generators for the algebra of polynomial invariants  $k[R(Q, \alpha)]^{GL(\alpha)_\theta}$  of the coordinate algebra  $k[R(Q, \alpha)]$ , where  $GL(\alpha)_\theta$  is the centralizer in  $GL(\alpha)$  of an element  $\theta$ . In particular we show that  $k[R(Q, \alpha)]^{GL(\alpha)}$  is generated by the coefficients of the characteristic polynomials of products over oriented cycles. In characteristic zero this is a result of Le Bruyn and Procesi, [4], Theorem 1. I am very grateful to Dr. W. W. Crawley-Boevey for bringing this problem to my attention.

By a quiver we mean a quadruple  $Q = (V, A, h, t)$ , consisting of the vertex set  $V = \{1, 2, \dots, n\}$ , a finite set  $A$  of arrows and maps  $h : A \rightarrow V$ ,  $t : A \rightarrow V$  which assign to an arrow  $a \in A$  its head,  $h(a)$ , and tail,  $t(a)$ .

Let  $E_1, E_2, \dots, E_n$  be finite dimensional  $k$ -vector spaces and let  $E = E_1 \oplus E_2 \oplus \dots \oplus E_n$ . Let  $\alpha_i = \dim_k E_i$ ,  $1 \leq i \leq n$  and let  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ . We write  $GL(\alpha) = GL(E_1) \times GL(E_2) \times \dots \times GL(E_n)$  and identify  $GL(\alpha)$  with a subgroup of  $GL(E)$ . Thus  $GL(\alpha)$  is the centralizer in  $GL(E)$  of a linear endomorphism  $\sigma$  of  $E$  which acts as multiplication by  $c_i$  on  $E_i$ ,  $1 \leq i \leq n$ , for distinct scalars  $c_1, c_2, \dots, c_n \in k$ . Then  $R(Q, \alpha) = \prod_{a \in A} \text{Hom}_k(E_{t(a)}, E_{h(a)})$  is the space of all  $k$ -representations of  $Q$  on the spaces  $E_1, E_2, \dots, E_n$ . Now  $GL(\alpha)$  acts rationally on  $R(Q, \alpha)$  by  $g \cdot (y_a)_{a \in A} = (g_{h(a)} y_a g_{t(a)}^{-1})_{a \in A}$ , for  $g = (g_1, g_2, \dots, g_n) \in GL(\alpha)$  and  $(y_a)_{a \in A} \in R(Q, \alpha)$ . For an endomorphism  $z$  of a  $k$ -vector space  $E$  of finite dimension  $d$  and non-negative integer  $s \leq d$  we let  $\chi_s(z)$  denote  $(-1)^s$  times the coefficient of  $t^{d-s}$  in the characteristic polynomial  $\det(tI - z)$  of  $z$  (where  $I$  is the identity map on  $E$ ). In the case in which  $Q$  has only one vertex the following becomes the description of generators of matrix invariants given in [3], §2, Theorem 1.

**PROPOSITION.** *The algebra of invariants  $k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  is generated by the functions  $(y_a)_{a \in A} \mapsto \chi_s(y_{a_r} \cdots y_{a_2} y_{a_1})$ , where  $(a_1, a_2, \dots, a_r)$  is an oriented cycle and  $s \geq 0$ .*

