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**Autor:** Dyer, Michael N.  
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## Groups with no infinite perfect subgroups and aspherical 2-complexes

MICHEAL N. DYER

*Abstract.* The purpose of this paper is to generalize a theorem of J. F. Adams. He showed in [A] that if  $X$  is a subcomplex of an aspherical 2-complex and the fundamental group  $G$  of  $X$  has no non-trivial perfect subgroups, then  $X$  is aspherical. We weaken the hypothesis on  $G$  to “no infinite perfect subgroups.”

### 1. Introduction

In [W], J. H. C. Whitehead, asked the following question: *Is a subcomplex of an aspherical 2-complex aspherical?*

A  $[G, 2]$ -complex  $X$  is a connected two-dimensional CW-complex with fundamental group  $\pi_1 X \cong G$ . If  $N$  is a subgroup of  $\pi_1 X$ , let  $X_N$  denote the covering of  $X$  corresponding to  $N$ . For any group  $G$ , let  $H_i G$  denote the  $i$ th homology of  $G$  with coefficients in the integers  $\mathbb{Z}$ . A group  $G$  is said to be *perfect* if the abelianization  $H_1 G$  of  $G$  is trivial;  $G$  is *superperfect* if  $H_1 G = H_2 G = 0$ .

A  $[G, 2]$ -complex  $X$  is *aspherical* iff its second homotopy group  $\pi_2 X$  vanishes. If  $X$  is a  $[G, 2]$ -complex which is a subcomplex of an aspherical 2-complex, then J. F. Adams showed in [A] that  $X$  is aspherical provided  $G$  has no non-trivial perfect subgroups. In this note we show that  $X$  is aspherical provided  $G$  is finitely presented and has no *infinite* perfect subgroups.

The idea of the proof is to show that if  $X$  is a  $[G, 2]$ -complex and  $G$  is a finitely presented group which has a finite, non-trivial, normal, superperfect subgroup  $P$  such that  $Q = G/P$  has cohomological dimension 1 or 2, then the Hurewicz homomorphism  $\pi_2 X \rightarrow H_2 X_P$  is non-trivial.

### 2. Basic definitions

If  $X$  is a connected 2-complex and  $N$  is a subgroup of  $\pi_1 X$  then  $X$  is  *$N$ -Cockcroft* if the Hurewicz homomorphism  $\pi_2 X = \pi_2(X_N) \rightarrow H_2(X_N)$  is trivial. The  $N$ -Cockcroft property has been extensively studied in [Bo, BD, BDS, D, GH, H].

Let  $N$  be a subgroup of  $G$ . Then we say that  $G$  is  *$N$ -Cockcroft* if there is a  $[G, 2]$ -complex  $X$  and an isomorphism  $\varphi : G \rightarrow \pi_1 X$  such that  $X$  is  $\varphi N$ -Cockcroft.

The following is the main theorem of this paper.

**2.1 THEOREM.** *Let  $P$  be a non-trivial, finite, superperfect, normal subgroup of a finitely presented group  $G$  such that  $Q = G/P$  has cohomological dimension 1 or 2. Then  $G$  is not  $P$ -Cockcroft.*

Note that the theorem is false if  $Q = 1$ . In this case,  $G = P$  is finite and superperfect. Let  $G$  be the binary icosahedral group. In this case,  $G$  admits a presentation with 2 generators and 2 relators. The realization of this presentation as a  $[G, 2]$ -complex has  $H_2X = 0 = H_1X$ , so  $X$  is  $P$ -Cockcroft.

If  $G$  is a group, the *maximal perfect subgroup*  $PG$  of  $G$  is defined as the normal subgroup of  $G$  generated by all perfect subgroups; it is also the intersection of the (transfinite) derived series of  $G$ .

**2.2 COROLLARY.** *Let  $G$  be a finitely presented group with maximal perfect subgroup  $PG$  finite. Then any  $[G, 2]$ -complex  $X$  which is the subcomplex of an aspherical 2-complex is aspherical.*

*Proof.* If  $G$  is finite, the result is well known (see [BD]). Hence we will assume that  $Q$  is infinite. If the  $[G, 2]$ -complex  $X$  is a subcomplex of an aspherical 2-complex and  $X$  is not aspherical, then by the main theorem of [BDS], we see that there must exist a superperfect, normal, non-trivial subgroup  $P$  of  $G$  such that  $G$  is  $P$ -Cockcroft and the quotient  $Q$  has  $\text{cd } Q \leq 2$ . The group  $Q$  is infinite, so the cohomological dimension of  $Q$  is 1 or 2. But the maximal perfect subgroup of  $G$  is finite, so  $P$  is infinite. The theorem then says that  $G$  cannot be  $P$ -Cockcroft. Thus  $X$  must be aspherical.  $\square$

### 3. Two lemmas

In this section we will prove two lemmas preliminary to giving a proof of the theorem.

