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# Witt group of hyperelliptic curves

#### R. PARIMALA and R. SUJATHA

## Introduction

Let k be a perfect field of characteristic  $\neq 2$ . Let X be a smooth projective curve over k. Let  $W(k(X), \Omega_{k(X)})$  denote the Witt group of the function field of X with values in the module of differentials  $\Omega_{k(X)}$  of k(X). A residue homomorphism

$$\partial: W(k(X),\,\Omega_{k(X)}) \to \bigoplus_{x \in X} W(k(x))$$

was defined in [7], k(x) denoting the residue field at points  $x \in X$  and a residue theorem was proved; namely the composite

$$W(k(X), \Omega_{k(X)}) \xrightarrow{\partial} \bigoplus_{x \in X} W(k(x)) \xrightarrow{\operatorname{trace}} W(k)$$

is zero. Thus image  $\partial$  is contained in the subgroup  $(\bigoplus_{x \in X} W(k(x)))^0$  consisting of tuples  $(\mu_x)$  with  $\Sigma_{x \in X}$  trace  $\mu_x = 0$ . The kernel and cokernel of  $\partial$  are well understood if  $X = \mathbf{P}^1$  [13] or if X is an anisotropic conic over k [14]. To have an intrinsic description of these groups for curves of higher genus is an interesting question posed by Milnor in [13].

In this paper, we study this problem for smooth hyperelliptic curves X with a rational point of ramification over  $\mathbf{P}^1$ . Let  $\pi: X \to \mathbf{P}^1$  be a covering defined over  $\mathbf{A}^1$  by the equation  $y^2 = f(T)$ . We exhibit an exact sequence (§3)

$$0 \to W(X) \to W(k(X)) \xrightarrow{\partial^0} \left( \bigoplus_{x \in X} W(k(x)) \right)^0 \to \frac{W(k[T]_f)}{\langle 1, -f \rangle W(k)} \to W(X) \to 0.$$

where  $\partial^0$  is simply the residue map  $\partial$  through an identification of  $W(k(X), \Omega_{k(X)})$  with W(k(X)) for a suitable choice of a differential as basis for  $\Omega_{k(X)}$ . We derive, as a corollary, that if all the ramification points of  $\pi$  are k-rational, W(X) is generated by one-dimensional forms. This exact sequence may be viewed in two ways: Firstly as characterising coker  $\partial^0$  as a subgroup of  $\bigoplus_{x \in S} W(k(x))$ , S denoting the set of ramification points of  $\pi$  and secondly, as giving the defining relations for expressing W(X) as a quotient of  $\bigoplus_{x \in S} W(k(x))$ .

Using the exact sequence above, we give a more precise description of coker  $\partial^0$ . It contains a subgroup  $V_r$  which is a quotient of  $\bigoplus_{x \in S} W(k(x))$ , which we call the ramified part of coker  $\partial^0$ . Under the rationality assumption that  ${}_4\mathrm{Pic}\,X = {}_4\mathrm{Pic}\,X_{\bar{k}},\bar{k}$  denoting the algebraic closure of k, the group  $V_r$  is zero. We call  $V_{nr} = \operatorname{coker}\,\partial^0/V_r$ , the unramified part of coker  $\partial^0$ . This group is 2-torsion (§5). It can be computed in terms of certain cohomology groups if  ${}_4\mathrm{Pic}\,X = {}_4\mathrm{Pic}\,X_{\bar{k}}$ , and supposing further that the curve Y = X or  $\mathbf{P}^1$  has the following property: 'Graded Witt group of Y is isomorphic to the cohomology ring'; Curves over local and global fields have this property [16]. We in fact show that under these assumptions on X, coker  $\partial^0$  is isomorphic to (Pic X'/2)  $\oplus NH^3(X')$ , where  $X' = X \setminus S$ , S denoting the set of ramification points of  $\pi$  and  $NH^n(X')$  denotes the kernel of the map  $H^n_{et}(X', \mu_2) \to H^n_{et}(k(X'), \mu_2)$ . For a smooth projective hyperelliptic curve over a local field with good reduction, if  ${}_4\mathrm{Pic}\,X = {}_4\mathrm{Pic}\,X_{\bar{k}}$ , coker  $\partial$  is isomorphic to  $W(k) \oplus (\mathbb{Z}/2)^{4g}$  where g is the genus of the curve (Theorem 7.1). Further, W(X) is also isomorphic to the group  $(\mathbb{Z}/2)^{4g} \oplus W(k)$ ! (Theorem 7.6).

The computations yield, as a by-product, that for any smooth projective curve X over a local field with good reduction, if  ${}_{4}\text{Pic }X = {}_{4}\text{Pic }X_{\bar{k}}$ , the classical invariants determine the class of a quadratic space in W(X).

We record here that J. E. Shick [19] has some independent computations of coker  $\partial$  for power series fields over  $\mathbb{R}$  and of  $\mathbb{C}$ .

We thank D. S. Nagaraj for carefully going through the manuscript.

# 1. Kernel of the residue homomorphism

Let k be a perfect field of characteristic  $\neq 2$ . Let X be a smooth projective curve defined over k. For a line bundle  $\mathcal{L}$  on X, let  $W(X, \mathcal{L})$  denote the Witt group of quadratic spaces on X with values in  $\mathcal{L}$  [9]. Let  $W(X) = W(X, \mathcal{O}_X)$ .

LEMMA 1.1. The group  $W(X, \mathcal{L})$  depends upto isomorphism, only on the class of  $\mathcal{L}$  in Pic X/2. In particular,  $W(X, \mathcal{L}^2) \xrightarrow{\sim} W(X)$ .

*Proof.* Let  $\mathcal{M} \in \operatorname{Pic} X$  and  $(\mathscr{E}, q)$  be a quadratic space with values in  $\mathcal{L} \otimes \mathcal{M}^2$ , i.e.,  $q : \mathscr{E} \to \mathscr{E}^* \otimes \mathcal{L} \otimes \mathcal{M}^2$ , where for any bundle  $\mathscr{F}$ ,  $\mathscr{F}^*$  denotes the dual of  $\mathscr{F}$ , is an isomorphism such that  $q' \otimes 1_{\mathscr{L} \otimes \mathscr{M}^2} = q$ . The assignment

$$(\mathscr{E}, q) \rightarrow (\mathscr{E} \otimes \mathscr{M}^*, q \otimes 1_{\mathscr{M}^*})$$

defines an isomorphism

$$W(X, \mathcal{L} \otimes \mathcal{M}^2) \simeq W(X, \mathcal{L}).$$

Let  $\Omega_X$  denote the sheaf of differentials on X. Let

$$\partial: W(k(X),\,\Omega_{k(X)}) \to \bigoplus_{x \in X} W(k(x))$$

be the residue homomorphism defined in [7], k(x) denoting the residue field at the closed point x of X. (Throughout, the notation  $x \in X$  stands for the set of all closed points x in X).

LEMMA 1.2. The kernel of the residue map

$$\partial: W(k(X),\,\Omega_{k(X)}) \to \bigoplus_{x\in X} W(k(x))$$

is  $W(X, \Omega_X)$ .

*Proof.* Let q be a quadratic space over k(X) with values in  $\Omega_{k(X)}$ , whose class belongs to ker  $\partial$ . Let x be a closed point of X and  $\pi_x$  a local parameter at x. Identifying W(k(X)) with  $W(k(X), \Omega_{k(X)})$  through  $d\pi_x$ , the residue map  $\partial_x$ :  $W(k(X)) \to W(k(X))$  is simply the second residue homomorphism with respect to  $\pi_x$ . Thus q which maps to zero under  $\partial_x$  (cf. [17], p. 207) is isometric to  $q_x \otimes_{\sigma_{X,x}} k(X)$  for some  $q_x \in W(\mathcal{O}_{X,x})$ . The spaces  $q_x \cdot d\pi_x$  over  $\mathcal{O}_{X,x}$  with values in  $\Omega_{X,x}$  become isometric to q over k(X). They patch up to yield a quadratic space  $q_X$  over X with values in X0 in view of the following

LEMMA 1.3. Let  $\mathcal{L}$  be a line bundle on X, q a quadratic space over k(X) with values in  $\mathcal{L}_{k(X)}$ . Suppose, for every  $x \in X$ , there exists a quadratic space  $q_x$  over  $\mathcal{O}_{X,x}$  with value in  $\mathcal{L} \otimes \mathcal{O}_{X,x}$  such that  $q_x \otimes k(X) \xrightarrow{\sim} q$ . Then there exists a quadratic space  $q_x$  over X with values in  $\mathcal{L}$  such that  $q_x \otimes k(X) \xrightarrow{\sim} q$ .

*Proof.* The proof of ([6], Corollary 2.7) in the case  $\mathcal{L} = \mathcal{O}_X$  goes through verbatim for any line bundle  $\mathcal{L}$ .

