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## Groups with no infinite perfect subgroups and aspherical 2-complexes

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Abstract. The purpose of this paper is to generalize a theorem of J. F. Adams. He showed in [A] that if X is a subcomplex of an aspherical 2-complex and the fundamental group G of X has no non-trivial perfect subgroups, then X is aspherical. We weaken the hypothesis on G to "no infinite perfect subgroups."

## 1. Introduction

In [W], J. H. C. Whitehead, asked the following question: Is a subcomplex of an aspherical 2-complex aspherical?

A [G, 2]-complex X is a connected two-dimensional CW-complex with fundamental group  $\pi_1 X \cong G$ . If N is a subgroup of  $\pi_1 X$ , let  $X_N$  denote the covering of X corresponding to N. For any group G, let  $H_iG$  denote the ith homology of G with coefficients in the integers  $\mathbb{Z}$ . A group G is said to be perfect if the abelianization  $H_1G$  of G is trivial; G is superperfect if  $H_1G = H_2G = 0$ .

A [G, 2]-complex X is aspherical iff its second homotopy group  $\pi_2 X$  vanishes. If X is a [G, 2]-complex which is a subcomplex of an aspherical 2-complex, then J. F. Adams showed in [A] that X is aspherical provided G has no non-trivial perfect subgroups. In this note we show that X is aspherical provided G is finitely presented and has no *infinite* perfect subgroups.

The idea of the proof is to show that if X is a [G, 2]-complex and G is a finitely presented group which has a finite, non-trivial, normal, superperfect subgroup P such that Q = G/P has cohomological dimension 1 or 2, then the Hurewicz homomorphism  $\pi_2 X \to H_2 X_P$  is non-trivial.

### 2. Basic definitions

If X is a connected 2-complex and N is a subgroup of  $\pi_1 X$  then X is N-Cockcroft if the Hurewicz homomorphism  $\pi_2 X = \pi_2(X_N) \to H_2(X_N)$  is trivial. The N-Cockcroft property has been extensively studied in [Bo, BD, BDS, D, GH, H].

Let N be a subgroup of G. Then we say that G is N-Cockcroft if there is a [G, 2]-complex X and an isomorphism  $\varphi : G \to \pi_1 X$  such that X is  $\varphi$ N-Cockcroft.

The following is the main theorem of this paper.

2.1 THEOREM. Let P be a non-trivial, finite, superperfect, normal subgroup of a finitely presented group G such that Q = G/P has cohomological dimension 1 or 2. Then G is not P-Cockcroft.

Note that the theorem is false if Q = 1. In this case, G = P is finite and superperfect. Let G be the binary icosahedral group. In this case, G admits a presentation with 2 generators and 2 relators. The realization of this presentation as a [G, 2]-complex has  $H_2X = 0 = H_1X$ , so X is P-Cockcroft.

If G is a group, the maximal perfect subgroup PG of G is defined as the normal subgroup of G generated by all perfect subgroups; it is also the intersection of the (transfinite) derived series of G.

2.2 COROLLARY. Let G be a finitely presented group with maximal perfect subgroup PG finite. Then any [G, 2]-complex X which is the subcomplex of an aspherical 2-complex is aspherical.

*Proof.* If G is finite, the result is well known (see [BD]). Hence we will assume that Q is infinite. If the [G, 2]-complex X is a subcomplex of an aspherical 2-complex and X is not aspherical, then by the main theorem of [BDS], we see that there must exist a superperfect, normal, non-trivial subgroup P of G such that G is P-Cockcroft and the quotient Q has  $\operatorname{cd} Q \leq 2$ . The group Q is infinite, so the cohomological dimension of Q is 1 or 2. But the maximal perfect subgroup of G is finite, so P is infinite. The theorem then says that G cannot be P-Cockcroft. Thus X must be aspherical.  $\square$ 

## 3. Two lemmas

In this section we will prove two lemmas preliminary to giving a proof of the theorem.

Let G be a group and let  $\mathbb{C}$  be a projective  $\mathbb{Z}G$ -resolution of the trivial  $\mathbb{Z}G$ -module  $\mathbb{Z}$ . To each integer  $i \geq 0$  we have an associated kernel  $K_i = \ker \left\{ \partial_i : C_i \to C_{i-1} \right\}$   $(C_{-1} = \mathbb{Z})$ . For any [G, 2]-complex X, let  $\tilde{X}$  be the universal covering of X. Then  $C_*\tilde{X}$ , the cellular chain complex of  $\tilde{X}$ , can be thought of as a partial resolution (of length two) of free left  $\mathbb{Z}G$ -modules. For any [G, 2]-complex X, the kernel  $K_1 = \ker \left\{ \partial_1 : C_1\tilde{X} \to C_0\tilde{X} \right\}$  is called the relation module determined by X.

