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On four-dimensional s -cobordisms, II

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In this note we complete the topological classification of s -cobordisms of 3-dimensional quaternionic space forms. Let Q_r be the quaternionic group of order 2^{r+2} and let $M_r = S^3/Q$ be the quotient of usual action of Q_r on S^3 .

THEOREM. *There exist precisely 2^{2^r-r-1} distinct topological s -cobordisms of M_r to itself.*

In [CS 1] (see also [CS 2]) it was shown that this number is a lower bound. Additional study using topological surgery, valid in this situation by the work of Freedman [F], showed that there are at most 2^{2^r-r} distinct s -cobordisms. The further paper [KwS] on s -cobordisms of space forms also left this ambiguity unresolved. In [CS 3] we erroneously claimed that there are precisely 2^{2^r-r} distinct s -cobordisms, and the new invariant was used to detect the topological non-triviality of an explicitly constructed smooth s -cobordism. In part [CS 3] used various exact sequences in Witt and L -theory, and the present note is the result of reconsideration of this material in light of [Ra] and the visible symmetric L -theory of Michael Weiss.

We will actually consider only the case $r = 1$, and will prove that every s -cobordism of $M = M_1$ to itself is homeomorphic to a product. From [KwS], the general result can easily be seen to follow from this case; alternatively, the argument to be given readily generalizes. (Similarly, the present methods also apply to the other space forms studied in [KwS].)

By [F], the topological surgery sequence ($Q = Q_1$)

$$\begin{aligned} [\Sigma^2 M_+; G/\text{TOP}] \xrightarrow{\theta} L_5(Q) \rightarrow \mathcal{S}(M \times I/\partial) \xrightarrow{\eta} \\ \rightarrow [\Sigma M_+; G/\text{TOP}] \rightarrow L_4(Q) \end{aligned}$$

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for structures on $M \times I$ relative the boundary is valid in this case. It is well known that all the elements in

$$[\Sigma M_+; G/TOP] \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

can be realized as the images under the normal invariant η of homotopy equivalences of $M \times I$ to itself that are the identity on the boundary. It is not difficult to construct these directly and to compute their normal invariants from the obvious characteristic variety for $M \times I$ rel boundary; this is also proven by a homotopy theoretic analysis in [KwS]. By [CS2], the image of θ is a copy of \mathbb{Z}_2 in

$$L_5(Q) = L_5^s(Q) = L_5^h(Q) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2.$$

Therefore there exists at most one non-trivial s -cobordism of M to itself.

Now let h be the composite

$$M \times I \rightarrow (M \times I) \vee S^4 \rightarrow M \times I,$$

where the first map is obtained by pinching off a cell in the interior and the second is the identity on $M \times I$ and the non-trivial element of $\pi_4(M) \cong \mathbb{Z}_2$ on S^4 . It is also well known that $\eta[h] = 0$. (This is not essential, as we could compose with one of the self equivalences mentioned above.) Hence the theorem follows if it can be shown that the element $[h] \in \mathcal{S}((M \times I)/\partial)$ is non-trivial. In fact, it will be shown that the surgery obstruction $\xi \in L_5(Q)$ of a normal cobordism of h to the identity is not in the image of θ .

Let $VL^5(\mathbb{Z}Q)$ denote the visible symmetric L -group of Michael Weiss [We], but using chain complexes of stably free modules. The visible L -groups are refinement of the symmetric L -groups of Ranicki [Ra], have the same formal properties, and cobordism classes of finite Poincare complexes and degree one maps of them determine elements in the visible groups in the same way. Let

$$j_*: L_5(Q) \rightarrow VL^5(\mathbb{Z}Q)$$

be the natural map. The normal map (Ω, b) of [CS 2],

$$\Omega: M \times T^2 \rightarrow M \times S^2$$

has surgery obstruction $\sigma(\Omega, b)$ the non-trivial element in the image of θ . The map Ω obviously bounds a degree one map of manifolds

