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## The Novikov conjecture and low-dimensional topology

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It is well known that many interesting manifolds can be obtained by cutting some standard manifold along a separating codimension-one submanifold and glueing the pieces together by a homeomorphism (or diffeomorphism) homotopic to the identity. (These manifolds will be simple homotopy equivalent.) For instance, in dimension at least five, all smooth homotopy spheres can be obtained from the standard sphere in this fashion. In [We 1] we studied the question of identifying those homotopy equivalences that can be so produced as well as related problems. In high dimensions, e.g. dimension at least five, a complete solution is often possible. Through dimension three, the result is trivial, no “new” homotopy equivalences are obtainable since for surfaces homeomorphisms homotopic to the identity are in fact isotopic to the identity. Thus, a gap is left in our knowledge in dimension four.

It is known that for a large class of three-manifolds, homotopy implies isotopy for homeomorphisms, and no example is known of this failing for any three-manifold. This suggests that the situation for four-manifolds should be no different from that for three manifolds and should therefore be very different from the higher dimensional theory.

Most of this paper is devoted to studying homotopy equivalences  $h: S^1 \times L_1 \rightarrow S^1 \times L_2$  where  $L_1$  and  $L_2$  are classical lens spaces. In §1 we review enough of [We 1] to get the flavor of the high dimensional theory and see why it would be anomalous for  $h$  not to be obtainable by cutting and pasting. (In fact,  $h \times 1_{S^1}: T^2 \times L_1 \rightarrow T^2 \times L_2$  can be obtained in such a manner.) In §2 we show by low dimensional techniques that if the codimension-one (three-) manifold cut along lies in the Poincaré category then  $h$  cannot result. In §4 we remove this restriction by an algebraic technique that also shows that many other homotopy equivalences are not cut-pastable. It is in this algebra (§3) that the Novikov conjecture enters as an ingredient in calculating the image of the  $L$ -theory of three-manifold groups in the  $L$ -theory of a certain class of groups.

This paper is an extension of part of the author's thesis. It is a pleasure to

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thank Sylvain Cappell for many useful conversations on surgery theory and Novikov’s conjecture, as well as providing valuable encouragement.

**1. Partial review of [We 1]**

Let  $M$  be a manifold and  $N$  a codimension one submanifold which divides  $M$  into two components, i.e.  $M = M_+ \cup_N M_-$ . The homotopy equivalence obtained by cutting and pasting with  $(N, h, H)$ , where  $h : N \rightarrow N$  is a homeomorphism and  $H : N \times I \rightarrow N \times I$  is a homotopy from  $h$  to the identity, is

$$\bar{H} : M(N, h) \cong M_+ \cup N \times I \cup_{h^{-1}} M_- \xrightarrow{1_M \cup H \cup 1_M} M_+ \cup N \times I \cup M_- = M.$$

Observe that if  $H$  is an isotopy,  $\bar{H}$  is a homeomorphism. A homotopy equivalence  $g : M' \rightarrow M$  is *CP* (*cut-pastable*) if there is a triple  $(N, h, H)$  as above and a homeomorphism  $G : M' \rightarrow M(N, H)$  such that  $g \sim \bar{H} \circ G$ . In [We 1] the following is proven:

**THEOREM A.** *Let  $h : M' \rightarrow M^n$  be a homotopy equivalence between closed  $n$  manifolds,  $n \geq 5$ , and suppose that*

- (a)  $H^2(\pi_1 M; \mathbb{Z}_2) = 0$  and  $H_*(\pi_1 M; \mathbb{Z}_{(2)}) = 0$  for  $* \geq n - 3$  or
- (b)  $Sq^2 : H^2(M; \mathbb{Z}_2) \rightarrow H^4(M; \mathbb{Z}_2)$  is injective, or
- (c)  $\pi_1 M$  is cyclic.