REMARK. If  $X = \mathbb{P}^1$ , ker  $\partial \longrightarrow W(X) \longrightarrow W(k)$ . ([13], Proposition 5.3). If X is an anisotropic conic, ker  $\partial = W(X, \Omega_X)$  is computed as  $\mathscr{B}_{\mathscr{C}}$  in ([14], Theorem 6.2).

PROPOSITION 1.4. Let X be a smooth hyperelliptic curve with a rational point of ramification over  $\mathbf{P}^1$ . Then  $\ker \partial \simeq W(X)$ .

*Proof.* By (1.1) and (1.2), it suffices to show that  $\Omega_X$  is the square of a line bundle on X. Let  $\pi: X \to \mathbf{P}^1$  be a covering, defined over  $\mathbf{A}^1$  by the equation  $y^2 = f(T)$ , degree f = 2g + 1, g being the genus of X. The divisor of the differential dT/y is  $(2g - 2)P_{\infty}$ ,  $P_{\infty}$  being the point of X lying over  $\infty$  in  $\mathbf{P}^1$ . Let  $\mathcal{L}$  be the line bundle corresponding to the divisor  $(g - 1)P_{\infty}$ . Then  $\Omega_X \xrightarrow{\sim} \mathcal{L}^2$ .

REMARK. As observed by M. Rost, one could define more generally, a residue map

$$\partial_{\mathcal{L}}: W(k(X), \mathcal{L}_{k(X)}) \to \bigoplus_{x \in X} (W(k(x)), (\mathcal{L} \otimes \Omega_X)(x)),$$

where  $(\mathscr{L} \otimes \Omega_X)(x)$  denotes the fibre of the line bundle  $(\mathscr{L} \otimes \Omega_X)$  at x. If  $X = \mathbf{P}^1$ , and  $\mathscr{L} = \mathscr{O}_X$ ,  $\partial_{\mathscr{O}_X} = \partial$  is the residue homomorphism discussed above, since  $\Omega_X$  is a square. If  $\mathscr{L} = \mathscr{O}_X(1)$ ,  $\partial_{\mathscr{O}_X(1)}$  is an isomorphism. In the case of an anisotropic conic, X we have  $\ker \partial_{\mathscr{O}_X} \longrightarrow W(X) \longrightarrow W(k)/\langle 1, -a, -b, ab \rangle W(k)$ , (cf. [1]), where X is defined by the equation  $aX^2 + bY^2 - Z^2 = 0$ . One can identify coker  $\partial$  with a subgroup of the Witt group of the residue field at the ramified point of the covering  $X \to \mathbf{P}^1$ .

## 2. Some auxiliary results on trace, transfer and residue homomorphisms

Let  $\pi: X \to \mathbf{P}^1$  be a double covering, defined over  $\mathbf{A}^1$  by the equation  $y^2 = f(T)$ , degree f = 2g + 1, g being the genus of X. We identify W(k(X)) and W(k(T)) with  $W(k(X), \Omega_{k(X)})$  and  $W(k(T), \Omega_{k(T)})$  through the basis dT/2y and dT respectively. For  $y \in \mathbf{A}^1$ , if  $p \in k[T]$  is the monic irreducible polynomial which gives a parameter at y, the composite map

$$W(k(T)) \stackrel{dT}{\longrightarrow} W(k(T), \Omega_{k(T)}) \stackrel{\partial_y}{\longrightarrow} W(k(y))$$

is the second residue homomorphism with respect to the parameter pp', p' denoting the derivative of p with respect to T. Similarly, one can verify that if  $x \in X$  lies over  $y \in \mathbf{P}^1$  corresponding to p(T), and x unramified over y, on choosing p(T) again as the parameter at y, the composite

$$W(k(X)) \xrightarrow{dT/2y} W(k(X), \Omega_{k(X)}) \xrightarrow{\partial_x} W(k(x))$$

is the second residue homomorphism with respect to the parameter 2pp'y. We again denote by  $\partial$  this residue map.

For any finite separable extension L/K, let  $tr: W(L) \to W(K)$  be the map induced by the linear map trace:  $L \to K$  and  $i: W(K) \to W(L)$  the map induced by the inclusion of K in L. Let  $s: W(k(X)) \to W(k(T))$  be the transfer homomorphism induced by the linear map  $s: k(X) \to k(T)$  defined by s(1) = 0, s(y) = 1 where  $\{1, y\}$  is a basis for k(X) over k(T) ([17], p. 47).

LEMMA 2.1. The diagram

$$W(k(X)) \xrightarrow{(\partial_X)} \bigoplus_{x/y} W(k(x))$$

$$\downarrow r$$

$$W(k(T)) \xrightarrow{\partial_y} W(k(y))$$

is commutative.

Proof. Since the diagram

$$W(\Omega_{k(X)}) \xrightarrow{\partial} \bigoplus_{x/y} W(k(x))$$

$$\downarrow r \qquad \qquad \downarrow r$$

$$W(\Omega_{k(T)}) \xrightarrow{\partial} W(k(y))$$

is commutative ([7], §1), it suffices to show that the diagram

$$W(k(X)) \xrightarrow{dT/2y} W(k(X), \Omega_{k(X)})$$
 $\downarrow tr$ 
 $W(k(T)) \xrightarrow{dT} W(k(T), \Omega_{k(T)})$ 

is commutative. It is enough to check that

$$tr(\langle h_0 + h_1 y \rangle \cdot dT/2y) = s(\langle h_0 + h_1 y \rangle) \cdot dT,$$

for  $h_0$ ,  $h_1 \in k(T)$ . We have,

$$tr(\langle h_0 + h_1 y \rangle dT/2y) = \langle tr((h_0 + h_1 y)/2y) \rangle \cdot dT$$

$$= \begin{pmatrix} h_1 & h_0 \\ h_0 & h_1 f \end{pmatrix} \cdot dT$$

$$= s(\langle h_0 + h_1 y \rangle) \cdot dT.$$

LEMMA 2.2. The diagram

is commutative if y is an unramified point for  $\pi$  and i' is the composite

$$W(k(y)) \xrightarrow{i} W(k(x)) \xrightarrow{\overline{2y}-1} W(k(x)).$$

If x is a ramified point for  $\pi$ ,  $\partial_x \circ i$  is zero.

*Proof.* Let  $\langle h \rangle \in W(k(T))$  and  $x \in X$  such that x is an unramified point for  $\pi$  with  $\pi(x) = y$ . Let  $p \in k[T]$  be the monic polynomial corresponding to y. Suppose  $v_y(h) = 0$ . Then  $\partial_y(\langle h \rangle) = 0$  and  $\partial_x \circ i(\langle h \rangle) = \partial_x(\langle h \rangle) = 0$ , since  $v_x(h) = v_y(h) = 0$ . Suppose h = up with  $v_y(u) = 0$ . Since  $\partial_x$  is the second residue map with respect to the parameter 2pp'y and  $\partial_y$  the second residue map with respect to pp', we have

$$i' \circ \partial_{\nu}(\langle up \rangle) = i' \circ \partial_{\nu}\langle (u/p') \cdot pp' \rangle = i' \langle \overline{u/p'} \rangle = \langle \overline{u/p'2y} \rangle$$

and

$$\partial_x \circ i(\langle h \rangle) = \partial_x (\langle (u/2p'y) \cdot 2pp'y \rangle) = \langle \overline{u/2p'y} \rangle$$

Suppose  $x \in X$  is a ramified point, lying over  $y \in \mathbf{P}^1$ . For  $h \in k(T)$ ,  $v_x(ih) \equiv 0 \mod 2$ , since x has ramification index 2, and we have  $\partial_x \circ i(\langle h \rangle) = 0$ .

LEMMA 2.3. Let x/y be an unramified point for  $\pi$ . Then the diagram

$$W(k(T)) \xrightarrow{\partial_{y}} W(k(y))$$

$$\langle 1, -f \rangle \downarrow \qquad \qquad \downarrow \langle 1, -f \rangle$$

$$W(k(T)) \xrightarrow{\partial_{y}} W(k(y))$$

is commutative.

Proof. Clear.

We repeatedly use the following lemma which is a consequence of the Lam's exact triangle ([17], Chapter 2, 5.10).

LEMMA 2.4. The following triangles are exact.

$$W(k(T)) \xrightarrow{i} W(k(X))$$

$$\langle 1, -f \rangle^{r} \qquad \qquad \downarrow s$$

$$W(k(T))$$

$$W(k(y)) \xrightarrow{i'} W(k(x))$$

$$\langle 1, -f \rangle^{r} \qquad \qquad \downarrow t'$$

$$W(k(y))$$

where, in the second triangle, y is unramified for  $\pi$  and  $\pi(x) = y$ ; if y splits in X, we mean by W(k(x)), the direct sum  $W(k(x_1)) \oplus W(k(x_2))$  with  $\pi(x_i) = y$ .