$$M \times S^1 \times D^2 \rightarrow M \times D^3.$$

Hence the element $\sigma^*(\Omega) = j_*(\sigma(\Omega, b)) \in VL^5(\mathbb{Z}Q)$ vanishes i.e., $j_* \circ \theta = 0$. Hence it will suffice to show that

$$j_*(\xi) \neq 0.$$

Let $i: \pi \subset Q$ be the inclusion of the center. Recall the diagram [We] (compare [Ra], [WeI, II])

$$\begin{array}{ccccccc}
 & & VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) & \longrightarrow & VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) & \xrightarrow{\Delta} & L_5(Q, \pi) \\
 & & \downarrow & & \downarrow \theta & & \downarrow \\
 L_5(\pi) & \longrightarrow & VL^5(\mathbb{Z}\pi) & \xrightarrow{\omega} & VL^5(\mathbb{Z}\pi) & \longrightarrow & L_4(\pi) \\
 \downarrow & & \downarrow i_* & & \downarrow & & \\
 L_5(Q) & \xrightarrow{j_*} & VL^5(\mathbb{Z}Q) & \longrightarrow & VL^5(\mathbb{Z}Q) & &
 \end{array}$$

with exact rows and columns. The main theorem of [We] yields compatible decompositions

$$VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) \cong \bigoplus_m H_{6-m}(Q, \pi; \hat{L}^m(\mathbb{Z}))$$

and

$$VL^5(\mathbb{Z}G) \cong \bigoplus_m H_{5-m}(G; \hat{L}^m(\mathbb{Z})), \quad G = Q \text{ or } \pi,$$

with

$$\hat{L}^m(\mathbb{Z}) = \begin{cases} \mathbb{Z}_8 \\ \mathbb{Z}_2 \\ 0 \\ \mathbb{Z}_2 \end{cases} \text{ for } m \equiv \begin{cases} 0 \\ 1 \\ 2 \\ 3 \end{cases} \pmod{4}.$$

Let

$$f: M \times S^1 \rightarrow M \times S^1$$

be obtained from h by the obvious identification of boundary components. Performing the corresponding identifications on the boundary of a rel boundary normal cobordism of h to the identity and then gluing on copies of $M \times D^2$ yields a normal map (g, b) ,

$$g: W \rightarrow M \times D^2$$

with $\partial W = M \times S^1$, $g \mid \partial W = f$, and $\sigma(g, b) = \xi$. On the other hand, a simple homotopy-theoretic argument shows that f extends to a map

$$k: Y = (M \times D^2) \# E \rightarrow M \times D^2,$$

where E is the nontrivial linear bundle over S^2 with fiber S^3 , with k trivial on the image of a cross-section of E and with $k \mid S^3$ the covering projection of S^3 to $M \subset M \times D^2$. By a standard cobordism-theoretic argument, using e.g. (17.6) of [C], $W \cup_{\partial W} Y$ with the obvious map represents zero in the oriented cobordism of M . It follows readily that

$$g \cup k: W \cup_{\partial W} Y \rightarrow M \times D^2 \cup_{\partial} M \times D^2 = M \times S^2$$

is the boundary of a degree one map of an oriented manifold into $M \times D^3$. Hence the invariant $\sigma^*(g \cup k) \in VL^5(\mathbb{Z}Q)$ vanishes, and so

$$j_*(\xi) = -j_*(\xi) = -\sigma^*(g) = \sigma^*(k).$$

Let $P^3 = S^3/\pi$ be real projective 3-space and let

$$f_2: P^3 \times S^1 \rightarrow P^3 \times S^1$$

be defined, analogously to f , by twisting the top cell around the generator of $\pi_3(P^3)$, and define the extension