Let  $G$  be a group and let  $C$  be a projective  $\mathbb{Z}G$ -resolution of the trivial  $\mathbb{Z}G$ -module  $\mathbb{Z}$ . To each integer  $i \geq 0$  we have an associated kernel  $K_i = \ker \{\partial_i : C_i \rightarrow C_{i-1}\}$  ( $C_{-1} = \mathbb{Z}$ ). For any  $[G, 2]$ -complex  $X$ , let  $\tilde{X}$  be the universal covering of  $X$ . Then  $C_*\tilde{X}$ , the cellular chain complex of  $\tilde{X}$ , can be thought of as a partial resolution (of length two) of free left  $\mathbb{Z}G$ -modules. For any  $[G, 2]$ -complex  $X$ , the kernel  $K_1 = \ker \{\partial_1 : C_1\tilde{X} \rightarrow C_0\tilde{X}\}$  is called the *relation module determined by  $X$* .

For any left  $\mathbb{Z}G$ -module  $M$ , we let  $M^G$  denote the subgroup of elements fixed by the action of  $G$ ; we let  $M_G = \mathbb{Z} \otimes_{\mathbb{Z}G} M = M/IG \cdot M$  ( $IG$  is the augmentation ideal in  $\mathbb{Z}G$ ) be  $M$  with the  $G$ -action killed.

**3.1 LEMMA.** *If  $P$  is a finite, normal subgroup of a group  $G$  and  $Q = G/P$ , then  $H^i(G, \mathbb{Z}G) \cong H^i(G, \mathbb{Z}Q) \cong H^i(Q, \mathbb{Z}Q)$  for all  $i > 0$ . The first isomorphism is induced by  $\mathbb{Z}G \rightarrow \mathbb{Z}Q$  and the second by  $G \rightarrow Q$ .*

*Proof.* Because  $P$  is finite, we have  $H^j(P, \mathbb{Z}G) = 0$  for  $j > 0$ . By using the Lyndon–Hochschild–Serre spectral sequence, we see that  $H^i(G, \mathbb{Z}G) \cong H^i(Q, \mathbb{Z}G^P)$  for  $i > 0$ . But clearly  $\mathbb{Z}G^P \cong \bigoplus_{a \in Q} (\mathbb{Z}P)_a^P \cong \bigoplus_{a \in Q} (\mathbb{Z})_a \cong \mathbb{Z}Q$  as a  $\mathbb{Z}Q$ -module.  $\square$

**3.2 LEMMA.** *Let  $X$  be a  $[G, 2]$ -complex and suppose  $P$  is a superperfect, normal subgroup of  $\pi_1 X$  such that the Hurewicz map from  $\pi_2 X = \pi_2 X_P \rightarrow H_2 X_P$  is trivial (i.e.,  $G$  is  $P$ -Cockcroft with respect to  $X$ ). Let  $K_1 = \ker \{\partial_1 : C_1 \tilde{X} \rightarrow C_0 \tilde{X}\}$  be the relation module determined by  $X$ , where  $\tilde{X}$  is the universal covering of  $X$ . Then  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1 = \ker \{\partial_1(X_P) : C_1(X_P) \rightarrow C_0(X_P)\} \cong \mathbb{Z} \otimes_{\mathbb{Z}P} C_2 \tilde{X}$  is a relation module for  $Q = (\pi_1 X)/P$ . Furthermore, the surjection  $G \rightarrow Q$  induces an isomorphism  $H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) \cong H^2(Q, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)$ .*

*Proof.* Because  $P$  is a subgroup of  $\pi_1 X$  we have  $C_i X_P \cong \mathbb{Z} \otimes_{\mathbb{Z}P} C_i \tilde{X}$ . That  $P$  is superperfect and  $G$  is  $P$ -Cockcroft with respect to  $X$  implies that

$$0 \rightarrow C_2 X_P \rightarrow C_1 X_P \rightarrow C_0 X_P \rightarrow \mathbb{Z} \rightarrow 0$$

is an exact sequence of free  $\mathbb{Z}Q$ -modules (a free resolution of the trivial module  $\mathbb{Z}$ ). Tensoring the exact sequence (of  $\mathbb{Z}G$ -modules)  $0 \rightarrow \pi_2 X \rightarrow C_2 \tilde{X} \rightarrow K_1 \rightarrow 0$  with  $\mathbb{Z} \otimes_{\mathbb{Z}P}$ – and using the fact that  $X$  is  $P$ -Cockcroft, we see that  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1 \cong C_2 X_P$ .

