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On four-dimensional s-cobordisms, II

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In this note we complete the topological classification of s-cobordisms of 3-dimensional quaternionic space forms. Let Q_r be the quaternionic group of order 2^{r+2} and let $M_r = S^3/Q$ be the quotient of usual action of Q_r on S^3.

THEOREM. *There exist precisely* 2^{2^r-r-1} *distinct topological s-cobordisms of* M_r *to itself.*

In [CS 1] (see also [CS 2]) it was shown that this number is a lower bound. Additional study using topological surgery, valid in this situation by the work of Freedman [F], showed that there are at most 2^{2^r-r} distinct s-cobordisms. The further paper [KwS] on s-cobordisms of space forms also left this ambiguity unresolved. In [CS 3] we erroneously claimed that there are precisely 2^{2^r-r} distinct s-cobordisms, and the new invariant was used to detect the topological non-triviality of an explicitly constructed smooth s-cobordism. In part [CS 3] used various exact sequences in Witt and L-theory, and the present note is the result of reconsideration of this material in light of [Ra] and the visible symmetric L-theory of Michael Weiss.

We will actually consider only the case $r = 1$, and will prove that every s-cobordism of $M = M_i$ to itself is homeomorphic to a product. From [KwS], the general result can easily be seen to follow from this case; alternatively, the argument to be given readily generalizes. (Similarly, the present methods also apply to the other space forms studied in [KwS].)

By [F], the topological surgery sequence ($Q = Q_1$)

$$[\Sigma^2 M_+; G/\mathrm{TOP}] \xrightarrow{\theta} L_s(Q) \to \mathcal{S}(M \times I/\partial) \xrightarrow{\eta}$$

$$\to [\Sigma M_+; G/\mathrm{TOP}] \to L_4(Q)$$

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for structures on $M \times I$ relative the boundary is valid in this case. It is well known that all the elements in

$$[\Sigma M_+; G/\mathrm{TOP}] \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

can be realized as the images under the normal invariant η of homotopy equivalences of $M \times I$ to itself that are the identity on the boundary. It is not difficult to construct these directly and to compute their normal invariants from the obvious characteristic variety for $M \times I$ rel boundary; this is also proven by a homotopy theoretic analysis in [KwS]. By [CS2], the image of θ is a copy of $\mathbb{Z}_2$ in

$$L_5(Q) = L_5^s(Q) = L_5^h(Q) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2.$$

Therefore there exists at most one non-trivial s-cobordism of M to itself.

Now let h be the composite

$$M \times I \to (M \times I) \vee S^4 \to M \times I,$$

where the first map is obtained by pinching off a cell in the interior and the second is the identity on $M \times I$ and the non-trivial element of $\pi_4(M) \cong \mathbb{Z}_2$ on S^4. It is also well known that $\eta[h] = 0$. (This is not essential, as we could compose with one of the self equivalences mentioned above.) Hence the theorem follows if it can be shown that the element $[h] \in \mathcal{S}((M \times I)/\partial)$ is non-trivial. In fact, it will be shown that the surgery obstruction $\xi \in L_5(Q)$ of a normal cobordism of h to the identity is not in the image of θ.

Let $VL^5(\mathbb{Z}Q)$ denote the visible symmetric L-group of Michael Weiss [We], but using chain complexes of stably free modules. The visible L-groups are refinement of the symmetric L-groups of Ranicki [Ra], have the same formal properties, and cobordism classes of finite Poincare complexes and degree one maps of them determine elements in the visible groups in the same way. Let

$$j_*: L_5(Q) \to VL^5(\mathbb{Z}Q)$$

be the natural map. The normal map (Ω, b) of [CS 2],

$$\Omega: M \times T^2 \to M \times S^2$$

has surgery obstruction $\sigma(\Omega, b)$ the non-trivial element in the image of θ. The map Ω obviously bounds a degree one map of manifolds