### 3. An exact sequence

Let  $\pi: X \to \mathbf{P}^1$  be a hyperelliptic curve defined over  $\mathbf{A}^1$  by the equation  $y^2 = f(T)$ , degree f = 2g + 1, g being the genus of X. Let

$$\partial^0: W(k(X)) \to (\bigoplus_{x \in X} W(k(x)))^0$$

be the residue homomorphism as defined in §2, identifying W(k(X)) with  $W(k(X), \Omega_{k(X)})$  through the basis dT/2y,  $(\bigoplus_{x \in X} W(k(x)))^0$  denoting the kernel of the trace map  $\bigoplus_{x \in X} W(k(x)) \xrightarrow{r} W(k)$ . We fix the following notation:  $S = \text{set of ramification points for } \pi$ ,  $X' = X \setminus S$ ,  $Y = \mathbb{P}^1$ ,  $Y' = Y \setminus \pi(S)$ . We have the following commutative diagram with exact rows and columns, in view of (2.1), (2.3) and (2.4) and ([13], Theorem 5.3).

$$W(k(X)) \xrightarrow{\partial} \bigoplus_{x \in X} W(k(x))$$

$$s \downarrow \qquad \qquad \downarrow tr$$

$$0 \longrightarrow W(k) \longrightarrow W(k(T)) \xrightarrow{\partial} \bigoplus_{y \in Y'} W(k(y)) \bigoplus_{y \in \pi(S)} W(k(y)) \xrightarrow{\text{trace}} W(k) \longrightarrow 0$$

$$\downarrow \qquad \qquad \downarrow (1, -f) \downarrow \qquad \qquad \downarrow (1, -f)$$

$$0 \longrightarrow W(Y') \longrightarrow W(k(T)) \xrightarrow{\partial} \bigoplus_{y \in Y'} W(k(y)).$$

We define a homomorphism  $\alpha: (\bigoplus_{x \in X} W(k(x)))^0 \to W(A)/\langle 1, -f \rangle \cdot W(k)$ , where  $A = k[T]_f$  as follows. Let  $\theta \in (\bigoplus_{x \in X} W(k(x)))^0$ . Then there exists  $q \in W(kT)$  with

 $\partial(q) = tr \theta$ . We have for  $y \in Y'$ 

$$\begin{split} \partial_y(\langle 1, -f \rangle \cdot q) &= \langle 1, -\vec{f} \rangle \partial_y(q) \\ &= \langle 1, -\vec{f} \rangle tr(\theta_x) \\ &= 0. \end{split}$$

Hence  $\langle 1, -f \rangle q \in W(Y') = W(A)$ . Let  $\alpha(\theta)$  denote its class in  $W(A)/\langle 1, -f \rangle \cdot W(k)$ . If  $q_1, q_2$  and two lifts of  $tr\theta$  in  $W(k(T)), q_1 - q_2 \in W(k)$  and  $\langle 1, -f \rangle q_1$  and  $\langle 1, -f \rangle q_2$  define the same class in  $W(A)/\langle 1, -f \rangle W(k)$ . Thus  $\alpha$  is well-defined.

LEMMA 3.1. ker  $\alpha = \partial^0(W(k(X)))$ .

*Proof.* Since  $\partial \circ s = tr \circ \partial$  and  $\langle 1, -f \rangle \circ s = 0$ , we have

 $\partial(W(k(X))) \subset \ker \alpha$ .

Let  $\theta \in \bigoplus_{x \in X} W(k(x))^0$  with  $\alpha(\theta) = 0$ . Let  $q_1 \in W(k(T))$  be such that  $\partial(q_1) = tr\theta$ . Then  $\langle 1, -f \rangle q_1 \in \langle 1, -f \rangle W(k)$ . Replacing  $q_1$  by  $q_1 - q_0$  for a suitable  $q_0 \in W(k)$ , we assume that  $\langle 1, -f \rangle q_1 = 0$ . Thus, by (2.4), there exists  $q_2 \in W(k(X))$  such that  $s(q_2) = q_1$ . We have  $tr(\theta - \partial q_2) = tr\theta - \partial sq_2 = tr\theta - \partial q_1 = 0$ . The fact that  $\theta - \partial q_2 \in \partial W(k(X))$  follows from the following

LEMMA 3.2. Let  $\theta \in (\bigoplus_{x \in X} W(k(x)))^0$  with  $tr\theta = 0$  in  $(\bigoplus_{y \in Y} W(k(y)))^0$ . Then  $\theta \in \partial \circ i(W(k(T)))$ .

SUBLEMMA 3.3. Let  $(\mu_x) \in \bigoplus_{x \in X'} W(k(x))$  be such that  $tr(\mu_x) = 0$  in  $\bigoplus_{y \in Y'} W(k(y))$ . Then there exists  $q \in W(k(T))$  such that  $\partial_x(i(q)) = \mu_x$ , for  $x \in X'$ .

*Proof.* By (2.4), there exists  $(v_y) \in \bigoplus_{y \in Y'} W(k(y))$  such that  $i'(v_y) = \mu_x$ . Since  $Y' \subset \mathbf{A}^1$ , the residue map  $\partial : W(k(T)) \to \bigoplus_{y \in Y'} W(k(y))$  is surjective. Let  $q \in W(k(T))$  be such that  $\partial_y(q) = v_y$  for  $y \in Y'$ . Then, by (2.2),  $\partial \circ i(q) = i' \circ \partial_y(q) = \mu_x$  for  $x \in X'$ .

Proof of 3.2. By (3.3), there exists  $q \in W(k(T))$  such that  $\partial_x \circ i(q) = \theta_x$  for  $x \in X'$ . Further, by (2.2),  $\partial_x \circ i(q) = 0$  for  $x \in S = X \setminus X'$ . Since for  $x \in S$ ,  $\theta_x = tr \theta_x = 0$ , we have,  $\partial(i(q)) = \theta$ .

Let  $A = k[T]_f$ ,  $B = (k[T, y]/(y^2 - f))_f$  be the co-ordinate rings of Y' and X' respectively. Since for  $x \in S$ ,  $q \in W(A)$ ,  $\partial_x \circ i(q) = 0$  by (2.2), the natural map  $W(A) \xrightarrow{i} W(B)$  has its image contained in W(X). This map vanishes on  $\langle 1, -f \rangle \cdot W(k)$  and induces a map  $\beta : W(A)/\langle 1, -f \rangle W(k) \to W(X)$ .

THEOREM 3.4. The sequence

$$0 \longrightarrow W(X) \xrightarrow{i} W(k(X)) \xrightarrow{\partial^0} \left( \bigoplus_{x \in X} W(k(x)) \right)^0$$

$$\xrightarrow{\alpha} W(A)/\langle 1, -f \rangle W(k) \xrightarrow{\beta} W(X) \longrightarrow 0$$

is exact.

*Proof.* Exactness at W(X) (left) and W(k(X)) are proved in ([10], p. 277) noting that  $\partial^0$  is the second residue homomorphism at all points  $x \in X$ . The exactness at  $(\bigoplus_{x\in X} W(k(x)))^0$  is proved in (3.1). That  $\beta\circ\alpha=0$  follows from the fact that  $i \circ \langle 1, -f \rangle = 0$ , (2.4). We now prove the surjectivity of  $\beta$ . We identify W(X) with the subgroup of W(k(X)) which is the kernel of  $\partial^0$ . Let  $q \in W(X)$ . Then  $\partial \circ s(q) = tr \circ \partial q = 0$  so that  $s(q) \in W(k)$ . Further  $\langle 1, -f \rangle s(q) = 0$  (2.4). This implies that s(q) = 0 in view of the fact that for any anisotropic quadratic space q over  $k, q \not \to g \cdot q$  for any odd degree polynomial g. Thus, there exists  $q_1 \in W(k(T))$  with  $i(q_1) = q$ . We have  $i' \circ \partial_{\nu}(q_1) = \partial_{\kappa} \circ i(q_1) = 0$  for  $\gamma \in Y'$ . There exists  $\mu_{\nu} \in W(k(\gamma))$ such that  $\langle 1, -\bar{f} \rangle (\mu_{\nu}) = \partial_{\nu}(q_1)$ . Since  $\partial : W(k(T)) \to \bigoplus_{\nu \in Y'} W(k(\nu))$  is surjective, there exists  $q_2 \in W(k(T))$  such that  $\partial_{\nu}(q_2) = \mu_{\nu}$  for every  $\nu \in Y'$ . We have  $\partial_{\nu}(q_1 - \langle 1, -f \rangle q_2) = \langle 1, -\overline{f} \rangle \mu_{\nu} - \langle 1, -\overline{f} \rangle \partial_{\nu}(q_2) = 0$ for  $y \in Y'$ that  $q_1 - \langle 1, -f \rangle q_2 \in W(A)$  and maps to q under  $\beta$ . We now prove exactness at  $W(A)/\langle 1, -f \rangle \cdot W(k)$ . Let  $q \in W(A)$  be such that  $\beta(\bar{q}) = 0$  in W(k(X)). By (2.4), there exists  $q_1 \in W(k(T))$  such that  $\langle 1, -f \rangle \cdot q_1 = q$ . Since  $\langle 1, -\overline{f} \rangle \partial_{\nu}(q_1) = 0$  for  $y \in Y'$ , there exists  $\mu_x \in W(k(x))$ , x/y such that  $tr(\mu_x) = \partial_{\nu}(q_1)$ . For  $x \in S$ , we set  $\mu_x = \partial_v(q_1)$ . Clearly  $(\mu_x) \in (\bigoplus_{x \in X} W(k(x)))^0$  and  $\alpha((\mu_x)) = q$ .