$$k_2: Y_2 = (P^3 \times D^2) \# E \rightarrow P^3 \times D^2$$

similarly to the above as well. Then we claim that

$$i_*(\sigma^*(k_2)) = \sigma^*(k),$$

$i: \pi \rightarrow Q$ the inclusion as above. The crucial geometric fact that will be used in the proof of the claim is that for both k and k_2 , the obvious generator of the kernel group $K_3(k; \mathbb{Z}Q)$ or $K_3(k_2; \mathbb{Z}\pi)$ is represented by an immersed sphere in $Y \times \mathbb{R}$ or $Y_2 \times \mathbb{R}$ with the coefficient of the non-zero element $g \in \pi$ in its self-intersection number equal to ± 1 . The generator of $K_3(k; \mathbb{Z}Q)$ is represented by the sum of a generator of $\pi_3(M)$ and a fiber of the summand E . Hence to see this fact, it suffices to check the same thing for the generators of π_3 of $M \times \mathbb{R}^3$ or $P^3 \times \mathbb{R}^3$. This can be done using the immersions $(z_i \in \mathbb{C}, z_1 \bar{z}_1 + z_2 \bar{z}_2 = 1)$

$$g(z_1, z_2) = (p(z_1, z_2), z_1, \text{Re}(z_2)),$$

p the appropriate covering projection ($\mathbb{R}^3 = \mathbb{C} \times \mathbb{R}$) to compute the self-intersection number. The details are left to the reader.

The proof of the claim requires a more detailed review of some aspects of visible L -theory, taken from [We] and from some elaborations kindly provided to us in private communication by Weiss. A visible symmetric algebraic Poincare complex (VSAPC) of dimension n is a finite dimensional chain complex C of stably free finitely generated modules over a group ring $\mathbb{Z}G$, together with a class $[\varphi]$ in

$$VQ^n(C) = H_n(P \otimes_{\mathbb{Z}G} \text{Hom}_{\mathbb{Z}[\mathbb{Z}_2]}(W, C \otimes_{\mathbb{Z}} C)),$$

where P is a projective resolution of \mathbb{Z} over $\mathbb{Z}G$, and W is the usual resolution of \mathbb{Z} over $\mathbb{Z}[\mathbb{Z}_2]$. Further, the image of $[\varphi]$ in the group $Q^n(C)$ under the map induced by $P \rightarrow \mathbb{Z}$ is assumed to be non-degenerate in the sense of [Ra]. The group $VL^n(\mathbb{Z}G)$ is the algebraic bordism group of VSAPC's over $\mathbb{Z}G$. The obstruction theory for surgery to kill a class $x \in H_k(C)$ in a VSAPC is formally the same as in [Ra] for symmetric algebraic Poincare complexes – the image of $[\varphi]$ in $VQ^n(\Sigma^{n-k}\mathbb{Z}G)$ under a chain map $C \rightarrow \Sigma^{n-k}\mathbb{Z}G$ representing the Poincare dual of x , where the target is the chain complex consisting of $\mathbb{Z}G$ concentrated in dimension $n - k$. Using the exact sequences 2.2 and 2.9 of [We] and the facts [Ra] that $Q^n(\Sigma^{n-k}\mathbb{Z}G) = 0$ for $k > n/2$ and $Q_n(\Sigma^{n-k}\mathbb{Z}G) = 0$ for $k < n/2$, one derives

$$VQ^n(\Sigma^{n-k}\mathbb{Z}G) = \begin{cases} \hat{H}^{k+1}(\mathbb{Z}_2; \mathbb{Z}G) / \hat{H}^{k+1}(\mathbb{Z}_2; \mathbb{Z}) & \text{for } k > n/2 \\ \hat{H}^{k+1}(\mathbb{Z}_2; \mathbb{Z}) & \text{for } k < n/2 \end{cases}$$