$$M \times S^1 \times D^2 \to M \times D^3.$$

Hence the element $\sigma^*(\Omega)=j_*(\sigma(\Omega, b)) \in VL^5(\mathbb{Z}Q)$ vanishes i.e., $j_* \circ \theta = 0$. Hence it will suffice to show that

$$j_*(\xi) \neq 0.$$

Let $i:\pi \subset Q$ be the inclusion of the center. Recall the diagram [We] (compare [Ra], [WeI, II])

$$\begin{tikzcd}
 & VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) \arrow[r] \arrow[d] & V\hat{L}^6(\mathbb{Z}Q, \mathbb{Z}\pi) \arrow[r, "\Delta"] \arrow[d, "\theta"] & L_5(Q, \pi) \arrow[d] \\
L_5(\pi) \arrow[r] \arrow[d] & VL^5(\mathbb{Z}\pi) \arrow[r, "\omega"] \arrow[d, "i_*"] & V\hat{L}^5(\mathbb{Z}\pi) \arrow[r] \arrow[d] & L_4(\pi) \\
L_5(Q) \arrow[r, "j_*"] & VL^5(\mathbb{Z}Q) \arrow[r] & V\hat{L}^5(\mathbb{Z}Q) &
\end{tikzcd}$$

with exact rows and columns. The main theorem of [We] yields compatible decompositions

$$V\hat{L}^6(\mathbb{Z}Q, \mathbb{Z}\pi) \cong \bigoplus_m H_{6-m}(Q, \pi; \hat{L}^m(\mathbb{Z}))$$

and

$$V\hat{L}^5(\mathbb{Z}G) \cong \bigoplus_m H_{5-m}(G; \hat{L}^m(\mathbb{Z})), \qquad G = Q \text{ or } \pi,$$

with

$$\hat{L}^m(\mathbb{Z}) = \begin{cases} \mathbb{Z}_8 \\ \mathbb{Z}_2 \\ 0 \\ \mathbb{Z}_2 \end{cases} \text{for} \quad m \equiv \begin{cases} 0 \\ 1 \\ 2 \\ 3 \end{cases} \bmod 4.$$

Let

$$f: M \times S^1 \to M \times S^1$$

be obtained from h by the obvious identification of boundary components. Performing the corresponding identifications on the boundary of a rel boundary normal cobordism of h to the identity and then gluing on copies of $M \times D^2$ yields a normal map (g, b),

$$g: W \to M \times D^2$$

with $\partial W = M \times S^1$, $g \mid \partial W = f$, and $\sigma(g, b) = \xi$. On the other hand, a simple homotopy-theoretic argument shows that f extends to a map

$$k : Y = (M \times D^2)\#E \to M \times D^2,$$

where E is the nontrivial linear bundle over S^2 with fiber S^3, with k trivial on the image of a cross-section of E and with $k \mid S^3$ the covering projection of S^3 to $M \subset M \times D^2$. By a standard cobordism-theoretic argument, using e.g. (17.6) of [C], $W \cup_{\partial W} Y$ with the obvious map represents zero in the oriented cobordism of M. It follows readily that

$$g \cup k : W \cup_{\partial W} Y \to M \times D^2 \cup_{\partial} M \times D^2 = M \times S^2$$

is the boundary of a degree one map of an oriented manifold into $M \times D^3$. Hence the invariant $\sigma^*(g \cup k) \in VL^5(\mathbb{Z}Q)$ vanishes, and so

$$j_*(\xi) = -j_*(\xi) = -\sigma^*(g) = \sigma^*(k).$$

Let $P^3 = S^3/\pi$ be real projective 3-space and let

$$f_2 : P^3 \times S^1 \to P^3 \times S^1$$

be defined, analogously to f, by twisting the top cell around the generator of $\pi_3(P^3)$, and define the extension