COROLLARY 3.5. If all ramification points of X are defined over k, then W(X) is generated by discriminants.

*Proof.* Suppose  $f = \Pi_i$   $(T - \alpha_i)$ ,  $\alpha_i \in k$ . An immediate consequence of the Milnor sequence ([13] Theorem 5.3) is that  $W(k[T]_f)$  is generated by  $\langle \lambda(T - \alpha_i) \rangle$  and  $\langle \mu \rangle$ ,  $\mu \in k^*$ ,  $1 \le i \le 2g + 1$ . Since  $\beta$  is surjective, their images under  $\beta$ , which are precisely the discriminants of W(X), generate W(X).

## 4. Some computations for hyperelliptic curves

Let X be a smooth hyperelliptic curve defined over k. We assume throughout that X has a rational point of ramification. Let  $\pi: X \to \mathbf{P}^1$  be a double covering as

in §3. If genus X > 1, since any two double coverings  $\pi_1, \pi_2 : X \to \mathbf{P}^1$  differ by an automorphism of X, the space  $X' = X \setminus S$ , S denoting the set of ramification points of the covering  $\pi : X \to \mathbf{P}^1$  determines and is determined by X. Following notations of §3, let  $A = k[T]_f$  and  $B = (k[T, y]/(y^2 - f))_f$  be the co-ordinate rings of Y' and X' respectively.

LEMMA 4.1. The unit group U(B) is generated by  $k^*$ , y, and divisors of f. If f splits into linear factors over k,  $U(B) \simeq k^* \times \mathbb{Z}^{2g+1}$ .

Proof. Let  $h \in U(B)$ . Then div  $h = \sum n_i x_i$ ,  $x_i \in S$ , div h denoting the divisor of h. Let  $\sigma$  denote the nontrivial automorphism of k(X) over k(T). Then  $\sigma x_i = x_i$ , so that div  $\sigma h = \text{div } h$ . Thus  $h = \lambda \sigma h$ ,  $\lambda \in k^*$ . We have,  $h^2 = \lambda(h\sigma h) \in U(A)$ . Thus  $h\sigma h$  is upto a scalar from  $k^*$ , a power product of divisors of f. On the other hand, the only non-square in k(T) which becomes a square in k(X) is f. It follows that  $h^2 = \mu^2 \prod_i h_i^{2m_i} f$  or  $h^2 = \mu^2 \prod_i h_i^{2m_i}, m_i \in \mathbb{Z}, h_i$  divisors of f in k[T]. Thus,  $h = \pm \mu(\prod h_i^{m_i}) y$  or  $h = \pm \mu(\prod h_i^{m_i})$ . Further, if  $f = \prod_{1 \le i \le 2g+1} (T - \alpha_i), \alpha_i \in k^*$ , the homomorphism  $k^* \times \mathbb{Z}^{2g+1} \to U(B)$ , defined by

$$(\lambda,(n_i)) \rightarrow \lambda(T-\alpha_1)^{n_1} \cdots (T-\alpha_{2g})^{n_{2g}} y^{n_{2g+1}}$$

is surjective, by the above remarks. Suppose

$$\lambda (T - \alpha_1)^{n_1} \cdots (T - \alpha_{2g})^{2_{2g}} \cdot y^{n_{2g+1}} = 1$$

is a relation. Then the divisor

$$\sum_{1 \le i \le 2g} 2n_i x_i + n_{2g+1} \left( \sum_{1 \le i \le 2g+1} x_i \right) - \left( \sum 2n_i + n_{2g+1} (2g+1) \right) x_{\infty} = 0,$$

where  $x_i \in S$  lie over  $T - \alpha_i$  and  $x_\infty$  lies over  $\infty$ . This implies that  $n_i = 0$ ,  $1 \le i \le 2g + 1$  and  $\lambda = 1$ . Thus we have an isomorphism  $k^* \times \mathbb{Z}^{2g+1} \longrightarrow U(B)$ .

LEMMA 4.2. Suppose f splits into linear factors over k. Then the map  $\operatorname{Pic} X' \to \operatorname{Pic} X'_{\overline{k}}$  is injective.

*Proof.* Since the divisor classes of degree zero supported on the ramification locus S are precisely the elements of  $_2$ Pic X, we have the following commutative diagram with exact rows

$$0 \longrightarrow {}_{2}\operatorname{Pic} X \longrightarrow \operatorname{Pic}^{0} X \longrightarrow \operatorname{Pic} X' \longrightarrow 0$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$0 \longrightarrow {}_{2}\operatorname{Pic} X_{\bar{k}} \longrightarrow \operatorname{Pic}^{0} X_{\bar{k}} \longrightarrow \operatorname{Pic} X'_{\bar{k}} \longrightarrow 0$$

Here  $\operatorname{Pic}^0 X$  is the group of divisor classes of degree zero. The first two vertical maps are natural injections. Since f splits into a product of linear factors,  ${}_2\operatorname{Pic} X = {}_2\operatorname{Pic} X_{\bar{k}}$ . Since  $\operatorname{Pic}^0 X \subseteq \operatorname{Pic}^0 X_{\bar{k}}$  is an injection, it follows that  $\operatorname{Pic} X' \to \operatorname{Pic} X'_{\bar{k}}$  is injective.

LEMMA 4.3. The group  $_2 \operatorname{Pic} X' \xrightarrow{} (\mathbb{Z}/2)^l$  where  $l \leq 2g$  and l = 2g if and only if  $_4 \operatorname{Pic} X = _4 \operatorname{Pic} X_{\bar{k}}$ .

Proof. We have an exact sequence

$$0 \rightarrow_2 \operatorname{Pic} X \rightarrow_4 \operatorname{Pic} X \rightarrow_2 \operatorname{Pic} X' \rightarrow 0$$
.

Let  $_2\operatorname{Pic} X \longrightarrow (\mathbb{Z}/2)^l$ ,  $l \le 2g$ . Let m elements in  $_2\operatorname{Pic} X$  admit a square root over k. Then  $|_4\operatorname{Pic} X| = m \cdot 2^l$ , with  $m \le 2^l \le 2^{2g}$ . Therefore  $|_2\operatorname{Pic} X'| = m \le 2^{2g}$  and equality holds if and only if  $m = 2^l = 2^{2g}$ ; i.e., if and only if  $_4\operatorname{Pic} X = _4\operatorname{Pic} X_{\bar{k}}$ .

PROPOSITION 4.4. Let Disc denote the discriminant group of a scheme. Let f split into linear factors over k. Then the composite map Disc  $X' \xrightarrow{N}$  Disc  $Y' \to$  Disc Y'/Disc K is surjective if and only if  $_4$ Pic  $X = _4$ Pic  $X_{\bar{k}}$ , N denoting the norm map.

*Proof.* Since X'/Y' is étale quadratic, we have an exact sequence in étale cohomology groups with  $\mu_2$  coefficients ([12], p. 92),

$$0 \longrightarrow H^0(Y') \stackrel{\cup \chi_f}{\longrightarrow} H^1(Y') \stackrel{i}{\longrightarrow} H^1(X') \stackrel{tr}{\longrightarrow} H^1(Y')$$

Here,  $H^i(-)$  denotes  $H^i_{et}(-, \mu_2)$ . The group  $H^1(-)$  is simply the discriminant group so that we have an exact sequence

$$1 \longrightarrow \operatorname{Disc} Y'/\langle f \rangle \longrightarrow \operatorname{Disc} X' \stackrel{N}{\longrightarrow} \operatorname{Disc} Y'.$$

Since the only square class in k(T) which becomes trivial in k(X) is  $\langle f \rangle$ , this sequence yields the following exact sequence

$$1 \to \text{Disc } Y'/\langle f \rangle \text{Disc } k \to \text{Disc } X'/\text{Disc } k \to \text{Disc } Y'/\text{Disc } k$$
.

