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## On the embedding of 1-convex manifolds with 1-dimensional exceptional set

MIHNEA COLTOIU

### Introduction

Let  $X$  be a 1-convex manifold and  $S \subset X$  its exceptional set.  $X$  is called embeddable if there exists a holomorphic embedding of  $X$  into  $\mathbb{C}^k \times \mathbb{P}^l$  for suitable  $k, l \in \mathbb{N}$ . When  $X$  has dimension 2 a result of C. Bănică [1], proved also by Vo Van Tan [13c], asserts that  $X$  is embeddable (in fact in this case we may allow  $X$  to have singularities).

The purpose of the present paper is to generalize this result to higher dimensions. We consider a 1-convex manifold  $X$  such that its exceptional set  $S$  is an irreducible curve. Under the assumption that  $S$  is not rational (i.e. its normalization is not  $\mathbb{P}^1$ ) we prove that  $X$  is embeddable. A similar result holds if we assume that  $S \cong \mathbb{P}^1$  and  $\dim X \neq 3$  (see Theorem 5).

The technique of proof enables us to obtain also the following result:

If  $X$  is a complex manifold (not necessarily 1-convex) and  $S \subset X$  is an irreducible exceptional curve with the above properties then the fundamental class of  $S$  in  $X$  does not vanish (see Theorem 6).

### 1. Preliminaries

Throughout this paper we shall not distinguish between holomorphic line bundles and invertible sheaves.

If  $X$  is a complex manifold and  $L$  is a holomorphic line bundle on  $X$  given by transition functions  $\{g_{kl}\}$  corresponding to an open covering  $\{U_k\}$  of  $X$ , a hermitian metric on  $L$  is a system  $\{h_k\}$  of  $C^\infty$  functions  $h_k: U_k \rightarrow (0, \infty)$  such that  $h_k/h_l = |g_{kl}|^2$  on  $U_k \cap U_l$ .

$L$  is said to be Nakano semipositive if there exists a hermitian metric  $h = (h_k)$  on  $L$  such that  $-\log h_k$  is plurisubharmonic on  $U_k$  for any  $k$ .

Let now  $X$  be a 1-convex manifold and  $S \subset X$  its exceptional set.  $X$  is said to be embeddable if it can be realized as a closed analytic submanifold of some  $\mathbb{C}^k \times \mathbb{P}^l$ .

The following theorem of M. Schneider [12], proved also by Vo Van Tan [13a], gives sufficient and necessary conditions for a 1-convex manifold to be embeddable.

**THEOREM 1.** *Let  $X$  be a 1-convex manifold and  $S \subset X$  its exceptional set. Then  $X$  is embeddable iff there exists a holomorphic line bundle  $L$  on  $X$  such that  $L|_S$  is ample.*

If  $X$  is a complex manifold we denote by  $K = K_X$  the canonical line bundle on  $X$ . In order to prove our results we shall need also the following “precise vanishing theorems”:

**THEOREM 2** [10] [13b]. *Let  $X$  be a 1-convex manifold with exceptional set  $S$  and let  $L$  be a holomorphic line bundle on  $X$  such that  $L|_S$  is ample. Then  $H^q(X, K \otimes L) = 0$  for  $q \geq 1$ .*

**THEOREM 3** [5]. *Let  $X$  be a Kählerian manifold and  $L$  a Nakano semipositive line bundle on  $X$ . If  $D \subset X$  is a relatively compact strongly pseudoconvex domain with smooth boundary then  $H^q(D, K \otimes L) = 0$  for  $q \geq 1$ .*

## 2. Main results

**DEFINITION.** Let  $S$  be an irreducible curve and  $\pi: \tilde{S} \rightarrow S$  its normalization.  $S$  is called a rational curve iff  $\tilde{S} = \mathbb{P}^1$ .

The following theorem explains us the behaviour of the canonical bundle in the neighbourhood of an exceptional irreducible curve.

**THEOREM 4.** *Let  $X$  be a 1-convex manifold and assume that its exceptional set  $S$  is an irreducible curve. Suppose that:*

- a)  $S$  is not a rational curve or
- b)  $S \cong \mathbb{P}^1$  and  $\dim X \geq 4$

*Then  $K|_S$  is ample.*

The proof of Theorem 4 is based on several lemmas.