For any left  $\mathbb{Z}G$ -module M, we let  $M^G$  denote the subgroup of elements fixed by the action of G; we let  $M_G = \mathbb{Z} \otimes_{\mathbb{Z}G} M = M/IG \cdot M$  (IG is the augmentation ideal in  $\mathbb{Z}G$ ) be M with the G-action killed.

3.1 LEMMA. If P is a finite, normal subgroup of a group G and Q = G/P, then  $H^i(G, \mathbb{Z}G) \cong H^i(G, \mathbb{Z}Q) \cong H^i(Q, \mathbb{Z}Q)$  for all i > 0. The first isomorphism is induced by  $\mathbb{Z}G \to \mathbb{Z}Q$  and the second by  $G \to Q$ .

*Proof.* Because P is finite, we have  $H^{j}(P, \mathbb{Z}G) = 0$  for j > 0. By using the Lyndon-Hochschield-Serre spectral sequence, we see that  $H^{i}(G, \mathbb{Z}G) \cong H^{i}(Q, \mathbb{Z}G^{P})$  for i > 0. But clearly  $\mathbb{Z}G^{P} \cong \bigoplus_{a \in Q} (\mathbb{Z}P)_{a}^{P} \cong \bigoplus_{a \in Q} (\mathbb{Z})_{a} \cong \mathbb{Z}Q$  as a  $\mathbb{Z}Q$ -module.  $\square$ 

3.2 LEMMA. Let X be a [G, 2]-complex and suppose P is a superperfect, normal subgroup of  $\pi_1 X$  such that the Hurewicz map from  $\pi_2 X = \pi_2 X_P \to H_2 X_P$  is trivial (i.e., G is P-Cockcroft with respect to X). Let  $K_1 = \ker \left\{ \partial_1 : C_1 \widetilde{X} \to C_0 \widetilde{X} \right\}$  be the relation module determined by X, where  $\widetilde{X}$  is the universal covering of X. Then  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1 = \ker \left\{ \partial_1(X_P) : C_1(X_P) \to C_0(X_P) \right\} \cong \mathbb{Z} \otimes_{\mathbb{Z}P} C_2 \widetilde{X}$  is a relation module for  $Q = (\pi_1 X)/P$ . Furthermore, the surjection  $G \to Q$  induces an isomorphism  $H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) \cong H^2(Q, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)$ .

*Proof.* Because P is a subgroup of  $\pi_1 X$  we have  $C_i X_P \cong \mathbb{Z} \otimes_{\mathbb{Z}P} C_i \tilde{X}$ . That P is superperfect and G is P-Cockcroft with respect to X implies that

$$0 \to C_2 X_P \to C_1 X_P \to C_0 X_P \to \mathbb{Z} \to 0$$

is an exact sequence of free  $\mathbb{Z}Q$ -modules (a free resolution of the trivial module  $\mathbb{Z}$ ). Tensoring the exact sequence (of  $\mathbb{Z}G$ -modules)  $0 \to \pi_2 X \to C_2 \widetilde{X} \to K_1 \to 0$  with  $\mathbb{Z} \otimes_{\mathbb{Z}P}$  and using the fact that X is P-Cockcroft, we see that  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1 \cong C_2 X_P$ . The isomorphism  $H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) \cong H^2(Q, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)$  follows from the LHS

$$1 \rightarrow P \rightarrow G \rightarrow Q \rightarrow 1$$

spectral sequence for the extension

together with the facts that P is superperfect and that  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is a trivial  $\mathbb{Z}P$ -module.  $\square$ 

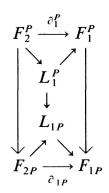
#### 4. Proof of Theorem 2.1

From now on we assume that X is a [G, 2]-complex with fundamental group equal to G. We let P be a finite, superperfect, normal subgroup of G so that the Hurewicz map  $\pi_2 X \to H_2 X_P$  is trivial. We let Q = G/P have cohomological dimension 1 or 2 and  $K_1$  be the relation module determined by X. The proof by contradiction is given in a series of steps as follows.