$$k_2 : Y_2 = (P^3 \times D^2)\#E \to P^3 \times D^2$$

similarly to the above as well. Then we claim that

$$i_*(\sigma^*(k_2)) = \sigma^*(k),$$

$i : \pi \to Q$ the inclusion as above. The crucial geometric fact that will be used in the proof of the claim is that for both k and k_2, the obvious generator of the kernel group $K_3(k; \mathbb{Z}Q)$ or $K_3(k_2; \mathbb{Z}\pi)$ is represented by an immersed sphere in $Y \times \mathbb{R}$ or $Y_2 \times \mathbb{R}$ with the coefficient of the non-zero element $g \in \pi$ in its self-intersection number equal to ± 1. The generator of $K_3(k; \mathbb{Z}Q)$ is represented by the sum of a generator of $\pi_3(M)$ and a fiber of the summand E. Hence to see this fact, it suffices to check the same thing for the generators of π_3 of $M \times \mathbb{R}^3$ or $P^3 \times \mathbb{R}^3$. This can be done using the immersions ($z_i \in \mathbb{C}$, $z_1\bar{z}_1 + z_2\bar{z}_2 = 1$)

$$g(z_1, z_2) = (p(z_1, z_2), z_1, \operatorname{Re}(z_2)),$$

p the appropriate covering projection ($\mathbb{R}^3 = \mathbb{C} \times \mathbb{R}$) to compute the self-intersection number. The details are left to the reader.

The proof of the claim requires a more detailed review of some aspects of visible L-theory, taken from [We] and from some elaborations kindly provided to us in private communication by Weiss. A visible symmetric algebraic Poincare complex (VSAPC) of dimension n is a finite dimensional chain complex C of stably free finitely generated modules over a group ring $\mathbb{Z}G$, together with a class $[\varphi]$ in

$$VQ^n(C) = H_n(P \otimes_{\mathbb{Z}G} \mathrm{Hom}_{\mathbb{Z}[\mathbb{Z}_2]}(W, C \otimes_{\mathbb{Z}} C)),$$

where P is a projective resolution of $\mathbb{Z}$ over $\mathbb{Z}G$, and W is the usual resolution of $\mathbb{Z}$ over $\mathbb{Z}[\mathbb{Z}_2]$. Further, the image of $[\varphi]$ in the group $Q^n(C)$ under the map induced by $P \to \mathbb{Z}$ is assumed to be non-degenerate in the sense of [Ra]. The group $VL^n(\mathbb{Z}G)$ is the algebraic bordism group of VSAPC's over $\mathbb{Z}G$. The obstruction theory for surgery to kill a class $x \in H_k(C)$ in a VSAPC is formally the same as in [Ra] for symmetric algebraic Poincare complexes – the image of $[\varphi]$ in $VQ^n(\Sigma^{n-k}\mathbb{Z}G)$ under a chain map $C \to \Sigma^{n-k}\mathbb{Z}G$ representing the Poincare dual of x, where the target is the chain complex consisting of $\mathbb{Z}G$ concentrated in dimension $n-k$. Using the exact sequences 2.2 and 2.9 of [We] and the facts [Ra] that $Q^n(\Sigma^{n-k}\mathbb{Z}G) = 0$ for $k > n/2$ and $Q_n(\Sigma^{n-k}\mathbb{Z}G) = 0$ for $k < n/2$, one derives

$$VQ^n(\Sigma^{n-k}\mathbb{Z}G) = \begin{cases} \hat{H}^{k+1}(\mathbb{Z}_2; \mathbb{Z}G)/\hat{H}^{k+1}(\mathbb{Z}_2; \mathbb{Z}) & \text{for } k > n/2 \\ \hat{H}^{k+1}(\mathbb{Z}_2; \mathbb{Z}) & \text{for } k < n/2 \end{cases}.$$