We denote U(B) and U(A) by U(X') and U(Y') respectively. By our hypothesis on f, Disc  $Y' \longrightarrow U(Y')/2 \longrightarrow (\mathbb{Z}/2)^{2g+1} \times \text{Disc } k$  so that Disc  $Y'/\langle f \rangle \text{Disc } k \longrightarrow (\mathbb{Z}/2)^{2g}$  and Disc  $Y'/\text{Disc } k \longrightarrow (\mathbb{Z}/2)^{2g+1}$ . Further, the exact sequence

$$1 \rightarrow U(X')/2 \rightarrow \text{Disc } X' \rightarrow {}_{2}\text{Pic } X' \rightarrow 0$$

gives, by (4.1) and (4.3) that  $\operatorname{Disc} X'/\operatorname{Disc} k \longrightarrow (\mathbb{Z}/2)^{2g+1+l}$ , where  ${}_{2}(\operatorname{Pic} X') \xrightarrow{\sim} (\mathbb{Z}/2)^{l}$ . Clearly, the map  $\operatorname{Disc} X'/\operatorname{Disc} k \to \operatorname{Disc} Y'/\operatorname{Disc} k$  is surjective if and only if l = 2g; i.e., if and only if  ${}_{4}\operatorname{Pic} X = {}_{4}\operatorname{Pic} X_{F}$ .

# 5. Ramified and unramified parts of coker $\partial^0$

Let  $(\bigoplus_{x \in S} W(k(x)))^0$  denote the subgroup of  $(\bigoplus_{x \in X} W(k(x)))^0$  with non-zero entries only at  $x \in S$ . Let  $V_r$  be the subgroup of coker  $\partial^0$ , defined by

$$V_{r} = \left(\bigoplus_{x \in S} W(k(x))\right)^{0} / \left(\partial W(k(X)) \cap \left(\bigoplus_{x \in S} W(k(x))\right)^{0}\right)$$
$$= \left(\bigoplus_{x \in S} W(k(x))\right)^{0} / \left(\partial W(X')\right)$$

We define  $V_{nr} = \operatorname{coker} \partial^0/V_r$ . If  $p: (\bigoplus_{x \in X} W(k(x)))^0 \to \bigoplus_{x \in X'} W(k(x))$  denotes the restriction of the projection, p is surjective, since  $S = X \setminus X'$  contains a rational point. Thus,

$$V_{nr} \xrightarrow{\sim} \bigoplus_{x \in X'} W(k(x))/(p \circ \partial)W(k(X)).$$

LEMMA 5.1. The map  $\alpha : \operatorname{coker} \partial^0 \to W(A)/\langle 1, -\rangle W(k)$  maps V, onto  $\langle 1, -f \rangle W(A)/\langle 1, -f \rangle W(k)$ .

*Proof.* Let  $\theta \in (\bigoplus_{x \in S} W(k(x))^0$ . Let  $q \in W(k(T))$  be such that  $\partial(q) = tr \theta$ . Since  $\partial_y((q) = tr(\theta_y) = 0$  for  $y \notin \pi(S)$ ,  $q \in W(A)$  and  $\alpha(\overline{\theta}) = \overline{\langle 1, -f \rangle q} \in \langle 1, -f \rangle W(A)/\langle 1, -f \rangle W(k)$ .

We now show that  $\alpha(V_r) = \langle 1, -f \rangle W(A)/\langle 1, -f \rangle W(k)$ . Let  $q \in W(A)$ . Let  $\mu = (\mu_x) \in (\bigoplus_{x \in X} W(k(X)))^0$  be defined by  $\mu_x = 0$  for  $x \in X'$ ,  $\mu_x = \partial_y(q)$ , for  $x \in S$ ,  $\pi(x) = y$ . Then  $\mu \in (\bigoplus_{x \in S} W(k(x)))^0$  and  $\alpha(\bar{\mu}) = \langle 1, -f \rangle q$  in  $\langle 1, -f \rangle W(A)/\langle 1, -f \rangle W(k)$ . We thus have an exact sequence

$$0 \to V_{nr} \xrightarrow{\alpha} W(A)/\langle 1, -f \rangle W(A) \xrightarrow{\beta} W(X) \to 0.$$

PROPOSITION 5.2. The group  $V_{nr}$  is 2-torsion.

*Proof.* Let  $\theta \in \bigoplus_{x \in X'} W(k(x))$ . Since  $\pi(S)$  has a rational point of ramification, there exists  $q \in W(k(T))$  such that  $\partial(q) = tr \theta$ . We have,  $\langle 1, -f \rangle (\langle 1, f \rangle q) = 0$  so

that there exists  $q_1 \in W(k(X))$  with  $s(q_1) = (\langle 1, f \rangle q)$ . Since for  $x \in X'$  with  $\pi(x) = y$ ,

$$\partial_{\nu}(\langle 1, -f \rangle q) = \langle 1, -\overline{f} \rangle \partial_{\nu}(q) = \langle 1, -\overline{f} \rangle tr(\theta_{\kappa}) = 0,$$

we have  $tr(2\theta) = \partial(\langle 1, f \rangle q) = \partial(s(q_1)) = tr(\partial(q_1))$ . Thus, by (3.3), there exists  $q_2 \in W(k(T))$  such that  $\partial_x \circ i(q_2) = 2\theta_x - \partial_x q_1$  for  $x \in X'$  and  $\partial_x \circ i(q_2) = 0$  for  $x \in S$ . Thus  $2\theta - \partial(q_1 - i(q_2)) \in (\bigoplus_{x \in S} W(k(x)))^0$  and its image under the projection map p is zero. Thus the class of  $2\theta$  in  $V_{nr}$  is zero.

Therefore coker  $\partial^0$  is an extension of  $V_r$  by the 2-torsion group  $V_{nr}$ . We now show that under the rationality assumption  ${}_4\mathrm{Pic}\ X = {}_4\mathrm{Pic}\ X_{\bar k},\ V_r = 0$ . We observe that  ${}_4\mathrm{Pic}\ X_{\bar k}$  being a finite group, there exists a finite separable extension l/k such that  $V_r = 0$  for  $X_l$ .

PROPOSITION 5.3. Suppose  $_2\operatorname{Pic} X = _2\operatorname{Pic} X_{\bar{k}}$ . Then the group  $V_r = 0$  if and only if  $_4\operatorname{Pic} X = _4\operatorname{Pic} X_{\bar{k}}$ .

*Proof.* We show that the map  $\partial: W(X') \to (\bigoplus_{x \in S} W(k(x)))^0$  is surjective if and only if  ${}_4\text{Pic } X = {}_4\text{Pic } X_{\bar{k}}$ . In view of the commutative diagram

$$W(X') \longrightarrow \left(\bigoplus_{x \in S} W(k(x))\right)^{0}$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \parallel$$

$$0 \longrightarrow W(k) \longrightarrow W(Y') \longrightarrow \left(\bigoplus_{y \in \pi(S)} W(k(y))\right)^{0} \longrightarrow 0 \qquad (**)$$

with (\*\*) exact, we need to show that  $s: W(X') \to W(Y')/W(k)$  is surjective if and only if  $_4\mathrm{Pic}\ X = _4\mathrm{Pic}\ X_{\bar{k}}$ . By our assumption  $_2\mathrm{Pic}\ X = _2\mathrm{Pic}\ X_{\bar{k}}$ , f splits as a product  $\Pi_{1 \le i \le 2g+1}\ (T-\alpha_i)$  over k. Suppose  $_4\mathrm{Pic}\ X = _4\mathrm{Pic}\ X_{\bar{k}}$ . The exact sequence (\*\*) with each  $W(k(y)) \xrightarrow{} W(k)$  for  $y \in \pi(S)$  implies that W(Y') is generated by Disc k and  $\langle \lambda(T-\alpha_i) \rangle$ ,  $\lambda \in k^*$ ,  $1 \le i \le 2g+1$ . It is therefore enough to show that given  $\langle \lambda(T-\alpha_i) \rangle$ ,  $\lambda \in k^*$ , there exists  $\mu \in k^*$  such that  $\langle \mu, \lambda(T-\alpha_i) \rangle \in s(W(X'))$ . By (4.4), there exists  $\tilde{z} \in \mathrm{Disc}\ X'$  such that  $N(\tilde{z}) = \langle v(T-\alpha_i) \rangle$  for some  $v \in k^*$ . We have,  $s(\tilde{z}) = z_1 \langle 1, -v(T-\alpha_i) \rangle$  for some  $z_1 \in k(T)$ . Thus,

$$s(-z_1^{-1}v^{-1}\cdot\lambda\cdot\tilde{z})=\langle -v^{-1}\lambda,\lambda(T-\alpha_i)\rangle.$$

Conversely, suppose  $W(X') \to W(Y')/W(k)$  is surjective. Then the map restricted to the ideal I(X') of even dimensional forms surjects onto I(Y')/I(k). In view of the commutative diagram

$$I(X') \xrightarrow{s} I(Y')/I(k)$$

$$\downarrow \qquad \qquad \downarrow$$

$$Disc X' \xrightarrow{N} Disc Y'/Disc k$$

with the vertical maps surjective, it follows that  $N: \operatorname{Disc} X' \to \operatorname{Disc} Y'/\operatorname{Disc} k$  is surjective. This implies, by (4.4) that  ${}_{4}\operatorname{Pic} X = {}_{4}\operatorname{Pic} X_{\bar{k}}$ .

In the next section, under certain assumptions on k and X, we describe the unramified part  $V_{nr}$  of coker  $\partial^0$  cohomologically.