*Let  $\nu(h)$  denote the normal invariant of  $h$ . Then  $h$  is CP iff*

- (1)  $h$  is a simple homotopy equivalence, and
- (2)  $\nu(h) : M \rightarrow G/Top$  lifts to  $\sum \Omega(G/Top)$ .

*Remarks.* 1. Condition (a) holds if  $\pi$  is of odd order or a classical knot group; (b) holds for many manifolds e.g. projective spaces or  $S^1 \times L_p^{2k+1}$  for  $p$  odd. There are many other choices of technical hypotheses available to replace (a) and (b), but some condition is necessary. That  $h$  is CP always implies that (1) and (2) hold, but for  $M = \partial$  (regular neighborhood of the two-skeleton of  $T^7$ ), the converse fails (see [We 1]).

For the next result recall that extensions  $1 \rightarrow \mathbb{Z}_2 \rightarrow E \rightarrow \pi \rightarrow 1$  are naturally in a one to one correspondence with  $H^2(\pi; \mathbb{Z}_2)$ . Also recall that if  $h : M' \rightarrow M$  is a homotopy equivalence then  $k_2(h) \in H^2(M; \mathbb{Z}_2)$  is  $\nu(h)^*(k_2)$  where  $k_2$  is the (unique) nonzero element of  $H^2(G/Top; \mathbb{Z}_2)$ .

**THEOREM B.** *Define*

$$E(h) = \begin{cases} E & \text{if } k_2(h) \in \text{Im } H^2(\pi_1 M; \mathbb{Z}_2) \rightarrow H^2(M; \mathbb{Z}_2) \\ & \text{with preimage corresponding to } E \\ \pi_1 M & \text{otherwise} \end{cases}$$

Then the obstruction to the normal cobordism invariance within the class of  $h$  of a simple homotopy equivalence's being CP lies in a quotient of  $\text{cok} : L_{n+1}^s(E(h)) \rightarrow L_{n+1}^s(\pi_1 M)$ .

**COROLLARY C.** *Any simple homotopy equivalence  $h : M' \rightarrow M^n$ ,  $n \geq 5$  normally cobordant to the identity is CP.*

Concretely, let  $h : L_1 \rightarrow L_2$  be a homotopy equivalence between classical lens spaces. It is easy to see that  $\nu(h) = 0$ . However,  $h$  is not a simple homotopy equivalence, but  $h \times 1_{S^1} : L_1 \times S^1 \rightarrow L_2 \times S^1$  is simple (by the product formula for torsion). Corollary C suggests that  $h \times 1_{S^1}$  is CP. We will see in §4 that this is not true! However  $h \times 1_{T^2} : L_1 \times T^2 \rightarrow L_2 \times T^2$  is CP.

Interestingly, one can combine these results to show that diffeomorphisms of four-manifolds do not up to pseudoisotopy preserve many codimension-one submanifolds. For instance:

**THEOREM 3.** *There is a diffeomorphism of  $T^2 \times S^2 \# k(S^2 \times S^2)$  which is not pseudoisotopic to one preserving  $\{1\} \times S^1 \times S^2$ , for  $k$  large enough.*

In higher dimensions one can often show that diffeomorphisms up to pseudoisotopy preserve codimension-one submanifolds under appropriate  $\pi_1$  conditions by analogy to the ‘‘Straightening Lemma’’ techniques of [We 1]. (See also [Ch].) We will not pursue these ideas further here.

## 2. Low-dimensional considerations

**THEOREM 1.** *If  $L_1$  and  $L_2$  are nondiffeomorphic three-dimensional lens spaces, then  $S^1 \times L_1 \neq S^1 \times L_2(N, h)$  for any  $N^3 \subset S^1 \times L_2$  and  $h : N^3 \rightarrow N^3$  homotopic to the identity. In other words,  $h : S^1 \times L_1 \rightarrow S^1 \times L_2$  is not cut-pastable.*

In this section we prove this under the additional hypothesis that  $N$  is in the Poincaré category, i.e. that any  $C^3 \subset N$ , a contractible compact three manifold,

must be the three-disk. More precisely we prove:

**PROPOSITION.** *If  $M^4 \rightarrow S^1 \times L_2$  is CP in the Poincaré category, then  $S^1 \times \tilde{M}^4 \approx S^1 \times R \times L_2$  where  $\tilde{M}^4$  is the cover of  $M$  corresponding to  $Z_p \subset \pi_1 M$ .*

*Proof of Theorem 1 from the Proposition.* If  $S^1 \times L_1 \rightarrow S^1 \times L_2$  is CP, then by the proposition  $S^1 \times R \times L_1 \approx S^1 \times R \times L_2$  so  $S^1 \times L_1$  is  $h$ -cobordant to  $S^1 \times L_2$ . The classical proof that  $L_1$  and  $L_2$  are not  $h$ -cobordant was based on a multisignature calculation [AB] and therefore shows that  $T^k \times L_1$  is not  $h$ -cobordant to  $T^k \times L_2$  for any  $k$ .  $\square$

*Proof of the Proposition.* For clarity, we first give a proof when the codimension-one submanifold  $N$  of  $S^1 \times L_2$  is assumed prime, and then describe the changes necessary to reduce the general case to one that can be handled by the same technique.

There are two cases to consider;  $N$  sufficiently large or otherwise. Notice that  $N$  is orientable. For prime sufficiently large 3-manifolds diffeomorphisms homotopic to the identity are isotopic to it (see [Wd 1] for the irreducible case and [La] for  $S^1 \times S^2$ ), and therefore does not change the diffeomorphism type by cutting and pasting. If  $N$  is not sufficiently large then we can homotop  $N$  off  $L_2 \times \{1\}$ . If it were isotopic to an embedding not intersecting  $L_2 \times \{1\}$ , then all the cutting and pasting would take place in a regular neighborhood of  $L_2 \times \{-1\}$  and the resulting manifold would therefore be (by 5-dimensional surgery)  $s$ -cobordant to  $L_2 \times S^1$  and the conclusion would follow. Unfortunately, if it is not clear how to perform such an isotopy (or even if one exists) so instead look at the cover  $L_2 \times R$  of  $L_2 \times S^1$  and the induced cover of  $N$ , i.e.  $\bigcup_{i=-\infty}^{\infty} T^i \tilde{N}$  where  $T$  generates the covering translates. The cover  $\tilde{M}^4$  is the result of cutting and pasting a  $L_2 \times R$  along  $\bigcup T^i N$ .

*Claim.* If  $K$  is obtained by CP of  $L \times R$  along a collection  $\mathcal{C}$  of compact submanifolds such that  $\mathcal{C}_1 = \{C \in \mathcal{C} \mid C \cap L \times [-1, 1] \neq \emptyset\}$  is finite, then  $S^1 \times K \approx S^1 \times L \times R$ .

*Proof of claim.* First CP  $S^1 \times K$  along  $S^1 \times \bar{\mathcal{C}}_1$ . The inclusion of  $S^1 \times L \times 0$  in the resulting manifold is a homotopy equivalence, and each end has  $\pi_1^\infty$  mapping isomorphically to the fundamental group of the ambient manifold, so by Siebenmann's criterion [Si] the diffeomorphism type remains  $S^1 \times L \times R$ . Now embed a copy of  $S^1 \times L$  disjoint from the elements of  $\mathcal{C}_1$  and the same argument applies proving the claim.  $\square$

Now for the general case, write  $N = N_1 \# N_2$  where  $N_1$  is the connected sum of the sufficiently large connect summands of  $N$  and  $N_2$  the remaining summands.

The argument in [La V S.4] shows that the glueing map  $h$  is isotopic to a diffeomorphism, still called  $h$ , which is the identity on  $N_1$ . (Here we are using the assumption that  $N$  lie in the Poincaré category.)