Now consider $(C(k), \varphi)$ representing $\sigma^*(k)$; then $H_i(C(k))$ vanishes for $i \neq 2, 3$, and for $i = 2, 3$, $H_i(C(k)) = K_i(k)$ is free on generators x_i , say, with intersection number $x_2 \cdot x_3 = 1$. In this case the obstruction to killing x_2 can be interpreted as the evaluation of $w_2 Y$ on x_2 and hence is the non-trivial element in $VQ^5(\Sigma^3\mathbb{Z}Q) = \mathbb{Z}_2$. Similarly, the obstruction to killing x_3 can be interpreted as the coefficient mod 2 of g in the self intersection number of an immersion of S^3 in $Y \times \mathbb{R}$ representing x_3 and so is also non-trivial in this case. Exactly the same discussion applies to $C(k_2)$; in fact the inclusion induces isomorphisms $VQ^n(\Sigma^{n-k}\mathbb{Z}\pi) \cong VQ^n(\Sigma^{n-k}\mathbb{Z}Q)$. It follows that $i_*(\sigma^*(k_2))$ is represented by a VSAPC (C, ψ) whose homology is free on generators y_2 and y_3 in dimensions two and three and is trivial elsewhere, with $y_2 \cdot y_3 = 1$ and with the obstructions to killing y_i are non-trivial. Thus in the VSAPC $(C(k), \varphi) \oplus -(C, \psi)$, the homology class $x_2 + y_2$ can be killed by algebraic surgery. The resulting VSAPC has homology free of rank one in dimensions two and three and zero elsewhere (by a standard argument), and the class $x_3 - y_3$ survives to represent a generator of H_3

of this complex. Hence this class can also be killed, and the result is a contractible complex; i.e. the above sum represents zero in $VL^5(\mathbb{Z}Q)$, the claim is proven, and hence

$$j_*(\xi) = i_*(\sigma^*(k_2)).$$

To analyze $\sigma^*(k_2) = \beta$, consider the Poincare complex

$$X = P^3 \times D^2 \cup_{f_2} P^3 \times D^2.$$

Since $(f_2)^2$ is homotopic to the identity, k_2 extends to a degree one map

$$g : (P^3 \times S^2) \# E \rightarrow X$$

with $\sigma^*(g) = \beta$. Hence

$$\beta = \sigma^*(X) - \sigma^*((P^3 \times S^2) \# E) = \sigma^*(X),$$

as the domain of g is obviously an oriented boundary. As in [WeII, 7.1], there is a commutative square

$$\begin{array}{ccc} \Omega'_5(B\pi) & \longrightarrow & H_5(B\pi; \text{MSG}) \\ \downarrow \sigma^* & & \downarrow \\ VL^5(\mathbb{Z}\pi) & \xrightarrow{\omega} & H_5(B\pi; \hat{\mathbb{L}}^*) \cong VL^5(\mathbb{Z}\pi). \end{array}$$

(The above decompositions of VL^5 come from a decomposition of $\hat{\mathbb{L}}^*$ into Eilenberg–Maclane spectra.) The top horizontal arrow is an (easy) map in the Levitt–Jones–Quinn exact sequence for oriented Poincare bordism, but in the notation used in [Ra 1]. Since the spectrum MSG has trivial homotopy in negative dimensions, this first of all implies that

$$\omega(\beta) \in \bigoplus_{m=0}^5 H_{5-m}(B\pi; \hat{\mathbb{L}}^m(\mathbb{Z})).$$

Further, let $\kappa \in H^2(G/O; \mathbb{Z}_2)$ be the non-trivial element, and let $\lambda \in H^3(BSG; \mathbb{Z}_2)$ be the image of κ under (all \mathbb{Z}_2 coefficients)

$$H^2(G/O) \cong H^2(SG, SO) \cong H^3(BSG, BSO) \rightarrow H^3(BSG).$$

The image $T\lambda$ of λ in $H^3(\text{MSG}; \mathbb{Z}_2) = H^{k+3}(\text{MSG}_k; \mathbb{Z}_2)$, k large, under the Thom

isomorphism detects the non-trivial element of

$$\pi_3(\text{MSG}) \cong \hat{L}^3(\mathbb{Z}) \cong L_2(e);$$

these isomorphisms are induced by maps in the sequences of Levitt and Ranicki. Hence, at least with respect to a suitable choice of the splitting of the spectrum \hat{L}^* , the diagram