# 6. The unramified part of coker $\partial^0$

Let Y be any scheme over k. Let the properties PQ(1), PQ(2) for Y be the following.

**PQ(1)**: For every geometric point  $y \in Y$ , the invariant theorem for quadratic spaces,  $I^n(k(y))/I^{n+1}(k(y)) \xrightarrow{\sim} H^n_{et}(k(y), \mu_2)$  holds for all  $n \geq 0$ .

**PQ(2)**: Y satisfies PQ(1) and the maps  $e_n: I_n(Y) \to \Gamma(Y, \mathcal{H}^n)$  defined in ([15, §1) are surjective for  $n \ge 0$ .

Here,  $\mathcal{H}^n$  denotes the Zariski sheaf associated to the presheaf  $U \to H^n_{et}(U, \mu_2)$ . The class of schemes which satisfy PQ(2) include all smooth quasi projective curves over local fields, in view of [2] and [16]. Conjecturally, all smooth projective curves over any field satisfy PQ(2).

We follow the same notations as in §4 and denote by  $\pi: X \to \mathbf{P}^1$  a double cover, X being a smooth hyperelliptic curve with a rational point of ramification. Under the assumptions that  $X' = X \setminus S$ ,  $Y' = Y \setminus \pi(S)$  satisfy PQ(2), we shall describe  $V_{nr}$  as a certain cohomology group.

LEMMA 6.1. Let  $Y \subseteq \mathbf{P}^1$  be any subscheme. Then Y satisfies PQ(2) if Y satisfies PQ(1).

*Proof.* We have the following commutative diagram (cf. [5], [10])

$$\begin{array}{cccc}
0 & 0 & 0 \\
\downarrow & & \downarrow \\
I_n(Y) & \xrightarrow{e_n} & \Gamma(Y, \mathcal{H}^n) \\
\downarrow & & \downarrow \\
0 \longrightarrow I^{n+1}(k(T)) \longrightarrow & I^n(k(T)) & \longrightarrow & H^n(k(T)) & \longrightarrow 0
\end{array}$$

$$\begin{array}{ccccc}
0 & & \downarrow & & \downarrow \\
\downarrow & & \downarrow & & \downarrow \\
0 \longrightarrow \left(\bigoplus_{y \in Y} I^n(k(y))\right)^0 \longrightarrow \left(\bigoplus_{y \in Y} I^{n-1}(k(y))\right)^0 \longrightarrow \left(\bigoplus_{y \in Y} H^{n-1}(k(y))\right)^0.
\end{array}$$

Here  $(\bigoplus_{y\in Y} I^m(k(y)))^0$  (resp.  $(\bigoplus_{y\in Y} H^m(k(y)))^0$  denotes the subgroup consisting of trace zero elements. The two vertical columns are exact, by ([10], p. 277) and [5]. By the assumption on Y, the two rows are exact. The surjectivity of  $e_n: I_n(Y) \to \Gamma(Y, \mathcal{H}^n)$  follows from the surjectivity of the residue map  $\partial: I^{n+1}(k(T)) \to (\bigoplus_{y\in Y} I^n(k(y)))^0$  [13], Theorem 5.3).

LEMMA 6.2. Suppose  $P^1$  and X satisfy PQ(1). Then the sequence

$$I_n(A) \xrightarrow{i} I_n(B) \xrightarrow{s} I_n(A)$$

is exact for  $n \ge 0$ .

*Proof.* Since B/A is unramified, by (2.1), (2.2) and (2.3), we have the following commutative diagram:

The vertical columns are exact by ([10], p. 277). Exactness of the rows is a consequence of the assumption PQ(1) for X and  $P^1$  [3]. Exactness of the top row follows from the surjectivity of  $\partial: I^{n-1}(k(T)) \to \bigoplus_{y \in Y'} I^{n-2}(k(y))$ , Y' being contained in  $A^1$ .

LEMMA 6.3. Suppose X', and Y' satisfy PQ(2). Then

$$(\langle 1, -f \rangle W(A)) \cap I_n(A) \xrightarrow{\sim} \langle 1, -f \rangle I_{n-1}(A).$$

*Proof.* We assume, by induction, that

$$(\langle 1, -f \rangle W(A)) \cap I_m(A) = \langle 1, -f \rangle I_{m-1}(A)$$

for  $m \le n-1$ . Let  $q \in (\langle 1, -f \rangle W(A)) \cap I_n(A)$ . By induction, we may write  $q = \langle 1, -f \rangle q_1, q_1 \in I_{n-2}(A)$ . Since X', Y' satisfy PQ(1), and B/A is étale quadratic,

we have the following commutative diagram

$$I_{n-2}(B) \xrightarrow{s} I_{n-2}(A) \xrightarrow{\langle 1, -f \rangle} I_{n-1}(A)$$

$$\downarrow^{e_{n-2}} \qquad \downarrow^{e_{n-1}}$$

$$H^{n-2}(B) \xrightarrow{tr} H^{n-2}(A) \xrightarrow{\cup \chi_f} H^{n-1}(A)$$

with the bottom row exact. Since

$$\langle 1, -f \rangle q_1 = q \in I_n(A), e_{n-1}(q) = 0; \text{ i.e., } \chi_f \cup e_{n-2}(q_1) = 0.$$

Therefore, there exists  $\theta \in H^{n-2}(B)$  such that  $tr\theta = e_{n-2}(q_1)$ . Let  $\tilde{\theta} \in \Gamma(B, \mathcal{H}^{n-2})$  be the image of  $\theta$  in  $H^{n-2}(k(X))$ . By the assumption that X' satisfies PQ(2), there exists  $q_2 \in I_{n-2}(B)$  such that  $e_{n-2}(q_2) = \tilde{\theta}$ . The diagram

$$I_{n-2}(B) \xrightarrow{s} I_{n-2}(A)$$

$$\downarrow^{e_{n-2}} \qquad \downarrow^{e_{n-2}}$$

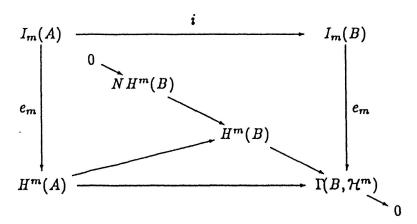
$$\Gamma(B, \mathcal{H}^{n-2}) \xrightarrow{tr} \Gamma(A, \mathcal{H}^{n-2}) = H^{n-2}(A)$$

can be verified to be commutative, so that  $e_{n-2}(q_1 - sq_2) = 0$ . Thus  $q_1 - sq_2 \in I_{n-1}(A)$  and  $\langle 1, -f \rangle (q_1 - sq_2) = \langle 1, -f \rangle q_1 \in \langle 1, -f \rangle I_{n-1}(A)$ . This proves the lemma.

We now assume that X' and Y' satisfy PQ(2). The group  $V_{nr} = \ker(W(A)/\langle 1, -f \rangle W(A) \to W(X))$  has a filtration induced by the filtration  $\{I_m(A)\}$  on W(A). Since the map  $W(X) \to W(B)$  is injective and since i preserves filtration, by (6.3), we have,

$$(V_{nr})_m = \ker (I_m(A)/(\langle 1, -f \rangle W(A) \cap I_m(A)) \xrightarrow{i} I_m(B))$$
  
=  $\ker (I_m(A)/\langle 1, -f \rangle I_{m-1}(A) \xrightarrow{i} I_m(B)).$ 

We now define a map  $\eta_m: (V_{nr})_m \to NH^m(B) = \ker (H^m(B) \to \Gamma(B, \mathcal{H}^m))$  as follows. Consider the following commutative diagram:



Let  $x \in I_m(A)$  be such that i(x) = 0. Then the element  $i(e_m(x)) \in H^m(B)$  maps to zero in  $\Gamma(B, \mathcal{H}^m)$ , by the commutativity of the above diagram. Hence  $i(e_m(x)) \in NH^m(B)$ . We define  $\eta_m(\bar{x}) = i \circ e_m(x)$ . To show that  $\eta_m$  is well-defined, we need to check that for  $x \in \langle 1, -f \rangle I_{m-1}(A)$ ,  $\eta_m(\bar{x}) = 0$ . Let  $x = \langle 1, -f \rangle x'$ ,  $x' \in I_{m-1}(A)$ . We have,  $i(e_m(x)) = i(\chi_f \cup e_{m-1}(x')) = \chi_{i(f)} \cup i \circ e_{m-1}(x') = 0$  since f is a square in B. Thus we have a well-defined homomorphism

$$\eta_m: (V_{nr})_m \to NH^m(B).$$

LEMMA 6.4. Ker  $\eta_m = (V_{nr})_{m+1}$ .