*Proof.* First suppose that  $Q$  is the quiver which has  $m$  arrows between each pair of vertices, i.e. there is a positive integer  $m$  such that for each  $(p, q) \in V \times V$  there are precisely  $m$  arrows  $a \in A$  with  $t(a) = p$  and  $h(a) = q$ . We write  $A$  as a disjoint union  $A = A_1 \cup A_2 \cup \cdots \cup A_m$  in such a way that for each  $(p, q) \in V \times V$  and  $1 \leq r \leq m$  there is exactly one element  $a \in A_r$  with  $t(a) = p$  and  $h(a) = q$ . We regard  $\text{End}_k(E)$  as a  $\text{GL}(\alpha)$ -module via conjugation and  $\text{End}_k(E)^m$  as the direct sum  $\text{End}_k(E) \oplus \text{End}_k(E) \oplus \cdots \oplus \text{End}_k(E)$ . Then we have an isomorphism of  $\text{GL}(\alpha)$ -modules (and varieties)  $\phi : R(Q, \alpha) \rightarrow \text{End}_k(E)^m$  given by  $\phi((y_a)_{a \in A}) = (\bar{y}_1, \bar{y}_2, \dots, \bar{y}_m)$ , for  $(y_a)_{a \in A} \in R(Q, \alpha)$ , where  $\bar{y}_r = \sum_{a \in A_r} y_a$ , for  $1 \leq r \leq m$ . Now the comorphism  $\phi^* : k[\text{End}_k(E)^m] \rightarrow k[R(Q, \alpha)]$  induces an isomorphism  $k[\text{End}_k(E)^m]^{\text{GL}(\alpha)} \rightarrow k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  on invariants. By [3], §2 Theorem 2, we have that  $k[\text{End}_k(E)^m]^{\text{GL}(\alpha)}$  is generated by the functions  $(\bar{y}_1, \bar{y}_2, \dots, \bar{y}_m) \mapsto \chi_u(\sigma^{q_1} \bar{y}_{i_1} \sigma^{q_2} \bar{y}_{i_2} \cdots \sigma^{q_r} \bar{y}_{i_r})$ , for  $r \geq 1$ ,  $q_1, q_2, \dots, q_r \geq 0$ ,  $u \geq 0$  and  $(i_1, i_2, \dots, i_r)$  an  $r$ -tuple with entries in  $\{1, 2, \dots, m\}$ . Therefore  $k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  is generated by functions of the form

$$(y_a)_{a \in A} \mapsto \chi_u \left( \sigma^{q_1} \left( \sum_{a \in A_{i_1}} y_a \right) \sigma^{q_2} \left( \sum_{a \in A_{i_2}} y_a \right) \cdots \sigma^{q_r} \left( \sum_{a \in A_{i_r}} y_a \right) \right)$$

with  $r, (i_1, i_2, \dots, i_r)$ , and  $q_1, q_2, \dots, q_r$  as above. However,  $\sigma y_a = c_{h(a)} y_a$  for  $a \in A$ , so the above function is

$$(y_a)_{a \in A} \mapsto \chi_u \left( \left( \sum_{a \in A_{i_1}} c_{h(a)}^{q_1} y_a \right) \left( \sum_{a \in A_{i_2}} c_{h(a)}^{q_2} y_a \right) \cdots \left( \sum_{a \in A_{i_r}} c_{h(a)}^{q_r} y_a \right) \right).$$

However, as is well known, a signed coefficient  $\chi_u(z_1 + z_2)$ , of the characteristic polynomial of a sum of endomorphisms  $z_1, z_2$ , is a linear combination of products of the coefficients of the characteristic coefficients in monomials in  $z_1$  and  $z_2$ . (Also, this follows from the main result of [3], since the function  $(z_1, z_2) \mapsto \chi_u(z_1 + z_2)$  is a polynomial invariant for the action of the general linear group by simultaneous conjugation on pairs on endomorphisms.) Hence the above function is a linear combination of products of functions of the form

$$(y_a)_{a \in A} \mapsto \chi_s(y_{a_r} \cdots y_{a_2} y_{a_1})$$

for  $a_1, a_2, \dots, a_r \in A, s \geq 0$ . Now, for  $a, b \in A$ , we have  $y_a \in \text{Hom}_k(E_{t(a)}, E_{h(a)})$  and  $y_b \in \text{Hom}_k(E_{t(b)}, E_{h(b)})$  so that  $y_a y_b$  is zero unless  $h(a) = t(b)$ . Thus  $y_{a_r} \cdots y_{a_2} y_{a_1}$  is

zero unless  $h(a_1) = t(a_2), h(a_2) = t(a_3), \dots, h(a_r) = t(a_1)$ . Moreover,  $y_{a_r} \cdots y_{a_2} y_{a_1}$  belongs to  $\text{Hom}_k(E_{t(a_1)}, E_{h(a_r)}) \leq \text{End}_k(E)$  and, for an element  $z$  of  $\text{Hom}_k(E_i, E_j)$ , we have  $\chi_s(z) = 0$  for all  $s > 0$  unless  $i = j$ . Hence  $k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  is generated by the functions  $(y_a)_{a \in A} \mapsto \chi_s(y_{a_r} \cdots y_{a_2} y_{a_1})$ , where  $(a_1, a_2, \dots, a_r)$  is an oriented cycle and  $s \geq 0$ .