Note that  $h|_{\partial N_2=S^2}$  is the identity so we choose a base point  $*$  in  $\partial N_2$ . Consider  $N_2 \# -N_2$  with the autodiffeomorphism  $h \# 1$ . This diffeomorphism induces the identity on  $\pi_1(N_2 \# -N_2, *)$  and homology of all covers. Regarding  $N_2 \# -N_2$  as being the boundary of a regular neighborhood of  $N_2 \subset S^1 \times L_2$ , it is easy to see  $S^1 \times L_2(N, h) \approx S^1 \times L_2(N_2 \# -N_2, h \# 1)$ . Now we argue as before to get the desired conclusion, since it is easy to justify the claim in the more general context where the pasting maps induce the identity on  $\pi_1$  and homology of all covers.

*Remark.* In the next two sections we develop a proof of Theorem 1 without any restriction on the submanifold. The above proof has the virtue of being more geometric and not throwing away 2-torsion. For instance the above proof implies that if  $M^4$  is obtained by CP in the Poincaré category from  $S^1 \times S^3$ , then  $M^4$  is  $s$ -cobordant to  $S^1 \times S^3$ . On the other hand, the results of the next two sections yield no information on this.

### 3. Algebraic preliminaries

Let  $Z = \langle x \mid \rangle$  and  $p \in \pi$ . Define  $h_p : Z \rightarrow \pi$  by  $h_p(x) = p$ .  $L_1(Z) = Z$  with canonical generator  $t$ . Let  $I : \pi \rightarrow L_1(\pi)$  denote the function given by  $I(p) = h_{p*}(t)$  where  $h_{p*} : L_1(Z) \rightarrow L_1(\pi)$  is the induced map on  $L$ -theory.

**PROPOSITION 1.**  *$I$  is a homomorphism, and any other natural homomorphism  $\pi \rightarrow L_1(\pi)$  is a multiple of  $I$ .*

*Proof.* Notice that  $I$  is defined to be natural. Therefore, by considering  $h_{g,g'} : Z * Z \rightarrow \pi$  defined as above, it suffices to show  $I(xy) = I(x) + I(y)$  for  $Z * Z = \langle x, y \mid \rangle$ . Cappell [C1] has shown that  $L_1(Z * Z) \xrightarrow{p_{1*} \oplus p_{2*}} L_1(Z) \oplus L_1(Z) = Z \oplus Z$  is an isomorphism. Now

$$\begin{aligned} p_{1*}I(xy) &= I(p_1(xy)) = I(p_1(x)) = I(p_1(x)) + I(p_1(y)) \\ &= p_{1*}(I(x) + I(y)) \end{aligned}$$

Similarly for  $p_{2*}$ .

The last statement is obvious.  $\square$

*Remarks.* 1. An elementary proof of this proposition based just on first

principles can be given. (I would like to thank Andrew Ranicki for pointing this out to me.) However, the above is shorter and more natural.

2. The above proof also yields a homomorphism  $K : \pi \rightarrow L_3(\pi)$  but this will not concern us here.

The significance of  $I$  is two-fold. First, the invariant of homotopy equivalences that is relevant to us lies in  $G(\pi) = \text{cok } I \otimes Q$ . Second, since  $L$ -groups are abelian,  $I$  actually factors through group homology and lifts the first of a sequence of homomorphisms  $I_i : H_i(\pi; Q) \rightarrow L_i(\pi) \otimes Q$  defined first by Wall [Wa 1].