Now consider $(C(k), \varphi)$ representing $\sigma^*(k)$; then $H_i(C(k))$ vanishes for $i \neq 2, 3$, and for $i = 2, 3$, $H_i(C(k)) = K_i(k)$ is free on generators x_i, say, with intersection number $x_2 \cdot x_3 = 1$. In this case the obstruction to killing x_2 can be interpreted as the evaluation of $w_2 Y$ on x_2 and hence is the non-trivial element in $VQ^5(\Sigma^3\mathbb{Z}Q) = \mathbb{Z}_2$. Similarly, the obstruction to killing x_3 can be interpreted as the coefficient mod 2 of g in the self intersection number of an immersion of S^3 in $Y \times \mathbb{R}$ representing x_3 and so is also non-trivial in this case. Exactly the same discussion applies to $C(k_2)$; in fact the inclusion induces isomorphisms $VQ^n(\Sigma^{n-k}\mathbb{Z}\pi) \cong VQ^n(\Sigma^{n-k}\mathbb{Z}Q)$. It follows that $i_*(\sigma^*(k_2))$ is represented by a VSAPC (C, ψ) whose homology is free on generators y_2 and y_3 in dimensions two and three and is trivial elsewhere, with $y_2 \cdot y_3 = 1$ and with the obstructions to killing y_i are non-trivial. Thus in the VSAPC $(C(k), \varphi) \oplus -(C, \psi)$, the homology class $x_2 + y_2$ can be killed by algebraic surgery. The resulting VSAPC has homology free of rank one in dimensions two and three and zero elsewhere (by a standard argument), and the class $x_3 - y_3$ survives to represent a generator of H_3

of this complex. Hence this class can also be killed, and the result is a contractible complex; i.e. the above sum represents zero in $VL^5(\mathbb{Z}Q)$, the claim is proven, and hence

$$j_*(\xi) = i_*(\sigma^*(k_2)).$$

To analyze $\sigma^*(k_2) = \beta$, consider the Poincare complex

$$X = P^3 \times D^2 \cup_{f_2} P^3 \times D^2.$$

Since $(f_2)^2$ is homotopic to the identity, k_2 extends to a degree one map

$$g : (P^3 \times S^2) \# E \to X$$

with $\sigma^*(g) = \beta$. Hence

$$\beta = \sigma^*(X) - \sigma^*((P^3 \times S^2)\# E) = \sigma^*(X),$$

as the domain of g is obviously an oriented boundary. As in [WeII, 7.1], there is a commutative square

$$\begin{array}{ccc} \Omega_5^P(B\pi) & \longrightarrow & H_5(B\pi ; \mathbb{MSG}) \\ \big\downarrow{\scriptstyle \sigma^*} & & \big\downarrow \\ VL^5(\mathbb{Z}\pi) & \xrightarrow{\omega} & H_5(B\pi ; \hat{\mathbb{L}}^*) \cong V\hat{L}^5(\mathbb{Z}\pi). \end{array}$$

(The above decompositions of $V\hat{L}$ come from a decomposition of $\hat{\mathbb{L}}^*$ into Eilenberg–Maclane spectra.) The top horizontal arrow is an (easy) map in the Levitt–Jones–Quinn exact sequence for oriented Poincare bordism, but in the notation used in [Ra 1]. Since the spectrum $\mathbb{MSG}$ has trivial homotopy in negative dimensions, this first of all implies that

$$\omega(\beta) \in \bigoplus_{m=0}^{5} H_{5-m}(B\pi ; \hat{L}^m(\mathbb{Z})).$$

Further, let $\kappa \in H^2(G/O; \mathbb{Z}_2)$ be the non-trivial element, and let $\lambda \in H^3(BSG; \mathbb{Z}_2)$ be the image of κ under (all $\mathbb{Z}_2$ coefficients)

$$H^2(G/O) \cong H^2(SG, SO) \cong H^3(BSG, BSO) \to H^3(BSG).$$

The image $T\lambda$ of λ in $H^3(\mathbb{MSG}; \mathbb{Z}_2) = H^{k+3}(MSG_k; \mathbb{Z}_2)$, k large, under the Thom isomorphism detects the non-trivial element of