To conclude we make use of the elementary remark that if  $L = M \oplus N$  is a direct sum decomposition of a finite dimensional rational  $H$ -module  $L$ , where  $H$  is an affine algebraic group over  $k$ , then the restriction map  $k[L] \rightarrow k[M]$  is a split surjection of  $H$ -modules and hence the induced map  $k[L]^H \rightarrow k[M]^H$  is surjective. So now let  $Q$  be arbitrary. Let  $m$  be a positive integer such that for each  $(p, q) \in V \times V$  the number of arrows  $a \in A$  satisfying  $t(a) = p$  and  $h(a) = q$  is at most  $m$ . Let  $\hat{Q}$  be a (complete) quiver on the same vertex set  $V$  with set of arrows  $\hat{A}$  containing  $A$  such that for each  $(p, q) \in V \times V$  there are exactly  $m$  arrows  $a \in \hat{A}$  with  $t(a) = p$  and  $h(a) = q$ . Let  $Q'$  be the complement of  $Q$  in  $\hat{Q}$ , i.e. the quiver on vertex set  $V$  with arrows  $A' = \hat{A} \setminus A$ . We identify  $R(Q, \alpha)$  with the subspace of  $R(\hat{Q}, \alpha)$  consisting of the elements  $(y_a)_{a \in \hat{A}}$  such that  $y_a = 0$  for  $a \notin A$ . We similarly identify  $R(Q', \alpha)$  with a subspace of  $R(\hat{Q}, \alpha)$ . Then  $R(\hat{Q}, \alpha) = R(Q, \alpha) \oplus R(Q', \alpha)$  is a decomposition of  $\text{GL}(\alpha)$ -modules. Hence the map  $k[R(\hat{Q}, \alpha)]^{\text{GL}(\alpha)} \rightarrow k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  is surjective. By the case already considered  $k[R(\hat{Q}, \alpha)]^{\text{GL}(\alpha)}$  is generated by the functions  $(y_a)_{a \in \hat{A}} \mapsto \chi_s(y_{a_r} \cdots y_{a_2} y_{a_1})$ , where  $(a_1, a_2, \dots, a_r)$  is an oriented cycle in  $\hat{A}$ , and  $s \geq 0$ . By restricting these functions to  $R(Q, \alpha)$  we get that  $k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  is generated by the functions  $(y_a)_{a \in A} \mapsto \chi_s(y_{a_r} \cdots y_{a_2} y_{a_1})$ , where  $(a_1, a_2, \dots, a_r)$  is an oriented cycle in  $A$ , and  $s \geq 0$ .

We now generalize this result to give generators for the invariants of  $k[R(Q, \alpha)]$ , for the action of a centralizer in  $\text{GL}(\alpha)$ . In the case  $[V] = 1$  this is [3], §2, Theorem 2, and in general follows from the Proposition above in the same way that [3], §2, Theorem 2 follows from [3], §2, Theorem 1.

Let  $\theta_i \in \text{End}_k(E_i)$  and let  $\text{GL}(E_i)_{\theta_i}$  be the centralizer of  $\theta_i$  in  $\text{GL}(E_i)$ ,  $1 \leq i \leq n$ . Let  $\theta = (\theta_1, \theta_2, \dots, \theta_n) \in \text{End}_k(E_1) \oplus \text{End}_k(E_2) \oplus \cdots \oplus \text{End}_k(E_n)$  and let  $\text{GL}(\alpha)_\theta = \text{GL}(E_1)_{\theta_1} \times \text{GL}(E_2)_{\theta_2} \times \cdots \times \text{GL}(E_n)_{\theta_n}$ .

We recall, from [2], the notion of a good pair of varieties. Let  $G$  be a reductive affine algebraic group over  $k$ . By a *good filtration* of a rational  $G$ -module  $M$  we mean an ascending filtration  $0 = M_0 \leq M_1 \leq M_2 \leq \cdots$  such that, for each  $i > 0$ , the section  $M_i/M_{i+1}$  is either zero or isomorphic to a module induced from a one dimensional module for a Borel subgroup of  $G$ . We call  $Z$  a good  $G$ -variety if the coordinate algebra  $k[Z]$  admits a good  $G$ -module filtration. By a *good pair* of  $G$  varieties we mean a pair  $(Z, T)$ , where  $Z$  is an affine  $G$ -variety,  $T$  is a closed  $G$ -stable subvariety of  $Z$  and the  $G$ -modules  $k[Z]$  and  $I_T$  admit good  $G$ -module filtrations, where  $I_T \leq k[Z]$  is the ideal of  $T$ . Recall from [2], §1.3, that if  $(Z, T)$  is

a good pair of  $G$ -varieties then  $Z$  and  $T$  are good  $G$ -varieties. The main point about good pairs, as far as invariant theory is concerned, is that the restriction map on fixed points  $k[Z]^G \rightarrow k[T]^G$  is surjective (by [2], Proposition 1.4a).