**LEMMA 1.** *Let  $X$  be a 1-convex manifold,  $S \subset X$  its exceptional set and  $k = \dim S$ . Then for every  $\mathcal{F} \in \text{Coh}(X)$  it follows that  $H^q(X, \mathcal{F}) = 0$  for  $q > k$ .*

*Proof.* By a theorem of Narasimhan [9]  $H^q(X, \mathcal{F}) \cong H^q(S, \mathcal{F}|_S)$  for any  $q > 0$ .

Here  $\mathcal{F}|_S$  denotes the topological restriction of  $\mathcal{F}$  to  $S$ , hence  $\mathcal{F}|_S$  is not a coherent sheaf on  $S$ . However, by a result of Reiffen [11 Satz 2] the cohomology groups  $H^q(S, \mathcal{F}|_S)$  vanish for  $q > k$  and the lemma is proved.

**LEMMA 2.** *Let  $X$  be a 1-convex manifold such that its exceptional set  $S$  is 1-dimensional. Then  $S$  has a Kählerian neighbourhood.*

A proof of this lemma can be found in [10 p. 165]. In fact it is shown that  $S$  has an embeddable neighbourhood.

If  $S$  is an irreducible curve we denote by  $\pi: \tilde{S} \rightarrow S$  its normalization. There is an injective morphism of sheaves  $\mathcal{O}_S \xrightarrow{i} \pi_* \mathcal{O}_{\tilde{S}}$  where  $\pi_* \mathcal{O}_{\tilde{S}}$  is the 0-direct image of  $\mathcal{O}_{\tilde{S}}$  (i.e. the sheaf of weakly holomorphic functions on  $S$ ). Let  $\mathbb{R}_S$  be the sheaf on  $S$  of locally constant real valued functions and similarly define  $\mathbb{R}_{\tilde{S}}$  on  $\tilde{S}$ . If  $\mathbb{R}_S \xrightarrow{j} \mathcal{O}_S$  is the natural inclusion map then  $k = i \circ j$  is an injective morphism of sheaves. Let  $k^*: H^1(S, \mathbb{R}_S) \rightarrow H^1(S, \pi_* \mathcal{O}_{\tilde{S}})$  denote the induced map on cohomology.

**LEMMA 3.** *The map  $k^*$  is surjective.*

*Proof.* Consider first the commutative diagram

$$\begin{array}{ccc} H^1(\tilde{S}, \mathbb{R}_{\tilde{S}}) & \xrightarrow{\alpha} & H^1(\tilde{S}, \mathcal{O}_{\tilde{S}}) \\ \gamma \uparrow & & \uparrow \delta \\ H^1(S, \pi_* \mathbb{R}_{\tilde{S}}) & \xrightarrow{\beta} & H^1(S, \pi_* \mathcal{O}_{\tilde{S}}) \end{array}$$

Remark that:

the map  $\delta$  is bijective since  $R^q \pi_*(\mathcal{O}_{\tilde{S}}) = 0$  for  $q > 0$  ( $\pi$  is a finite morphism).

the map  $\gamma$  is bijective since  $R^q \pi_*(\mathbb{R}_{\tilde{S}}) = 0$  for  $q > 0$

(if  $U \subset S$  is contractible it follows easily that  $H^q(\pi^{-1}(U), \mathbb{R}_{\tilde{S}}) = 0$  for  $q > 0$ ; since any point in  $S$  has a fundamental system of contractible open neighbourhoods we deduce that  $R^q \pi_*(\mathbb{R}_{\tilde{S}}) = 0$  for  $q > 0$ ).

the map  $\alpha$  is bijective since  $\tilde{S}$  is Kählerian.

It follows from the commutativity of this diagram that  $\beta$  is bijective.

Consider now the commutative diagram:

$$\begin{array}{ccc} H^1(S, \pi_* \mathbb{R}_{\tilde{S}}) & \xrightarrow{\beta} & H^1(S, \pi_* \mathcal{O}_{\tilde{S}}) \\ \uparrow v & \nearrow k^* & \uparrow i^* \\ H^1(S, \mathbb{R}_S) & \xrightarrow{j^*} & H^1(S, \mathcal{O}_S) \end{array}$$



The map  $v$  is surjective because  $\text{supp}(\pi_* \mathbb{R}_{\tilde{S}}/\mathbb{R}_S)$  is a finite set. Hence  $k^*$  is surjective and Lemma 3 is proved.