$$\pi_3(\mathbb{MSG}) \cong \hat{L}^3(\mathbb{Z}) \cong L_2(e);$$

these isomorphisms are induced by maps in the sequences of Levitt and Ranicki. Hence, at least with respect to a suitable choice of the splitting of the spectrum $\hat{\mathbb{L}}^*$, the diagram

$$\begin{array}{ccc} H_5(B\pi; \mathbb{MSG}) & \longrightarrow & H_5(B\pi; \Omega^{-3}\mathbb{K}(\mathbb{Z}_2)) \\ & & \| \\ \downarrow & & H_2(B\pi; \mathbb{Z}_2) \\ & & \| \\ H_5(B\pi; \hat{\mathbb{L}}^*) & \longrightarrow & H_2(B\pi; \hat{L}^3(\mathbb{Z})), \end{array}$$

where the bottom arrow is given by the splitting above, commutes.

It follows from standard arguments and the above diagrams that the component of $\omega(\sigma^*(X)) = \omega(\beta)$ in $H_2(B\pi; \hat{L}^3(\mathbb{Z}))$ is given by the image in $H_2(B\pi; \mathbb{Z}_2)$ of the class $[X] \cap v^*\lambda \in H_2(X; \mathbb{Z}_2)$ under the obvious map, where

$$v: X \to BSG$$

classifies the Spivak normal fiber space of X. Clearly the restriction of v to the copies of $P^3 \times D^2$ is trivial, and a map of trivial stable spherical fibrations covering f_2 can be viewed as a "clutching function" for v. Further, its classifying map

$$P^3 \times S^1 \to SG$$

is a lift of the normal invariant

$$\eta(f_2): P^3 \times S^1 \to G/O.$$

It follows that $v^*\lambda$ is the image of $\eta(f_2)^*\kappa$ under the connecting homomorphism

$$\delta: H^2(P^3 \times S^1; \mathbb{Z}_2) \to H^3(X; \mathbb{Z}_2)$$

of the Mayer–Vietoris sequence. It follows that $[X] \cap v^*\lambda$ is the image of

$$\lambda = [P^3 \times S^1] \cap \eta(f_2)^*\kappa \in H_2(P^3 \times S^1; \mathbb{Z}_2)$$

under the map induced by inclusion.

It is well-known that the normal invariant $\eta(f_2)$ is non-trivial; one method of proof is to show that the transverse inverse image under f_2 of a framed $S^1 \times S^2 \subset P^3 \times S^1$ has non-trivial Kervaire invariant. (A proof that $\eta(f)$ is trivial can be based on the fact that similar constructions for f using tori in $M \times S^1$ yield two copies of a 2-dimensional Kervaire manifold.) In fact, the above class λ is represented by the inclusion

$$P^2 \times \{pt\} \subset P^3 \times S^1$$

and hence has non-trivial image in $H_2(B\pi; \mathbb{Z}_2)$. Therefore the component of $\omega(\beta)$ in $H_2(B\pi; \hat{L}^3(\mathbb{Z}))$ is non-trivial.

It follows readily from the fact that $\pi_4(P^3) \to \pi_4(P^4)$ is trivial that the canonical map $X \to B\pi = P^\infty$ factors through P^4 and hence $[X]$ maps trivially in $H_5(B\pi)$. This then implies that the component of $\omega(\beta)$ in $H_5(B\pi; \hat{L}^\theta)$ is trivial. Since $i_*(\beta) = j_*(\xi)$, we therefore know that $\omega(\beta) = \partial(\gamma)$,

$$\gamma \in \bigoplus_{m=1}^{5} H_{6-m}(BQ, B\pi; \hat{L}^m(\mathbb{Z})).$$

Let γ_m be the component in $H_{6-m}(BQ, B\pi; \hat{L}^m(\mathbb{Z}))$. The diagram

$$\begin{array}{ccc} \bigoplus_m H_{6-m}(BQ, B\pi; L^m(\mathbb{Z})) & \longrightarrow & \bigoplus_m H_{6-m}(BQ, B\pi; \hat{L}^m(\mathbb{Z})) \\ \downarrow & & \downarrow \Delta \\ VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) & \longrightarrow & VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) \end{array}$$