**THEOREM.** *The algebra of invariants  $k[R(Q, \alpha)]^{\text{GL}(\alpha)_\theta}$  is generated by the functions  $(y_a)_{a \in A} \mapsto \chi_s(\theta_{h(a_r)}^{q_r} y_{a_r} \cdots \theta_{h(a_2)}^{q_2} y_{a_2} \theta_{h(a_1)}^{q_1} y_{a_1})$ , for  $(a_1, a_2, \dots, a_r)$  an oriented cycle,  $q_1, q_2, \dots, q_r \geq 0$  and  $s \geq 0$ .*

*Proof.* Let  $Q^+$  be the quiver  $(V^+, A^+, t^+, h^+)$  obtained from  $Q$  by adding  $n$  extra loops, one at each vertex. Thus we have  $V^+ = V$ ,  $A^+ = A \cup L$ , the disjoint union of  $A$  and  $L = \{l_1, l_2, \dots, l_n\}$ ,  $t^+|_A = t$ ,  $h^+|_A = h$ , and  $t^+(l_i) = h^+(l_i) = i$ , for  $1 \leq i \leq n$ . We let  $\alpha^+ = \alpha$  and take

$$R(Q^+, \alpha^+) = \prod_{a \in A^+} \text{Hom}_k(E_{t(a)}, E_{h(a)}) = R(Q, \alpha) \times S$$

where  $S = \prod_{l \in L} \text{Hom}_k(E_{t(l)}, E_{h(l)}) = \bigoplus_{i=1}^n \text{End}_k(E_i)$ .

For any affine  $\text{GL}(\alpha)$ -variety  $X$  we have

$$k[X]^{\text{GL}(\alpha)_\theta} \cong (k[X] \otimes \text{Ind}_{\text{GL}(\alpha)_\theta}^{\text{GL}(\alpha)} k)^{\text{GL}(\alpha)}$$

by Frobenius reciprocity and the tensor identity for induction (see also [5], (1.4)). But we have the natural isomorphism  $\text{Ind}_{\text{GL}(\alpha)_\theta}^{\text{GL}(\alpha)} k \cong k[\overline{C(\theta)}]$ , by [2], Theorem 2.2a (iii), where  $\overline{C(\theta)}$  is the Zariski closure of the  $\text{GL}(\alpha)$  conjugacy class  $C(\theta)$  of  $\theta$ . Thus we get  $k[X]^{\text{GL}(\alpha)_\theta} \cong k[\overline{\text{GL}(\alpha) \times C(\theta)}]^{\text{GL}(\alpha)}$ . Explicitly, we have the isomorphism  $\xi : k[X \times \overline{C(\theta)}]^{\text{GL}(\alpha)} \rightarrow k[X]^{\text{GL}(\alpha)}$ ,  $\xi(f)(x) = f(x, \theta)$ , for  $f \in k[X \times \overline{C(\theta)}]^{\text{GL}(\alpha)}$ ,  $x \in X$ . Assume now that  $X$  is a good  $G$ -variety. Then  $(S, \overline{C(\theta)})$  is a good pair of  $\text{GL}(\alpha)$ -varieties by [2], Theorem 2.2a (ii), so that  $(X \times S, X \times \overline{C(\theta)})$  is a good pair of  $\text{GL}(\alpha)$ -varieties, by [2], Proposition 1.3c (i). Hence the restriction  $k[X \times S]^{\text{GL}(\alpha)} \rightarrow k[X \times \overline{C(\theta)}]^{\text{GL}(\alpha)}$  is surjective, by [2], Lemma 2.3a (or Proposition 1.4a). Thus, for a good  $\text{GL}(\alpha)$ -variety  $X$ , we have the surjective map  $\eta : k[X \times S]^{\text{GL}(\alpha)} \rightarrow k[X]^{\text{GL}(\alpha)_\theta}$ ,  $\eta(f)(x) = f(x, \theta)$ , for  $f \in k[X \times \text{End}_k(E)^n]^{\text{GL}(\alpha)}$ ,  $x \in X$ .