**LEMMA 4.** *Let  $S$  be an irreducible curve and  $\pi: \tilde{S} \rightarrow S$  its normalization. Let  $L$  be a holomorphic line bundle on  $S$  which is topologically trivial. Then there exists a holomorphic line bundle  $L'$  on  $S$  which can be given by constant transition functions  $\{g_{kl}\}$  with  $|g_{kl}| = 1$  and such that  $\pi^*(L \otimes L')$  is the trivial line bundle on  $\tilde{S}$ .*

*Proof.* Let  $\mathcal{U} = \{U_i\}$  be a finite open covering of  $S$  such that  $L|_{U_i}$  is trivial and all intersections  $U_{i_0} \cap \dots \cap U_{i_r}$  are connected and contractible. Let  $h_{kl} \in \mathcal{O}^*(U_k \cap U_l)$  denote the transition functions for  $L$ . Since  $L$  is topologically trivial and the covering  $\mathcal{U}$  is topologically acyclic we can find holomorphic functions  $\lambda_{kl} \in \mathcal{O}(U_k \cap U_l)$  such that  $\exp(2\pi i \lambda_{kl}) = h_{kl}$  and  $\lambda_{kl} + \lambda_{ls} + \lambda_{sk} = 0$  on  $U_k \cap U_l \cap U_s$  for any  $k, l, s$ . Hence  $\{\lambda_{kl}\}$  defines a cocycle in  $Z^1(\mathcal{U}, \mathcal{O}_S)$ . Set:  $\hat{U}_i = \pi^{-1}(U_i)$ ,  $\hat{\mathcal{U}} = \{\hat{U}_i\}$  and  $\hat{\lambda}_{kl} = \lambda_{kl} \circ \pi \cdot \{\hat{\lambda}_{kl}\}$  is a cocycle in  $Z^1(\hat{\mathcal{U}}, \pi_* \mathcal{O}_{\tilde{S}})$ . Consider now the commutative diagram:

$$\begin{array}{ccc} H^1(\mathcal{U}, \mathbb{R}_S) & \xrightarrow{p} & H^1(\mathcal{U}, \pi_* \mathcal{O}_{\tilde{S}}) \\ \downarrow & & \downarrow \\ H^1(S, \mathbb{R}_S) & \xrightarrow{k^*} & H^1(S, \pi_* \mathcal{O}_{\tilde{S}}) \end{array}$$

Note that:

the map  $k^*$  is surjective by Lemma 3

the map  $m$  is bijective because  $\mathcal{U}$  is topologically acyclic

the map  $n$  is injective

It follows that  $p$  is surjective. This implies that one can find a cocycle  $\{c_{kl}\} \in Z^1(\mathcal{U}, \mathbb{R}_S)$  and holomorphic functions  $f_k \in \mathcal{O}(\hat{U}_k)$  such that  $\hat{\lambda}_{kl} - f_k + f_l = c_{kl}$  on  $\hat{U}_k \cap \hat{U}_l$  for any  $k, l$ .

If  $L'$  is the holomorphic line bundle on  $S$  with transition functions  $g_{kl} = \exp(-2\pi i c_{kl})$  it follows from our construction that  $\{\exp(2\pi i f_k)\}$  defines a nonvanishing section in  $\pi^*(L \otimes L')$ , hence  $\pi^*(L \otimes L')$  is the trivial line bundle and Lemma 4 is completely proved.

**LEMMA 5.** *Let  $S$  be an irreducible curve and  $\pi: \tilde{S} \rightarrow S$  its normalization. Suppose that there exists a holomorphic line bundle  $L$  on  $S$  such that  $H^1(S, L) = 0$  and  $\pi^*L$  is the trivial line bundle on  $\tilde{S}$ . Then  $S$  is a rational curve.*

*Proof.* There is a canonical morphism of sheaves  $L \rightarrow \pi_* \pi^* L$ . If we set  $\mathcal{F}_1 = \ker \phi$  and  $\mathcal{F}_2 = \text{Im } \phi$  we get an exact sequence

$$0 \rightarrow \mathcal{F}_1 \rightarrow L \rightarrow \mathcal{F}_2 \rightarrow 0$$

Since  $H^1(S, L) = 0$  by hypothesis and  $H^2(S, \mathcal{F}_1) = 0$  because  $\dim S = 1$  it follows from the long exact sequence of cohomology that  $H^1(S, \mathcal{F}_2) = 0$ .