$$\begin{array}{ccc} H_5(B\pi; \text{MSG}) & \longrightarrow & H_5(B\pi; \Omega^{-3}\mathbb{K}(\mathbb{Z}_2)) \\ & & \parallel \\ & & H_2(B\pi; \mathbb{Z}_2) \\ \downarrow & & \parallel \\ H_5(B\pi; \hat{L}^*) & \longrightarrow & H_2(B\pi; \hat{L}^3(\mathbb{Z})), \end{array}$$

where the bottom arrow is given by the splitting above, commutes.

It follows from standard arguments and the above diagrams that the component of $\omega(\sigma^*(X)) = \omega(\beta)$ in $H_2(B\pi; \hat{L}^3(\mathbb{Z}))$ is given by the image in $H_2(B\pi; \mathbb{Z}_2)$ of the class $[X] \cap \nu^*\lambda \in H_2(X; \mathbb{Z}_2)$ under the obvious map, where

$$\nu: X \rightarrow BSG$$

classifies the Spivak normal fiber space of X . Clearly the restriction of ν to the copies of $P^3 \times D^2$ is trivial, and a map of trivial stable spherical fibrations covering f_2 can be viewed as a “clutching function” for ν . Further, its classifying map

$$P^3 \times S^1 \rightarrow SG$$

is a lift of the normal invariant

$$\eta(f_2): P^3 \times S^1 \rightarrow G/O.$$

It follows that $\nu^*\lambda$ is the image of $\eta(f_2)^*\kappa$ under the connecting homomorphism

$$\delta: H^2(P^3 \times S^1; \mathbb{Z}_2) \rightarrow H^3(X; \mathbb{Z}_2)$$

of the Mayer–Vietoris sequence. It follows that $[X] \cap \nu^*\lambda$ is the image of

$$\lambda = [P^3 \times S^1] \cap \eta(f_2)^*\kappa \in H_2(P^3 \times S^1; \mathbb{Z}_2)$$

under the map induced by inclusion.

It is well-known that the normal invariant $\eta(f_2)$ is non-trivial; one method of

proof is to show that the transverse inverse image under f_2 of a framed $S^1 \times S^2 \subset P^3 \times S^1$ has non-trivial Kervaire invariant. (A proof that $\eta(f)$ is trivial can be based on the fact that similar constructions for f using tori in $M \times S^1$ yield two copies of a 2-dimensional Kervaire manifold.) In fact, the above class λ is represented by the inclusion

$$P^2 \times \{pt\} \subset P^3 \times S^1$$

and hence has non-trivial image in $H_2(B\pi; \mathbb{Z}_2)$. Therefore the component of $\omega(\beta)$ in $H_2(B\pi; \hat{L}^3(\mathbb{Z}))$ is non-trivial.

It follows readily from the fact that $\pi_4(P^3) \rightarrow \pi_4(P^4)$ is trivial that the canonical map $X \rightarrow B\pi = P^\infty$ factors through P^4 and hence $[X]$ maps trivially in $H_5(B\pi)$. This then implies that the component of $\omega(\beta)$ in $H_5(B\pi; \hat{L}^\theta)$ is trivial. Since $i_*(\beta) = j_*(\xi)$, we therefore know that $\omega(\beta) = \partial(\gamma)$,

$$\gamma \in \bigoplus_{m=1}^5 H_{6-m}(BQ, B\pi; \hat{L}^m(\mathbb{Z})).$$

Let γ_m be the component in $H_{6-m}(BQ, B\pi; \hat{L}^m(\mathbb{Z}))$. The diagram

$$\begin{array}{ccc} \bigoplus_m H_{6-m}(BQ, B\pi; L^m(\mathbb{Z})) & \longrightarrow & \bigoplus_m H_{6-m}(BQ, B\pi; \hat{L}^m(\mathbb{Z})) \\ \downarrow & & \downarrow \Delta \\ VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) & \longrightarrow & VL^6(\mathbb{Z}Q, \mathbb{Z}\pi) \end{array}$$