Now take  $X = R(Q, \alpha)$ . Then  $X \times S = R(Q^+, \alpha^+)$  so every  $\text{GL}(\alpha)_\theta$  invariant of  $k[R(Q, \alpha)]$  has the form  $f|_{y_l = \theta_{h(l)}, l \in L}$ , for some  $f \in k[R(Q^+, \alpha^+)]^{\text{GL}(\alpha)}$ . But by the Proposition  $k[R(Q^+, \alpha^+)]^{\text{GL}(\alpha)}$  is generated by the functions  $(y_b)_{b \in A^+} \mapsto \chi_s(y_{b_1} \cdots y_{b_2} y_{b_1})$ , with  $(b_1, b_2, \dots, b_l)$  an oriented cycle. Specializing  $y_l$  to  $\theta_{h(l)}$ , for  $l \in L$ , gives generators of the form described in the theorem.

*Remark.* We now deduce that the Proposition is valid over  $\mathbb{Z}$  and stable under base change. The arguments are entirely analogous to those of [3], §3,1 so we shall

be brief about the details. Let  $E_{i,\mathbb{Z}}$  be a free  $\mathbb{Z}$ -module of rank  $\alpha_i$ ,  $1 \leq i \leq n$ . Let  $\text{GL}(\alpha)$  be the product of general linear group schemes corresponding to  $E_{1,\mathbb{Z}}, E_{2,\mathbb{Z}}, \dots, E_{n,\mathbb{Z}}$ . Let  $R(Q, \alpha)_{\mathbb{Z}} = \prod_{a \in A} \text{Hom}_{\mathbb{Z}}(E_{t(a), \mathbb{Z}}, E_{h(a), \mathbb{Z}})$  and let  $\mathbb{Z}[R(Q, \alpha)]$  be the symmetric algebra on the dual abelian group  $\text{Hom}_{\mathbb{Z}}(R(Q, \alpha)_{\mathbb{Z}}, \mathbb{Z})$ . Let  $J = \mathbb{Z}[R(Q, \alpha)]^{\text{GL}(\alpha)}$  and let  $J'$  be the subring generated by the coefficients of the characteristic polynomials of products, taken over oriented cycles, of elements of  $\text{Hom}_{\mathbb{Z}}(E_{t(a), \mathbb{Z}}, E_{h(a), \mathbb{Z}})$ ,  $a \in A$ . Let  $A = \{a_1, a_2, \dots, a_m\}$  have cardinality  $m$ . Then there is a natural multigrading  $\mathbb{Z}[R(Q, \alpha)] = \bigoplus_{\omega \in \mathbb{N}_0^m} \mathbb{Z}[R(Q, \alpha)]_{\omega}$ , such that a non-zero element of the dual of  $\text{Hom}_{\mathbb{Z}}(E_{t(a_r), \mathbb{Z}}, E_{h(a_r), \mathbb{Z}})$  has degree  $(0, \dots, 0, 1, 0, \dots, 0)$  (1 in the  $r$ th position), for  $1 \leq r \leq m$ . This induces multigradings on  $J, J', k[R(Q, \alpha)]$  and  $k[R(Q, \alpha)]^{\text{GL}(\alpha)}$ . For  $\omega \in \mathbb{N}_0^m$ , the component  $k[R(Q, \alpha)]_{\omega}$  has a good  $\text{GL}(\alpha)$ -filtration (e.g. by [1], Corollary 3.2.6 and the fact that  $R(Q, \alpha)$  is a good  $\text{GL}(\alpha)$ -variety). Furthermore, the formal character of  $k[R(Q, \alpha)]_{\omega}$  is independent of the field  $k$  and determines  $\dim_k k[R(Q, \alpha)]_{\omega}^{\text{GL}(\alpha)}$ , which is therefore also independent of the field  $k$ . By the Proposition, the natural map  $k \otimes_{\mathbb{Z}} J'_{\omega} \rightarrow k[R(Q, \alpha)]_{\omega}^{\text{GL}(\alpha)}$  is surjective for every algebraically closed field  $k$ . It follows that  $J = J'$  and that the natural map  $k \otimes_{\mathbb{Z}} J \rightarrow k[R(Q, \alpha)]^{\text{GL}(\alpha)}$  is an isomorphism, for every algebraically closed field  $k$ .

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Received November 9, 1992