Consider now the exact sequence

$$0 \rightarrow \mathcal{F}_2 \rightarrow \pi_* \pi^* L \rightarrow \frac{\pi_* \pi^* L}{\mathcal{F}_2} \rightarrow 0$$

Since  $\text{supp}(\pi_* \pi^* L / \mathcal{F}_2)$  is a finite set it follows that  $H^1(S, \pi_* \pi^* L / \mathcal{F}_2) = 0$ , hence  $H^1(S, \pi_* \pi^* L) = 0$ . But  $H^1(S, \pi_* \pi^* L) \cong H^1(\tilde{S}, \pi^* L)$  because  $\pi$  is a finite morphism. We deduce that  $H^1(\tilde{S}, \mathcal{O}_{\tilde{S}}) = 0$  and consequently  $\tilde{S} \cong \mathbb{P}^1$ , i.e.  $S$  is a rational curve. Lemma 5 is completely proved.

We are now in a position to prove Theorem 4.

a) Suppose first that  $S$  is an irreducible curve which is not rational. We prove that  $K|_S$  is ample.

It is easy to verify that  $H^2(S, \mathbb{Z}) \cong H^2(\tilde{S}, \mathbb{Z}) \cong \mathbb{Z}$  for any irreducible curve and if  $F$  is a holomorphic line bundle on  $S$  then  $F$  is ample iff  $c(F)$  (the Chern class of  $F$ ) corresponds under the above isomorphisms to a strictly positive integer. Consequently we have to prove that  $c(K|_S) > 0$ .

We remark first that  $c(K|_S) \geq 0$ . Indeed, if  $c(K|_S) < 0$  then  $K^{-1}$  (the dual of  $K$ ) is ample when restricted to  $S$ . By Theorem 2 we obtain  $H^1(X, K \otimes K^{-1}) = 0$ , hence  $H^1(X, \mathcal{O}_X) = 0$ . If  $\mathcal{T}$  denotes the ideal sheaf of  $S$  there is an exact sequence of sheaves on  $X$ :

$$0 \rightarrow \mathcal{T} \rightarrow \mathcal{O}_X \rightarrow \mathcal{O}_X / \mathcal{T} \rightarrow 0$$

Since  $H^1(X, \mathcal{O}_X) = 0$  and  $H^2(X, \mathcal{T}) = 0$  (by Lemma 1) we deduce from the long exact sequence of cohomology that  $H^1(S, \mathcal{O}_S) = 0$  which implies  $S \cong \mathbb{P}^1$ . This contradicts our assumption that  $S$  is not a rational curve. So we must have  $c(K|_S) \geq 0$ .

In order to prove Theorem 4 in case a) we have only to verify that  $c(K|_S) \neq 0$ .

Suppose that  $c(K|_S) = 0$ , hence  $L := K|_S$  is topologically trivial. If  $\pi: \tilde{S} \rightarrow S$  denotes the normalization of  $S$  from Lemma 4 there exists a holomorphic line bundle  $L'$  on  $S$  which can be given by constant transition functions  $\{g_{kl}\}$  with  $|g_{kl}| = 1$  and such that  $\pi^*(L \otimes L')$  is the trivial line bundle on  $\tilde{S}$ .