STEP 1 is devoted to the proof of the following claim. Let p be the order of the finite group P and consider the inclusion  $K_1^P \to \mathbb{Z} \otimes_{\mathbb{Z}P} K_1 = K_{1P}$ .

CLAIM. If P is superperfect, then the image of  $K_1^P$  inside  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is  $p \cdot \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$ .

*Proof of the Claim.* Let  $F_2 \to F_1 \to \mathbb{Z}P \to \mathbb{Z} \to 0$  be a partial resolution of  $\mathbb{Z}$  over  $\mathbb{Z}P$  by finitely generated free modules. Let  $L_1$  denote the kernel of the map  $\partial_1: F_1 \to \mathbb{Z}P$ . Then the following diagram commutes:



The group P is finite implies that the vertical arrows are monomorphisms. The two outer vertical arrows are clearly multiplication by p because the modules are free. The group P is perfect implies that  $\partial_{1P}$  and  $\partial_{1}^{P}$  are epimorphisms and hence  $L_{1P} = F_{1P}$  and  $L_{1}^{P} = F_{1}^{P}$ . Thus the interior vertical arrow has image which is multiplication by p. Now one uses Schanuel's lemma and a simple argument to show that the same is true of  $K_{1}^{P} \to K_{1P}$ . This completes the proof of the claim.

Hence the  $\mathbb{Z}Q$ -module  $A = \mathbb{Z} \otimes_{\mathbb{Z}P} K_1/K_1^P = \mathbb{Z} \otimes_{\mathbb{Z}P} K_1/p \cdot \mathbb{Z} \otimes_P K_1$ . If we write  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1 \cong \mathbb{Z}Q^{\alpha}$  ( $= \mathbb{Z} \otimes_{\mathbb{Z}P} C_2 \tilde{X}$ ; this follows from lemma 3.2), then  $A \cong \mathbb{Z}_p Q^{\alpha}$ .

STEP 2. The following diagram is commutative, with top and vertical sequences exact:

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad$$

The horizontal maps f and f' are induced by  $K_1 \to \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  and  $K_1^P \to \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$ , respectively. By using a dimension shifting argument one shows that  $H^2(P, K_1) \cong \mathbb{Z}_p$  has trivial  $\mathbb{Z}Q$ -action. The fact that  $p \cdot A = 0$  shows that  $p \cdot H^2(Q, A) = 0$  also. The vertical sequences come from the LHS-spectral sequence. The left-most vertical sequence is exact, because  $\mathrm{cd}\ Q \le 2$  and  $H^1(P, K_1) = 0$  (this is a consequence of the finiteness of P). The fact that  $H^2(P, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) = 0$  follows because P is superperfect and  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is a trivial  $\mathbb{Z}P$ -module. We observe that the map f' is an isomorphism modulo torsion; that is to say, the kernel and the cokernel of f' are torsion groups. The group  $H^3(Q, K_1^P) = 0$  because Q is two dimensional. By lemma 3.2,  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is a free  $\mathbb{Z}Q$ -module, so f is an isomorphism, by lemma 3.1.

STEP 3. Let M be any  $\mathbb{Z}G$ -module and  $\rho(M): M \to \mathbb{Z} \otimes_{\mathbb{Z}P} M$  be the natural surjection. We will show that  $\rho(K_1): K_1 \to \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  induces a split epimorphism

$$f: H^2(G, K_1) \to H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1).$$

We will show that there is a map  $s: H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) \to H^2(G, K_1)$  such that fs is an isomorphism.

Now  $H^2(G, C_2\tilde{X}) \cong H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)$ , by lemma 3.1; the isomorphism is induced by  $\rho(K_1) \partial_2$ , where  $\partial_2 : \mathbb{Z}G^{\alpha} = C_2\tilde{X} \to K_1$ . This last follows because  $\rho(K_1)\partial_2 = (1 \otimes \partial_2)\rho(C_2\tilde{X})$ . The map  $1 \otimes \partial_2$  is an isomorphism because G is P-Cockcroft and  $\rho(C_2\tilde{X})$  induces an isomorphism on  $H^2(G, -)$  by lemma 3.1. Thus the map  $\partial_2$  induces a map  $g: H^2(G, C_2\tilde{X}) \to H^2(G, K_1)$  whose composite gf is induced by the natural map  $\mathbb{Z}G^{\alpha} \to \mathbb{Z}Q^{\alpha}$ . Thus gf is an isomorphism, again by 3.1. Hence f is a split epimorphism and the map g can be chosen as  $g = \partial_{2*}(\rho(K_1) \partial_2)^{-1}$ .