and the fact that $L^m(\mathbb{Z}) \to \hat{L}^m(\mathbb{Z})$ is an isomorphism for $m = 1, 2, 5$ implies that $\Delta(\gamma_m) = 0$ for $m = 1, 2, 5$. The facts that $L^4(\mathbb{Z}) = \mathbb{Z}$ maps onto $\hat{L}^4(\mathbb{Z}) = \mathbb{Z}_8$ and $H_1(B\pi) \to H_1(BQ)$ is trivial imply that γ may be chosen so that $\Delta(\gamma_4) = 0$ also.

Thus there is an least one γ with $\partial(\gamma) = \omega(\beta)$ and $\Delta(\gamma) = \Delta(\gamma_3)$; note that γ_3 is the unique non-trivial element in $H_3(BQ, B\pi; \hat{L}^3(\mathbb{Z}))$. If $\partial(\gamma) = \partial(\gamma')$, then $\gamma - \gamma'$ comes from $H_6(BQ; \hat{\mathbb{L}}^*)$. However, the composite of Δ and the map from $H_6(BQ; \hat{\mathbb{L}}^*)$ factors through the assembly map

$$A_*: H_6(BQ; \Omega^{-1}\mathbb{L}_*) = H_5(BQ; \mathbb{L}_*) \to L_5(Q).$$

Since the elements of $H_{4i+1}(BQ)$ and $H_{4i-1}(BQ; \mathbb{Z}_2)$ are represented by smooth oriented manifolds, everything in the image is the surgery obstruction of a closed manifold. Hence, by direct computation using representatives (see [CS2] or [Mi]) or by [H], this image is trivial at least when one maps to $L_5^p(Q)$. Therefore, for any γ with $\partial(\gamma)=\omega(\beta)$, we have

$$\Delta^p(\gamma)=\Delta^p(\gamma_3)\in L_5^p(Q,\pi),$$

where Δ^p is the composite of Δ and the map to $L_5^p(Q,\pi)$.

We will now show that $\Delta^p(\gamma_3)\neq 0$. From the above, this implies that β does not come from $VL^6(\mathbb{Z}Q,\mathbb{Z}\pi)$ and hence

$$j_*\xi=i_*\beta\neq 0$$

as was to be shown to prove the theorem. Let $H\subset Q$ be a subgroup of index four, and consider the diagram $(\hat{L}^3(\mathbb{Z})=\mathbb{Z}_2)$

$$\begin{array}{ccccc} & & H_2(BQ;\mathbb{Z}_2) & \longrightarrow & L_4^p(Q,-) \\ & & \big\downarrow p^! & & \big\downarrow p^! \\ H_3(BQ,B\pi;\mathbb{Z}_2) & \xrightarrow{\iota} & H_3(BQ,BH);\mathbb{Z}_2) & \longrightarrow & L_5^p(Q,H), \end{array}$$

where p is the projection of the nontrivial line bundle over BQ corresponding to BH. As in [CS 2] for L^h or in [H], $p^!$ on surgery groups is an isomorphism, and on homology groups $p^!$ is an isomorphism by the Thom isomorphism theorem. The elements of $H_2(BQ;\mathbb{Z}_2)$ are represented by Klein bottles, and a peeling argument similar to [CS 2] shows that $(p^!)^{-1}\iota(\gamma_3)$ has non-trivial image in $L_4^P(Q,-)\cong\mathbb{Z}_2$. Alternatively it is well-known (e.g. [H]) that the nontrivial element in $L_4^p(Q,-)$ is the surgery obstruction of a twisted product of a Klein bottle with the Arf invariant (the codimension two Arf invariant). The reader can easily check that the homology class represented by this Klein bottle maps under $p^!$ to the image of γ_3; this implies $\Delta^p(\gamma_3)\neq 0$ and so completes the proof.

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