*Proof.* Let  $\eta_m(\bar{x}) = 0$  with  $x \in I_m(A)$ . Then  $ie_m(x) = 0$  and the exactness of the sequence

$$H^{m-1}(A) \xrightarrow{\cup \chi_f} H^m(A) \xrightarrow{i} H_m(B) \xrightarrow{tr} H^m(A)$$
 (\*\*\*)

implies that there exists  $y \in H^{m-1}(A)$  such that  $\chi_f \cup y = e_m(x)$ . By (6.1), there exists  $z \in I_{m-1}(A)$  such that  $e_{m-1}(z) = y$ . We have,  $e_m(x - \langle 1, -f \rangle \cdot z) = 0$  so that  $x - \langle 1, -f \rangle \cdot z \in I_{m+1}(A)$  and its class in  $(V_{nr})_{m+1}$  is simply the class of x.

We thus have a filtration  $\{(V_{nr})_m\}$  on  $V_{nr}$  with successive quotients  $(V_{nr})_m/(V_{nr})_{m+1}$  injecting into  $NH^m(B)$ .

THEOREM 6.5. Under the assumption that X' and Y' have PQ(2),  $V_{nr} \simeq \bigoplus_{m\geq 2} NH^m(B)$ .

Proof. Since by (5.2),  $V_{nr}$  is a 2-torsion group, it is enough to show that  $\eta_m$  maps  $(V_{nr})_m$  onto  $NH^m(B)$ . Let  $x \in NH^m(B)$ . Since  $NH^n(A) = 0 \ \forall n, trx = 0$ , and the exact sequence (\*\*\*) implies that there exists  $y \in H^m(A)$  with i(y) = x. By (6.1), there exists  $z \in I_m(A)$  with  $e_m(z) = y$ . Then  $e_m \circ i(z) = \text{class of } x$  in  $\Gamma(B, \mathcal{H}^m)$  which is zero since  $x \in NH^m(B)$ . Thus  $i(z) \in I_{m+1}(B)$  and  $s \circ i(z) = 0$ . By (6.2), there exists  $z' \in I_{m+1}(A)$  with i(z') = i(z). Replacing z by z - z' which again maps to y under  $e_m$ , we have i(z) = 0; i.e.,  $\bar{z} \in (V_{nr})_m$  with  $\eta_m(\bar{z}) = x$ .

# 7. An example

THEOREM 7.1. Let X be a smooth projective hyperelliptic curve defined over a local field k with residue field characteristic  $\neq 2$ . Suppose X has a rational point of ramification, X has good reduction and  $_4\text{Pic }X = _4\text{Pic }X_{\bar{k}}$ . Then

coker 
$$\partial \longrightarrow W(k) \oplus (\mathbb{Z}/2)^{4g}$$
,

g being the genus of X.

In view of results of [2], any curve over a local field satisfies PQ(1). It is shown in [16] that any such curve also satisfies PQ(2). Therefore by our assumption  ${}_{4}\text{Pic }X = {}_{4}\text{Pic }X_{\bar{k}}$ , we have, coker  $\partial \longrightarrow W(k) \oplus (\bigoplus_{m \geq 2} NH^m(X'))$ . Let  $G = G(\bar{k}/k)$ ,  $\bar{k}$  denoting the algebraic closure of k. Then  $cd_2k \leq 2$  [18] and  $cd_2X'_{\bar{k}} \leq 1$ ,  $X'_{\bar{k}}$  being affine. The spectral sequence ([12], p. 105)

$$H^i(G, H^j(X'_{\bar{k}})) \Rightarrow H^n(X')$$

yields  $H^n(X') = 0$  for  $n \ge 4$ . Thus coker  $\partial^0 \longrightarrow NH^2(X') \oplus NH^3(X')$ . We shall now compute these groups.

LEMMA 7.2. Let X be any smooth projective curve of genus g (not necessarily hyperelliptic) over a local field k with residue field characteristic  $\neq 2$  and such that  $X(k) \neq \emptyset$  and  $_2\operatorname{Pic} X = _2\operatorname{Pic} X_{\bar{k}}$ . Then  $H^3(X) \simeq (\mathbb{Z}/2)^{2g+2}$  and  $\Gamma(X, \mathcal{H}^3) = 0$ .

*Proof.* The only two non-zero terms in the above spectral sequence contributing to  $H^3(X)$  are  $H^1(G, H^2(X_{\bar{k}}))$  and  $H^2(G, H^1(X_{\bar{k}}))$ . The only possible non-zero differential  $H^0(G, H^2(X_{\bar{k}})) \to H^2(G, H^1(X_{\bar{k}}))$  is zero, X(k) being non-empty, since  $H^2(X) \to H^0(G, H^2(X_{\bar{k}}))$  is surjective. Therefore

$$H^3(X) \xrightarrow{\sim} H^2(G, H^1(X_{\bar{k}})) \oplus H^1(G, H^2(X_{\bar{k}}))$$
  
 $\xrightarrow{\sim} (\mathbb{Z}/2)^{2g} \oplus (\mathbb{Z}/2)^2.$ 

In fact the action of G on  $H^1(X_{\bar{k}}) \xrightarrow{\sim} _2 \operatorname{Pic} X_{\bar{k}} \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$  is trivial by our assumption and  $H^2(X_{\bar{k}}) \xrightarrow{\sim} \operatorname{Pic} X_{\bar{k}}/2 \xrightarrow{\sim} \mathbb{Z}/2$  with trivial action again. Further, k being a local field,  $H^2(G, \mathbb{Z}/2) \xrightarrow{\sim} _2 Br(k) \xrightarrow{\sim} \mathbb{Z}/2$  and  $H^1(G, \mathbb{Z}/2) \xrightarrow{\sim} _k */k *^2 \xrightarrow{\sim} \mathbb{Z}/2 \times \mathbb{Z}/2$ . In view of [4],  $NH^3(X) \xrightarrow{\sim} _k */k *^2 \times J(k)/2J(k)$ . Since k is a local field, by [11], J(k) contains a subgroup  $\mathcal{M}$  isomorphic to copies of the valuation ring such that  $J(k)/\mathcal{M}$  is finite. The 2-primary part of  $J(k)/\mathcal{M}$  is isomorphic to  $\Pi_{1 \leq j \leq l}(\mathbb{Z}/2^{lj})$ , where  $l = \dim_{\mathbb{Z}/2}(_2\operatorname{Pic} X) = 2g$  by our assumption. Therefore  $J(k)/2J(k) \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$ , so that  $NH^2(X) \xrightarrow{\sim} (\mathbb{Z}/2)^{2g+2}$ . Thus  $NH^3(X) = H^3(X)$  and  $\Gamma(X, \mathcal{H}^3) = 0$ .

COROLLARY 7.3. Let X be a smooth projective curve over a local field k with residue field characteristic  $\neq 2$ . Suppose X has good reduction and  $_2$ Pic  $X = _2$ Pic  $X_{\bar{k}}$ . Then the classical invariants uniquely determine the class of a quadratic space in W(X).

*Proof.* In view of ([15], §1), we have injections  $rk : W(X)/I(X) \hookrightarrow \mathbb{Z}/2$ , disc:  $I(X)/I_2(X) \hookrightarrow H^1(X)$ ,  $c: I_2(X)/I_3(X) \hookrightarrow {}_2Br(X) = \Gamma(X, \mathcal{H}^2)$ , where rk, disc

and c stand for rank, discriminant and Hasse-Witt invariant maps. Since  $I_4(X) \subseteq I^4(k(X)) = 0$  [2] and  $I_3(X)$  injects into  $\Gamma(X, \mathcal{H}^3) = 0$  by (7.2), we have, rk, disc and c uniquely determine an element in W(X).

LEMMA 7.4. Let X be a hyperelliptic curve. Then  $NH^2(X') \longrightarrow (\mathbb{Z}/2)^{2g}$ , under the assumptions of (7.1) on X.

*Proof.* We have  $NH^2(X') \cong \operatorname{Pic} X'/2$ . The exact sequence

$$0 \rightarrow {}_{2}\operatorname{Pic} X \rightarrow \operatorname{Pic}^{0} X \rightarrow \operatorname{Pic} X' \rightarrow 0$$

yields the following long exact sequence

$$0 \rightarrow {}_{2}\operatorname{Pic} X \rightarrow {}_{2}\operatorname{Pic}^{0} X \rightarrow {}_{2}\operatorname{Pic} X' \rightarrow {}_{2}\operatorname{Pic} X/2 \rightarrow \operatorname{Pic}^{0} X/2 \rightarrow \operatorname{Pic} X'/2 \rightarrow 0.$$

We have  $_2\operatorname{Pic} X = _2\operatorname{Pic}^0 X$ ,  $_2\operatorname{Pic} X' \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$  (4.3),  $_2\operatorname{Pic} X/2 \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$  and  $\operatorname{Pic}^0 X/2 = J(k)/2J(k) \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$ , in view of (7.2). We therefore have  $\operatorname{Pic} X'/2 \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$ .

LEMMA 7.5. Let X be a hyperelliptic curve. Then  $NH^3(X') \simeq (\mathbb{Z}/2)^{2g}$ , under the assumptions of (7.1) on X.