The isomorphism  $H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) \cong H^2(Q, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)$  follows from the LHS spectral sequence for the extension

$$1 \rightarrow P \rightarrow G \rightarrow Q \rightarrow 1$$

together with the facts that  $P$  is superperfect and that  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is a trivial  $\mathbb{Z}P$ -module.  $\square$

#### 4. Proof of Theorem 2.1

From now on we assume that  $X$  is a  $[G, 2]$ -complex with fundamental group equal to  $G$ . We let  $P$  be a finite, superperfect, normal subgroup of  $G$  so that the Hurewicz map  $\pi_2 X \rightarrow H_2 X_P$  is trivial. We let  $Q = G/P$  have cohomological dimension 1 or 2 and  $K_1$  be the relation module determined by  $X$ . The proof by contradiction is given in a series of steps as follows.

STEP 1 is devoted to the proof of the following claim. Let  $p$  be the order of the finite group  $P$  and consider the inclusion  $K_1^P \rightarrow \mathbb{Z} \otimes_{\mathbb{Z}P} K_1 = K_{1P}$ .

CLAIM. *If  $P$  is superperfect, then the image of  $K_1^P$  inside  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is  $p \cdot \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$ .*

*Proof of the Claim.* Let  $F_2 \rightarrow F_1 \rightarrow \mathbb{Z}P \rightarrow \mathbb{Z} \rightarrow 0$  be a partial resolution of  $\mathbb{Z}$  over  $\mathbb{Z}P$  by finitely generated free modules. Let  $L_1$  denote the kernel of the map  $\partial_1 : F_1 \rightarrow \mathbb{Z}P$ . Then the following diagram commutes:

$$\begin{array}{ccc}
 F_2^P & \xrightarrow{\partial_1^P} & F_1^P \\
 \downarrow & \swarrow & \downarrow \\
 & L_1^P & \\
 & \downarrow & \\
 & L_{1P} & \\
 \downarrow & \swarrow & \downarrow \\
 F_{2P} & \xrightarrow{\partial_{1P}} & F_{1P}
 \end{array}$$

The group  $P$  is finite implies that the vertical arrows are monomorphisms. The two outer vertical arrows are clearly multiplication by  $p$  because the modules are free. The group  $P$  is perfect implies that  $\partial_{1P}$  and  $\partial_1^P$  are epimorphisms and hence  $L_{1P} = F_{1P}$  and  $L_1^P = F_1^P$ . Thus the interior vertical arrow has image which is multiplication by  $p$ . Now one uses Schanuel’s lemma and a simple argument to show that the same is true of  $K_1^P \rightarrow K_{1P}$ . This completes the proof of the claim.

Hence the  $\mathbb{Z}Q$ -module  $A = \mathbb{Z} \otimes_{\mathbb{Z}P} K_1 / K_1^P = \mathbb{Z} \otimes_{\mathbb{Z}P} K_1 / p \cdot \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$ . If we write  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1 \cong \mathbb{Z}Q^\alpha$  ( $= \mathbb{Z} \otimes_{\mathbb{Z}P} C_2 \tilde{X}$ ; this follows from lemma 3.2), then  $A \cong \mathbb{Z}_p Q^\alpha$ .

STEP 2. The following diagram is commutative, with top and vertical sequences exact:

$$\begin{array}{ccccccc}
 & & 0 & & & & \\
 & & \downarrow & & & & \\
 H^1(Q, A) & \longrightarrow & H^2(Q, K_1^P) & \xrightarrow{f'} & H^2(Q, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) & \longrightarrow & H^2(Q, A) \longrightarrow H^3(Q, K_1^P) \\
 & & \downarrow i & & \downarrow j \cong & & \parallel \\
 & & H^2(G, K_1) & \xrightarrow{f} & H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) & & 0 \quad (4.1) \\
 & & \downarrow h & & \downarrow & & \\
 \mathbb{Z}_p \cong H^2(P, K_1)^Q & \longrightarrow & H^2(P, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)^Q & = & 0 & & \\
 & & \downarrow & & & & \\
 & & 0 & & & & 
 \end{array}$$