By Lemma 2  $S$  has an open neighbourhood  $U$  which is Kählerian and shrinking  $U$  if necessary we may assume that there exists a continuous retract  $\rho: U \rightarrow S$ . Let  $S \subset U' \Subset U$  be a strongly pseudoconvex neighbourhood of  $S$  with smooth boundary and let  $\mathcal{V} = \{V_i\}$  be an open covering of  $S$  such that  $L'$  is given on  $V_k \cap V_l$  by the constants  $g_{kl}$  with  $|g_{kl}| = 1$ . Set  $\tilde{V}_k := \rho^{-1}(V_k) \subset U$  and on  $\tilde{V}_k \cap \tilde{V}_l$  consider the transition functions  $\tilde{g}_{kl} := g_{kl}$ . Since  $g_{kl}$  are constants it follows that

the cocycle  $\{\tilde{g}_{kl}\}$  defines a holomorphic line bundle  $\tilde{L}'$  on  $U$  and  $\tilde{L}'|_S = L'$ . Moreover  $\tilde{L}'$  is Nakano semipositive because  $|\tilde{g}_{kl}| = 1$  for any  $k, l$ . From Theorem 3 of Grauert and Riemenschneider we get  $H^1(U', K \otimes \tilde{L}') = 0$ .

Now consider the exact sequence on  $U'$ :

$$(*) \quad 0 \rightarrow \mathcal{I} \rightarrow \mathcal{O}_{U'} \rightarrow \mathcal{O}_{U'}/\mathcal{I} \rightarrow 0$$

where  $\mathcal{I}$  is the ideal sheaf of  $S$ . From  $(*)$  we get the exact sequence on  $U'$ :

$$(**) \quad 0 \rightarrow K \otimes \tilde{L}' \otimes \mathcal{I} \rightarrow K \otimes \tilde{L}' \rightarrow K \otimes \tilde{L}' \otimes \mathcal{O}/\mathcal{I} \rightarrow 0.$$

By Lemma 1  $H^2(U', K \otimes \tilde{L}' \otimes \mathcal{I}) = 0$ . Since  $\tilde{L}'|_S = L'$  the long exact sequence of cohomology implies that  $H^1(S, K|_S \otimes L') = 0$ . But  $\pi^*(K|_S \otimes L')$  is the trivial line bundle on  $\tilde{S}$  and from Lemma 5 it follows that  $S$  is a rational curve which contradicts our hypothesis. Consequently a) is proved.

b) Assume that  $S \cong \mathbb{P}^1$  and  $n = \dim X \geq 4$ . We shall prove that  $K|_S$  is ample.

Let  $N_{S|X}$  denote the normal bundle of  $S$  in  $X$  and  $K_S$  the canonical line bundle of  $S$ . If we use the adjunction formula  $K|_S = K_S \otimes \det(N_{S|X}^*)$  we obtain the following formula for the Chern class of  $K|_S$ :

$$c(K|_S) = c(K_S) - c(\det(N_{S|X}))$$

Since  $S \cong \mathbb{P}^1$  we have  $c(K_S) = -2$ . On the other hand a result of Laufer [6] gives the following estimation:  $c(\det(N_{S|X})) \leq -n + 1$ . Hence we obtain  $c(K|_S) \geq n - 3 > 0$  and Theorem 4 is completely proved.

*Remark.* If  $\dim X = 3$  and  $S \cong \mathbb{P}^1$  it may happen that  $K$  is trivial in the neighbourhood of  $S$ . If  $N_{S|X} = \mathcal{O}(c_1) \otimes \mathcal{O}(c_2)$ ,  $c_1 \leq c_2$ , is the decomposition of  $N_{S|X}$  into line bundles and  $K$  is trivial in the neighbourhood of  $S$  then  $(c_1, c_2) \in \{(-1, -1), (-2, 0), (-3, 1)\}$  (see Laufer [6]). Hence Theorem 4 does not hold if  $\dim X = 3$  and  $S \cong \mathbb{P}^1$ . If  $\dim X = 2$  and  $S \cong \mathbb{P}^1$  easy examples show us that  $K|_S$  may even be negative.

**THEOREM 5.** *Let  $X$  be a 1-convex manifold such that its exceptional set  $S$  is an irreducible curve. Assume that:*

a)  $S$  is not a rational curve

or

b)  $S \cong \mathbb{P}^1$  and  $\dim X \neq 3$ .

*Then  $X$  is embeddable.*

*Proof.* In case a) it follows from Theorem 4 that  $K|_S$  is ample. By Theorem 1  $X$  is embeddable. A similar argument shows us that  $X$  is embeddable if  $S \cong \mathbb{P}^1$

and  $\dim X \geq 4$ . If  $X$  has dimension 2 then  $S$  is a divisor and if we denote by  $[S]$  the corresponding line bundle it follows that  $[S]^{-1}$  (the dual of  $[S]$ ) is ample when restricted to  $S$ . Again by Theorem 1 we deduce that  $X$  is embeddable.