STEP 4. We will show that, if  $i: H^2(G, K_1^P) \to H^2(G, K_1)$  is the map in diagram 4.1, then im s = im i.

First we observe that, by definition, im  $s = \text{im } \partial_{2*}$ . Let  $K_2 = \text{ker } \partial_2$  and consider the long exact sequence arising from the short exact sequence  $0 \to K_2 \to C_2 \tilde{X} \to K_1 \to 0$ ;

$$\cdots \to H^2(G, C_2 \widetilde{X}) \xrightarrow{\ell_2 *} H^2(G, K_1) \to H^3(G, K_2) \to H^3(G, C_2 \widetilde{X}) = 0.$$

The group  $H^3(G, C_2 \tilde{X}) = 0$  by 3.1 and the fact that cd  $Q \le 2$  (3.2).

The commutativity of the diagram below (where we identify  $H^3(P, K_2)$  with  $H^2(P, K_1)$ ) shows that im  $i = \text{im } \partial_{2*} = \text{im } s$ :

$$H^{2}(G, K_{1}^{P})$$

$$\downarrow i$$

$$H^{2}(G, C_{2}\tilde{X}) \xrightarrow{\partial_{2}*} H^{2}(G, K_{1}) \longrightarrow H^{3}(G, K_{2}) \longrightarrow 0$$

$$\downarrow h \qquad \qquad \downarrow \cong$$

$$H^{2}(P, K_{1})^{Q} \longrightarrow H^{3}(P, K_{2})^{Q}.$$

$$(4.2)$$

STEP 5. We show that  $\mathbb{Z}_p \otimes H^2(Q, \mathbb{Z}Q) = 0$ .

The map fi (see 4.1) is an isomorphism because  $\ker f \cap \operatorname{im} i = \ker f \cap \operatorname{im} s = 0$ . This implies  $f' = j^{-1}fi$  is an isomorphism. Thus,  $H^2(Q, A) = 0$  and hence  $\mathbb{Z}_p \otimes H^2(Q, \mathbb{Z}Q) = H^2(Q, \mathbb{Z}_pQ) = 0$ .

STEP 6. The contradiction.

Case 1 (Q is free). The same proof above works (by simply reducing the dimension of the cohomology groups and the kernels by one in 4.1 and 4.2) to show that  $\mathbb{Z}_p \otimes H^1(Q, \mathbb{Z}Q) = 0$ . But this is impossible because  $H^1(Q, \mathbb{Z}Q)$  is known to be free abelian and non-trivial [Sw, corollary 3.7]. Thus, G is P-Cockcroft and Q free leads to a contradiction.

Case 2: (cd Q = 2). Because P is finite and cd Q = 2 we have that  $\mathbb{Z}_p \otimes H^2(Q, \mathbb{Z}Q) = 0$  by step 5.

Because G is finitely presented, so is Q. We observe that ([BE], theorem 5.2) Q is a free product of duality groups of dimension 1 or 2. Let R be one of the factors, and define  $D = H^2(Q, \mathbb{Z}Q)$  and  $E = H^2(R, \mathbb{Z}R)$ . Let q be any prime divisor of p. The fact that  $\mathbb{Z}_p \otimes D = 0$  implies that  $\mathbb{Z}_q \otimes D = 0$ . This in turn implies that  $\mathbb{Z}_q \otimes E = 0$ . If R is a duality group of dimension 2, we have, for any  $\mathbb{Z}_q Q$ -module M,  $H^2(R, M) \cong \mathbb{Z} \otimes_{\mathbb{Z}R} (M \otimes D)$ . But because M is a  $\mathbb{Z}_q$ -module, we have  $M \otimes D \cong \mathbb{Z}_q \otimes M \otimes D = 0$ . Hence, the cohomological dimension of  $R \leq 1$  over the ring  $\mathbb{Z}_q$ . This, together with the fact that R is torsion-free, shows that cd R = 1. Hence R is free and so Q is free. This brings us back to case 1. Hence no such group G can be P-Cockcroft. This finishes the proof of Theorem 2.1.  $\square$ 

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