These homomorphisms are crucial for calculation of surgery obstructions for problems on closed manifolds. They are also intimately related to the Novikov higher signature conjecture (cf. [Wa 2]). For our purposes we shall regard the Novikov conjecture as the statement that

$$\bigoplus_{i=n(4)} I_i : \bigoplus_{i=n(4)} H_i(\pi; Q) \rightarrow L_n(\pi) \otimes Q$$

is injective. The Novikov conjecture has been verified in many cases, cf. [C2], [FH 1], [Lu]. In particular,  $G(\pi)$  can be arbitrarily large. (For the group in [We 2],  $G(\pi)$  has infinite rank.) Actually, there is no known example of a torsion free group where the homomorphism  $\bigoplus I_i$  is not an isomorphism. We call this the strong Novikov conjecture. In fact [C2] and [FH 2] show that for the Waldhausen class [Wd 2] and Bieberbach groups respectively, this conjecture holds. Since fundamental groups of irreducible sufficiently large three manifolds lie in the Waldhausen class [C2] calculates their rational surgery groups.

**DEFINITION.** A group  $\pi$  is *very large* if for any infinite finitely generated subgroup  $H$  of  $\pi$ , there is a homomorphism  $h_H : H \rightarrow Z$  of  $H$  onto the infinite cyclic group  $Z$ .

**EXAMPLES.** 1. Finite groups are very large.

2. Free groups, abelian groups and surface groups are all very large.

3. Given an exact sequence

$$1 \rightarrow \pi \rightarrow \pi' \rightarrow G$$

with  $G$  a torsion free very large group, then  $\pi$  is very large iff  $\pi'$  is. In particular, Poly- $Z$  groups are very large.

4. Fundamental groups of irreducible sufficiently large three manifolds need not be very large. (For instance, the first Betti number can vanish.)

**THEOREM 2.** *If  $\pi$  is a very large group and  $K \approx \pi_1 M^3$ ,  $h : K \rightarrow \pi$  an arbitrary homomorphism, then  $h_* : G(K) \rightarrow G(\pi)$  vanishes. ( $G(\pi) = \text{cok } I \otimes Q$ .)*

*Proof.* By [C1]  $G(A * B) \cong G(A) \oplus G(B)$  so without loss of generality  $M$  can be assumed irreducible. If  $\text{Im } h$  is finite then  $h_*$  factors through  $G$  (finite group) which vanishes as odd Wall groups of finite groups are finite [Wa 3]. Thus, assume  $\text{Im } H$  is infinite, so that the composite  $K \xrightarrow{h} \text{Im } h \xrightarrow{h_{\text{im } h}} Z$  is onto which implies that  $K$  is in the Waldhausen class. If  $K \neq Z$ ,  $M$  is aspherical; in particular in all cases  $H_{4k+1}(K; Q) = 0$  for  $k > 0$ , and  $I: H_1(K; Q) \rightarrow L_1(K) \otimes Q$  is an isomorphism ([C2]) so that  $G(K) = 0$ .  $\square$

*Remarks.* 1. If the strong Novikov conjecture were true for all three-manifold groups, then the conclusion of Theorem 2 would hold for all groups.

2. For torsion free very large groups one can modify the above to calculate  $\text{Im} \bigoplus_{K=\pi_1 M^3} L_i(K) \otimes Q \rightarrow L_i(\pi) \otimes Q$ .

#### 4. An invariant of 4-dimensional homotopy equivalences

Let  $h: M' \rightarrow M^4$  be a homotopy equivalence. In this section we define an  $h$ -cobordism invariant  $\eta(h) \in G(\pi_1 M)$  which, for  $\pi_1 M$  very large, is an obstruction to  $h$  being CP. Stably, the entire image of  $L_1^s(\pi_1 M)$ , in  $G(\pi)$  is realizable as the  $\eta$ -invariant of some simple homotopy equivalence. It will be clear that for  $h \# 1_{k(S^2 \times S^2)}: M' \# k(S^2 \times S^1) \rightarrow M \# k(S^2 \times S^2)$ ;  $\eta(h \# 1_{k(S^2 \times S^2)}) = \eta(h)$  so that these are stable obstructions to CP, violating the usual philosophy that the stable geometric topology of dimension four is no different than the high dimensional theory. The proof of Theorem 1 will merely be the calculation that  $\eta(1_{S^1} \times h: S^1 \times L_1 \rightarrow S^1 \times L_2) \neq 0$ .