*Remark.* It seems very likely that Theorem 5 should hold for any curve  $S$ .

Let now  $X$  be a complex manifold,  $S \subset X$  an irreducible, compact curve and  $\pi: \tilde{S} \rightarrow S$  its normalization. The image of the fundamental class of  $\tilde{S}$  in  $H_2(X, \mathbb{Z})$  is called the fundamental class of  $S$  in  $X$ . A straightforward consequence of Theorem 4 is the following topological result:

**THEOREM 6.** *Let  $X$  be a complex manifold and  $S \subset X$  an irreducible exceptional curve such that:*

a)  $S$  is not a rational curve

or

b)  $S \cong \mathbb{P}^1$  and  $\dim X \neq 3$

*Then the fundamental class of  $S$  in  $X$  does not vanish.*

*Remarks.* i) In [13b] Vo Van Tan has proved that any 1-convex manifold with 1-dimensional exceptional set is Kählerian. Unfortunately, as we shall see, there is a gap in a main step of his proof.

According to his notations let  $\pi: X \rightarrow Y$  be the Remmert reduction of  $X$ . We assume also that the exceptional set  $S$  is a smooth curve and let  $T$  be any point of  $S$  and set  $Z := X \setminus T$ ,  $\check{S} := S \setminus T$ . If  $\hat{E}$  is a holomorphic line bundle on  $Y$  we set  $E := \pi^*(\hat{E})$  and  $L := E|_Z$ . The author asserts that if  $\hat{E}$  is positive then there exists a metric  $\{h_i\}$  on  $L$  such that:

$$(*) \quad \begin{cases} -\partial\bar{\partial} \log h_i(x) > 0 & \text{on } T_{\check{S},x} \\ -\partial\bar{\partial} \log h_i(x) \geq 0 & \text{on } N_{\check{S},x} \\ -\partial\bar{\partial} \log h_i(z) > 0 & \text{on } T_{Z,z} \text{ if } z \in Z \setminus \check{S} = X \setminus S \end{cases}$$

where  $T_{\check{S},x}$  is the tangent space to  $\check{S}$  at  $x$  and  $N_{\check{S},x}$  is the complement space of  $T_{\check{S},x}$  in  $T_{Z,x}$ .

We shall show that  $(*)$  does not hold. We take  $\hat{E}$  to be the trivial line bundle on  $Y$  which is positive since  $Y$  is Stein. It follows that  $L$  is also the trivial line bundle on  $Z$  and  $(*)$  implies the existence of a  $C^\infty$  function  $h: Z \rightarrow (0, \infty)$  such that  $-\log h$  is strongly plurisubharmonic on  $Z \setminus \check{S}$  and  $-\log h|_{\check{S}}$  is strongly plurisubharmonic. Since  $-\log h$  is strongly plurisubharmonic on  $Z \setminus \check{S}$  it follows from the continuity of second derivatives that  $-\log h$  is plurisubharmonic on  $Z$ . By a well known result concerning the extension of plurisubharmonic functions (see Grauert–Remmert [4]) there exists a plurisubharmonic function  $p$  on  $X$  such that  $p|_Z = -\log h$ . The maximum principle for plurisubharmonic

functions implies that  $p|_S = \text{constant}$ , hence  $-\log h|_S = \text{constant}$ . This contradicts the fact that  $-\log h|_S$  is strongly plurisubharmonic.

The gap in the proof of Vo Van Tan is the following: since  $\check{S} := S \setminus T$  is Stein the metric  $\{h_i\}$  can be suitably modified such that  $L|_{\check{S}}$  is Nakano positive [8] but this can be done only on  $\check{S}$  and there is no control outside  $\check{S}$ .

ii) Under the assumptions of Lemma 5 it follows that  $S$  is a rational curve with  $\dim_{\mathbb{C}} H^1(S, \mathcal{O}_S) \leq 1$ . This can easily be deduced from Riemann–Roch theorem for singular curves. Consequently all our theorems hold if we assume that  $S$  is a rational curve with  $\dim_{\mathbb{C}} H^1(S, \mathcal{O}_S) \geq 2$ .

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