and the fact that $L^m(\mathbb{Z}) \rightarrow \hat{L}^m(\mathbb{Z})$ is an isomorphism for $m = 1, 2, 5$ implies that $\Delta(\gamma_m) = 0$ for $m = 1, 2, 5$. The facts that $L^4(\mathbb{Z}) = \mathbb{Z}$ maps onto $\hat{L}^4(\mathbb{Z}) = \mathbb{Z}_8$ and $H_1(B\pi) \rightarrow H_1(BQ)$ is trivial imply that γ may be chosen so that $\Delta(\gamma_4) = 0$ also.

Thus there is at least one γ with $\partial(\gamma) = \omega(\beta)$ and $\Delta(\gamma) = \Delta(\gamma_3)$; note that γ_3 is the unique non-trivial element in $H_3(BQ, B\pi; \hat{L}^3(\mathbb{Z}))$. If $\partial(\gamma) = \partial(\gamma')$, then $\gamma - \gamma'$ comes from $H_6(BQ; \hat{L}^*)$. However, the composite of Δ and the map from $H_6(BQ; \hat{L}^*)$ factors through the assembly map

$$A_* : H_6(BQ; \Omega^{-1}\mathbb{L}_*) = H_5(BQ; \mathbb{L}_*) \rightarrow L_5(Q).$$

Since the elements of $H_{4i+1}(BQ)$ and $H_{4i-1}(BQ; \mathbb{Z}_2)$ are represented by smooth oriented manifolds, everything in the image is the surgery obstruction of a closed manifold. Hence, by direct computation using representatives (see [CS2] or [Mi]) or by [H], this image is trivial at least when one maps to $L_5^p(Q)$. Therefore, for

any γ with $\partial(\gamma) = \omega(\beta)$, we have

$$\Delta^p(\gamma) = \Delta^p(\gamma_3) \in L_5^p(Q, \pi),$$

where Δ^p is the composite of Δ and the map to $L_5^p(Q, \pi)$.

We will now show that $\Delta^p(\gamma_3) \neq 0$. From the above, this implies that β does not come from $VL^6(\mathbb{Z}Q, \mathbb{Z}\pi)$ and hence

$$j_*\xi = i_*\beta \neq 0$$

as was to be shown to prove the theorem. Let $H \subset Q$ be a subgroup of index four, and consider the diagram ($\hat{L}^3(\mathbb{Z}) = \mathbb{Z}_2$)

$$\begin{array}{ccccc} H_2(BQ; \mathbb{Z}_2) & \longrightarrow & L_4^p(Q, -) & & \\ \downarrow p' & & \downarrow p' & & \\ H_3(BQ, B\pi; \mathbb{Z}_2) & \xrightarrow{l} & H_3(BQ, BH); \mathbb{Z}_2 & \longrightarrow & L_5^p(Q, H), \end{array}$$

where p is the projection of the nontrivial line bundle over BQ corresponding to BH . As in [CS 2] for L^h or in [H], p' on surgery groups is an isomorphism, and on homology groups p' is an isomorphism by the Thom isomorphism theorem. The elements of $H_2(BQ; \mathbb{Z}_2)$ are represented by Klein bottles, and a peeling argument similar to [CS 2] shows that $(p')^{-1}i(\gamma_3)$ has non-trivial image in $L_4^p(Q, -) \cong \mathbb{Z}_2$. Alternatively it is well-known (e.g. [H]) that the nontrivial element in $L_4^p(Q, -)$ is the surgery obstruction of a twisted product of a Klein bottle with the Arf invariant (the codimension two Arf invariant). The reader can easily check that the homology class represented by this Klein bottle maps under p' to the image of γ_3 ; this implies $\Delta^p(\gamma_3) \neq 0$ and so completes the proof.

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