Zeitschrift: Commentarii Mathematici Helvetici

Herausgeber: Schweizerische Mathematische Gesellschaft

Band: 58 (1983)

Erratum: Correction to "The Boolean algebra of spectra", Vol. 54 (1979), pp. 368-377.

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Comment. Math. Helvetici **58** (1983) 599–600 0010–2571/83/040599–02$01.50 + 0.20/0

Correction to 'The Boolean algebra of spectra'

Commentarii Mathematici Helvetici 54/3 (1979) pp. 368–377

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J. M. Boardman and others have pointed out an error in my proof of Proposition 1.5 [1]. Namely, the presentation of the CW-spectrum B_λ/A as the homotopy cofibre of $1-g$ on p. 371 is incorrect for a general limit ordinal λ. The error arose when I wrongly simplified an earlier proof, and the proposition remains valid. As suggested by Boardman, the proof can be repaired by using the equivalence of B_λ/A with the homotopy colimit C of the transfinite sequence $\{B_s/A\}_{s<\lambda}$. The required theory of homotopy colimits can be found in [3], [4], [5]. In more detail, C can be obtained by imposing appropriate face identifications on the wedge of the $B_{s_0}/A \wedge (\Delta^n \cup *)$ running over all $(n+1)$-tuples of ordinals $s_0 < s_1 < \cdots < s_n < \lambda$ for all $n \geq 0$. Thus, C is a CW-spectrum with an increasing filtration by closed subspectra $\{F_nC\}$ such that $F_nC/F_{n-1}C$ is the wedge of the $B_{s_0}/A \wedge S^n$ running over all $(n+1)$-tuples of ordinals $s_0 < s_1 < \cdots < s_n < \lambda$. The associated spectral sequence for π_*C has $E^2_{n,t} \approx \operatorname{colim}^n \{\pi_t B_s/A\}$, and this derived colimit vanishes for $n > 0$ because it is indexed by a directed set. Thus there is an edge isomorphism $\operatorname{colim}_{s<\lambda} \pi_* B_s/A \approx \pi_* C$ and the canonical map $C \to B_\lambda/A$ is a weak equivalence of CW-spectra. Consequently $C \simeq B_\lambda/A$. This equivalence can also be shown by using the isomorphisms $\operatorname{colim}_T \pi_* C_T \approx \pi_* C$ and $\pi_* C_T \approx \pi_* B_{m(T)}/A$ where T runs over all finite nonempty sets of ordinals less than λ, where $C_T \subset C$ is the homotopy colimit of the finite sequence $\{B_t/A\}_{t \in T}$, and where $m(T)$ is the largest ordinal in T. Having shown $C \simeq B_\lambda/A$ one uses the Milnor cofibering $\bigvee_{n \geq 0} F_nC \to \bigvee_{n \geq 0} F_nC \to C$ together with the above wedge decomposition of $F_nC/F_{n-1}C$ to deduce that B_λ/A is $[E, \ ]_*$-colocal and belongs to Class-E as required for the proof of Proposition 1.5 [1] and for subsequent applications. A similar error appeared in the proof of Lemma 1.13 [2] and can be repaired similarly.

REFERENCES

[1] A. K. BOUSFIELD, *The Boolean algebra of spectra*. Comment. Math. Helvet. *54* (1979), 368–377.
[2] A. K. BOUSFIELD, *The localization of spectra with respect to homology*. Topology *18* (1979), 257–281.

[3] A. K. BOUSFIELD and D. M. KAN, *Homotopy limits, completions and localizations*. Lecuture Notes in Mathematics Vol. 304, Springer-Verlag, New York, 1972.
[4] G. SEGAL, *Classifying spaces and spectral sequences*. Publ. Math. I.H.E.S. *34* (1968), 105–112.
[5] R. M. VOGT, *Homotopy limits and colimits*. Math. Z. *134* (1973), 11–52.

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Received April 8, 1983