The horizontal maps  $f$  and  $f'$  are induced by  $K_1 \rightarrow \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  and  $K_1^P \rightarrow \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$ , respectively. By using a dimension shifting argument one shows that  $H^2(P, K_1) \cong \mathbb{Z}_p$  has trivial  $\mathbb{Z}Q$ -action. The fact that  $p \cdot A = 0$  shows that  $p \cdot H^2(Q, A) = 0$  also. The vertical sequences come from the LHS-spectral sequence. The left-most vertical sequence is exact, because  $\text{cd } Q \leq 2$  and  $H^1(P, K_1) = 0$  (this is a consequence of the finiteness of  $P$ ). The fact that  $H^2(P, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) = 0$  follows because  $P$  is superperfect and  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is a trivial  $\mathbb{Z}P$ -module. We observe that the map  $f'$  is an isomorphism modulo torsion; that is to say, the kernel and the cokernel of  $f'$  are torsion groups. The group  $H^3(Q, K_1^P) = 0$  because  $Q$  is two dimensional. By lemma 3.2,  $\mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  is a free  $\mathbb{Z}Q$ -module, so  $j$  is an isomorphism, by lemma 3.1.

STEP 3. Let  $M$  be any  $\mathbb{Z}G$ -module and  $\rho(M) : M \rightarrow \mathbb{Z} \otimes_{\mathbb{Z}P} M$  be the natural surjection. We will show that  $\rho(K_1) : K_1 \rightarrow \mathbb{Z} \otimes_{\mathbb{Z}P} K_1$  induces a split epimorphism

$$f : H^2(G, K_1) \rightarrow H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1).$$

We will show that there is a map  $s : H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1) \rightarrow H^2(G, K_1)$  such that  $fs$  is an isomorphism.

Now  $H^2(G, C_2\tilde{X}) \cong H^2(G, \mathbb{Z} \otimes_{\mathbb{Z}P} K_1)$ , by lemma 3.1; the isomorphism is induced by  $\rho(K_1)\partial_2$ , where  $\partial_2 : \mathbb{Z}G^\alpha = C_2\tilde{X} \rightarrow K_1$ . This last follows because  $\rho(K_1)\partial_2 = (1 \otimes \partial_2)\rho(C_2\tilde{X})$ . The map  $1 \otimes \partial_2$  is an isomorphism because  $G$  is  $P$ -Cockcroft and  $\rho(C_2\tilde{X})$  induces an isomorphism on  $H^2(G, -)$  by lemma 3.1. Thus the map  $\partial_2$  induces a map  $g : H^2(G, C_2\tilde{X}) \rightarrow H^2(G, K_1)$  whose composite  $gf$  is induced by the natural map  $\mathbb{Z}G^\alpha \rightarrow \mathbb{Z}Q^\alpha$ . Thus  $gf$  is an isomorphism, again by 3.1. Hence  $f$  is a split epimorphism and the map  $s$  can be chosen as  $s = \partial_{2*}(\rho(K_1)\partial_2)_*^{-1}$ .

STEP 4. We will show that, if  $i : H^2(G, K_1^P) \rightarrow H^2(G, K_1)$  is the map in diagram 4.1, then  $\text{im } s = \text{im } i$ .

First we observe that, by definition,  $\text{im } s = \text{im } \partial_{2*}$ . Let  $K_2 = \ker \partial_2$  and consider the long exact sequence arising from the short exact sequence  $0 \rightarrow K_2 \rightarrow C_2\tilde{X} \rightarrow K_1 \rightarrow 0$ ;

$$\cdots \rightarrow H^2(G, C_2\tilde{X}) \xrightarrow{\partial_{2*}} H^2(G, K_1) \rightarrow H^3(G, K_2) \rightarrow H^3(G, C_2\tilde{X}) = 0.$$

The group  $H^3(G, C_2\tilde{X}) = 0$  by 3.1 and the fact that  $\text{cd } Q \leq 2$  (3.2).

The commutativity of the diagram below (where we identify  $H^3(P, K_2)$  with  $H^2(P, K_1)$ ) shows that  $\text{im } i = \text{im } \partial_{2*} = \text{im } s$ :

$$\begin{array}{ccccccc}
 & & H^2(G, K_1^P) & & & & \\
 & & \downarrow i & & & & \\
 H^2(G, C_2 \tilde{X}) & \xrightarrow{\partial_{2*}} & H^2(G, K_1) & \longrightarrow & H^3(G, K_2) & \longrightarrow & 0 \\
 & & \downarrow h & & \downarrow \cong & & \\
 & & H^2(P, K_1)^Q & \longrightarrow & H^3(P, K_2)^Q & & \\
 & & & \cong & & & 
 \end{array} \tag{4.2}$$

STEP 5. We show that  $\mathbb{Z}_p \otimes H^2(Q, \mathbb{Z}Q) = 0$ .