*Proof.* We have an exact sequence

$$0 \rightarrow U(X'_{\overline{k}})/2 \rightarrow H^1(X'_{\overline{k}}) \rightarrow {}_{2}\operatorname{Pic} X'_{\overline{k}} \rightarrow 0.$$

By (4.1) and (4.3),  $U(X_{\bar{k}}')/2 \xrightarrow{\sim} (\mathbf{Z}/2)^{2g+1}$  and  $_2\operatorname{Pic} X_{\bar{k}}' \xrightarrow{\sim} (\mathbf{Z}/2)^{2g}$ . Therefore  $H^1(X_{\bar{k}}') \xrightarrow{\sim} (\mathbf{Z}/2)^{4g+1}$ . Further, since  $U(X_{\bar{k}}')/2$  is generated by  $\{y, T - \alpha_i\}$ ,  $1 \le i \le 2g$ , which are defined over k, and  $_2\operatorname{Pic} X_{\bar{k}}'$  is also defined over k under the assumption  $_4\operatorname{Pic} X' = _4\operatorname{Pic} X_{\bar{k}}'$ , the action of G on  $H^1(X_{\bar{k}}')$  is trivial. The only non-zero terms in the spectral sequence

$$H^i(G, H^j(X'_{\overline{k}})) \Rightarrow H^n(X')$$

contributing to  $H^3(X')$  is  $H^2(G, H^1(X'_{\bar{k}}))$  with all the differentials vanishing, as before. We therefore have

$$H^3(X') \xrightarrow{\sim} H^2(G, H^1(X'_{\overline{k}})) \xrightarrow{\sim} (\mathbb{Z}/2)^{4g+1}$$
.

We shall now compute  $\Gamma(X', \mathcal{H}^3)$ . The sequence

$$H^3(k(X)) \xrightarrow{tr} H^3(k(T)) \xrightarrow{\cup \chi_f} H^4(k(T))$$

is exact and since  $cd_2(k) \le 2$ ,  $cd_2(k(T)) \le 3$ ,  $H^4(k(T)) = 0$ . Thus

$$tr: H^3(k(X)) \rightarrow H^3(k(T))$$

is surjective. It induces a map

$$tr: \Gamma(X', \mathcal{H}^3) \to \Gamma(Y', \mathcal{H}^3) \stackrel{\sim}{\longrightarrow} H^3(Y').$$

We show that this map is surjective. Let  $\lambda \in H^3(Y')$  and  $\mu \in H^3(k(X))$  be such that  $tr \mu = \lambda$ , identifying  $H^3(Y')$  with a subgroup of  $H^3(k(T))$ . In view of the commutative diagram

$$H^{3}(k(T)) \xrightarrow{i} H^{3}(k(X)) \xrightarrow{tr} H^{3}(k(T))$$

$$\downarrow \emptyset \qquad \qquad \downarrow \emptyset$$

with exact rows,  $tr \circ \partial \mu = \partial \circ tr\mu = \partial(\lambda) = 0$  and hence there exists  $v \in \bigoplus_{y \in Y'} H^2(k(y))$  with  $i(v) = \partial(\mu)$ . Since  $Y' \subset A^1$ ,  $\partial : H^3(k(T)) \to \bigoplus_{y \in Y'} H^2(k(y))$  is surjective and hence there exists  $\tilde{v} \in H^3(k(T))$  with  $\partial(\tilde{v}) = v$ . We have  $\partial(\mu - iv) = 0$  so that  $(\mu - iv) \in \Gamma(X', \mathcal{H}^3)$  and maps to  $\lambda \in \Gamma(Y', \mathcal{H}^3) = H^3(A)$ . We thus have a surjection  $tr : \Gamma(X', \mathcal{H}^3) \to \Gamma(Y', \mathcal{H}^3)$ . We now compute its kernel. Since  $H^3(k) = 0$ , the map  $\partial : H^3(A) \to (\bigoplus_{y \in \pi(S)} H^2(k(y)))^0$  is an isomorphism. Since the square

$$\Gamma(X', \mathcal{H}^3) \xrightarrow{\partial} \left( \bigoplus_{x \in S} H^2(k(x)) \right)^0$$

$$\downarrow^{tr} \downarrow \qquad \qquad \parallel$$

$$H^3(A) \qquad \simeq \left( \bigoplus_{y \in \pi(S)} H^2(k(y)) \right)^0$$

is commutative, we have,  $\ker tr = \ker \partial = \Gamma(X, \mathcal{H}^3) = 0$ , by [5] and (7.2). Thus,  $\Gamma(X', \mathcal{H}^3) \xrightarrow{\sim} H^3(A) \xrightarrow{\sim} (\mathbb{Z}/2)^{2g+1}$ . Therefore  $NH^3(X') \xrightarrow{\sim} (\mathbb{Z}/2)^{2g}$ .

This completes the proof of Theorem 7.1. Finally, we use the exact sequence (§3) to compute the defining relations for W(X) as a quotient of  $\bigoplus_{x \in S} W(k(x))$ . More precisely, we have the following

THEOREM 7.6. Under the same hypothesis as in (7.1),

$$W(X) \longrightarrow (\mathbb{Z}/2)^{4g} \oplus W(k)$$
.

*Proof.* In view of (3.4) and (7.1), we have an exact sequence

$$0 \to (\mathbb{Z}/2)^{4g} \to W(A)/(\langle 1, -f \rangle W(k)) \to W(X) \to 0 \tag{*}$$

The residue map  $\partial: W(A) \to \bigoplus_{1 \le i \le 2g+1} W(k)$  is surjective, with kernel W(k). We have, in W(k(T)),  $(W(k) \cap \langle 1, -f \rangle \cdot W(k)) = 0$ . In fact, for  $q \in W(k) \cap \langle 1, -f \rangle \cdot W(k)$ , q extends to zero in W(X). Since X(k) is non-empty, specialising at a rational point yields q = 0 in W(k). We thus have an exact sequence

$$0 \to W(k) \to W(A)/(\langle 1, -f \rangle \cdot W(k)) \to \bigoplus_{2g+1} W(k)/\partial(\langle 1, -f \rangle W(k)) \to 0.$$

The image of the map  $\eta: W(k) \to \bigoplus_{2g+1} W(k)$  defined by

$$\eta(q) = (-f'(\alpha_1)q, -f'(\alpha_2)q, \ldots, -f'(\alpha_{2g+1})q)$$

is precisely  $\partial(\langle 1, -f \rangle \cdot W(k))$ . The map  $\eta$  is injective, since for  $q \in W(k)$ ,  $\eta(q) = 0$  implies that  $\partial(\langle 1, -f \rangle q) = 0$ ; i.e.,  $\langle 1, -f \rangle q \in W(k) \cap \langle 1, -f \rangle W(k) = 0$  and  $q \longrightarrow fq$ . Since degree f is odd, q = 0. Clearly  $\eta$  is a split injection, a section t being given by  $t(q_1, q_2, \ldots, q_{2g+1}) = -f'(\alpha_1) \cdot q_1$ . We thus have an isomorphism

$$\tilde{\eta}: W(A)/(\langle 1, -f \rangle W(k)) \to W(k) \oplus \left(\bigoplus_{2g} W(k)\right)$$

given by  $\tilde{\eta}(\bar{q}) = (\tilde{q}, (\partial_{x_i}q)), 2 \le i \le 2g+1, x_i \in S, \tilde{q}$  denoting specialisation at  $\infty$ . If  $\bar{q} \in W(A)/<1, -f > W(k)$ , maps to zero in W(X), specialising at  $\infty$ , we see that  $\tilde{q}=0$ , so that in the sequence (\*),  $(\mathbb{Z}/2)^{4g}$  injects into the factor  $\bigoplus_{2g} W(k) \xrightarrow{\sim} \bigoplus_{4g} W(F)$  where F denotes the residue field of k. If -1 is a square in F,  $W(F) \xrightarrow{\sim} (\mathbb{Z}/2)^2$  and if -1 is not a square in F,  $W(F) \xrightarrow{\sim} \mathbb{Z}/4$ . Therefore,

$$W(X) \simeq W(k) \oplus W(F)^{4g}/(\mathbb{Z}/2)^{4g}$$
  
 $\simeq W(k) \oplus (\mathbb{Z}/2)^{4g}.$ 

The above theorem leads one to the following natural questions.

QUESTION 1. For a smooth hyperelliptic curve X over an arbitrary ground field k, (with  $_4\text{Pic }X = _4\text{Pic }X_{\bar{k}}$ ), is W(X) isomorphic to  $W(k) \oplus (\mathbb{Z}/2)^{4g}$ ?

A positive answer to this question will also provide evidence to an affirmative answer to the following more general

QUESTION. (Scharlau) Let X be a smooth projective curve over a field k. If W(k) is finitely generated, is W(X) finitely generated?

QUESTION 2. For a smooth projective curve X over k with  $X(k) \neq \emptyset$ , is coker  $\partial \simeq W(X)$ ?

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