The map  $fi$  (see 4.1) is an isomorphism because  $\ker f \cap \text{im } i = \ker f \cap \text{im } s = 0$ . This implies  $f' = j^{-1}fi$  is an isomorphism. Thus,  $H^2(Q, A) = 0$  and hence  $\mathbb{Z}_p \otimes H^2(Q, \mathbb{Z}Q) = H^2(Q, \mathbb{Z}_p Q) = 0$ .

STEP 6. The contradiction.

*Case 1* ( $Q$  is free). The same proof above works (by simply reducing the dimension of the cohomology groups and the kernels by one in 4.1 and 4.2) to show that  $\mathbb{Z}_p \otimes H^1(Q, \mathbb{Z}Q) = 0$ . But this is impossible because  $H^1(Q, \mathbb{Z}Q)$  is known to be free abelian and non-trivial [Sw, corollary 3.7]. Thus,  $G$  is  $P$ -Cockcroft and  $Q$  free leads to a contradiction.

*Case 2:* ( $\text{cd } Q = 2$ ). Because  $P$  is finite and  $\text{cd } Q = 2$  we have that  $\mathbb{Z}_p \otimes H^2(Q, \mathbb{Z}Q) = 0$  by step 5.

Because  $G$  is finitely presented, so is  $Q$ . We observe that ([BE], theorem 5.2)  $Q$  is a free product of duality groups of dimension 1 or 2. Let  $R$  be one of the factors, and define  $D = H^2(Q, \mathbb{Z}Q)$  and  $E = H^2(R, \mathbb{Z}R)$ . Let  $q$  be any prime divisor of  $p$ . The fact that  $\mathbb{Z}_p \otimes D = 0$  implies that  $\mathbb{Z}_q \otimes D = 0$ . This in turn implies that  $\mathbb{Z}_q \otimes E = 0$ . If  $R$  is a duality group of dimension 2, we have, for any  $\mathbb{Z}_q Q$ -module  $M$ ,  $H^2(R, M) \cong \mathbb{Z} \otimes_{\mathbb{Z}R} (M \otimes D)$ . But because  $M$  is a  $\mathbb{Z}_q$ -module, we have  $M \otimes D \cong \mathbb{Z}_q \otimes M \otimes D = 0$ . Hence, the cohomological dimension of  $R \leq 1$  over the ring  $\mathbb{Z}_q$ . This, together with the fact that  $R$  is torsion-free, shows that  $\text{cd } R = 1$ . Hence  $R$  is free and so  $Q$  is free. This brings us back to case 1. Hence no such group  $G$  can be  $P$ -Cockcroft. This finishes the proof of Theorem 2.1.  $\square$

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*University of Oregon*  
*Eugene, Oregon 97403, USA*

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Preface – Contributors – Distance from an isometry to the Banach-Stone maps: **Jesús Araujo** – Pseudocompact and *P*-spaces in non-archimedean functional analysis: **Jesús Araujo, P. Fernández-Ferreirós and J. Martínez-Maurica** – Extension of isometries with values in nonarchimedean fields: **José M. Bayod** – Weak *c'*-compactness in (strongly) polar Banach spaces over a non archimedean, densely valued field: **Sabine Borrey** –  $C(E,F)$  as a dual space: **N. De Grand-De Kimpe** – Non integrally closed algebras  $H(D)$ : **A. Escassut und Bertin Diarra** – Continuous operators which commute with translations, on the space of continuous functions on  $Z_p$ : **Lucien van Hamme** – Dyadic frames for intermittency. Perturbed models: **O. Iordache** – Non-archimedean  $\Delta$ -nuclear spaces: **A. K. Katsaras** – The Locally *K*-convex spaces  $C^n(X)$ ,  $C^\infty(X)$ : **Samuel Navarro** – The Hahn-Banach extension property in *p*-adic analysis: **C. Pérez-García** – Banach algebra of *p*-adic valued almost periodic functions: **G. Rangan, M. S. Saleemullah** – The axiom of choice in *p*-adic functional analysis: **A. S. M. van Rooij** – The equation  $y' = \omega y$  and the meromorphic products: **Marie-Claude Sarmant, Alain Escassut** – The *p*-adic Krein-smulian Theorem: **W. H. Schikhof** – Topological fields and nonarchimedean analysis, Niel shell – Open problems: **A. C. M. van Rooij and C. H. Schikhof** – Appendix A: the space  $L^1(K)$  is not ultrametrizable: **José M. Bayod** – Appendix B: Zero sequences in *p*-adic compactoids: **W. H. Schikhof**.

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