Recall the surgery exact sequence of [Wa 2]

$$\cdots \rightarrow \left[ \sum M: G/\text{Top} \right] \rightarrow L_{n+1}(\pi_1 M) \xrightarrow{\phi} h \text{ Top} (M^n) \xrightarrow{\eta} [M: G/\text{Top}] \xrightarrow{\theta} L_n(\pi_1 M)$$

which is exact for  $n \geq 5$ . By [KS essay V Appendix C], there is an  $H$ -space structure on  $G/\text{Top}$  and an abelian group structure on  $h \text{ Top} (M)$  such that the surgery sequence becomes an exact sequence of groups and homomorphism (rather than merely pointed sets). For four manifolds  $\ker \theta: [M^4: G/\text{Top}] \rightarrow L_4(\pi_1 M)$  is easily seen to be a two-group. If  $h: M' \rightarrow M^4$  is a homotopy equivalence it determines an element  $[h] \in h \text{ Top} (M)$ . Taking the product with  $S^1$  we get an element  $[1_{S^1} \times h] \in h \text{ Top} (S^1 \times M)$ . Now  $2^k n([h]) = 0$  so  $0 = 2^k n([h \times 1_{S^1}]) = n(2^k [h \times 1_{S^1}])$ . Thus  $2^k [h \times 1_{S^1}] = \phi(\alpha)$  for some  $\alpha \in L_6(Z \times \pi_1 M)$ . Let  $S: L_6(Z \times \pi) \rightarrow L_5(\pi)$  denote the Shaneson homomorphism [Sh] and

$p : L_5(\pi) = L_1(\pi) \rightarrow G(\pi)$  the quotient map. We define

$$\eta(h) = \frac{1}{2^k} pS(\alpha) \in G(\pi).$$

**PROPOSITION 2.**  $\eta(h)$  is well defined and vanishes for CP homotopy equivalences, if  $\pi_1 M$  is very large.

*Proof.* First, the power of 2 used is clearly irrelevant since  $\phi$  is a homomorphism. Suppose  $\phi(\alpha) = \phi(\alpha')$ ; then  $(\alpha - \alpha') \in \text{Im} [\sum (S^1 \times M) : G/\text{Top}]$ . Thus it suffices to show that the composite

$$[\sum (S^1 \times M) : G/\text{Top}] \rightarrow L_6(Z \times \pi_1 M) \rightarrow L_5(\pi_1 M) \rightarrow G(\pi_1 M)$$

vanishes. Notice that the map

$$\begin{array}{c} [\sum (S^1 \times M) : G/\text{Top}] \rightarrow [\sum^2 M : G/\text{Top}] \oplus [\sum M : G/\text{Top}] \\ \downarrow \\ L_6(\pi_1 M) \oplus L_5(\pi_1 M) \end{array}$$

preserves the factorization so that it suffices to show that  $\text{Im} [\sum M : G/\text{Top}] \subset L_5(\pi_1 M)$  actually lies in  $\text{Im } I$ , at least rationally. Analogous to [Wa 2 §13 B] there is a factorization

$$\begin{array}{ccc} [\sum M : G/\text{Top}] \otimes Z[\frac{1}{2}] & \xrightarrow{\quad} & L_5(\pi_1 M) \otimes Z[\frac{1}{2}] \\ & \searrow & \nearrow \hat{\theta} \\ H_1(\pi; Z[\frac{1}{2}]) \xrightarrow{\cong} \tilde{\Omega}_4(K(\pi, 1) \times \Omega(G/\text{Top})) \otimes Z[\frac{1}{2}] & & \end{array}$$

The map  $H_1(\pi; Z[\frac{1}{2}]) \rightarrow L_5(\pi_1 M) \otimes Z[\frac{1}{2}]$  is a homomorphism and hence multiple of  $I$  (Proposition 1) so even away from 2 we are done. This proves well definedness.  $\square$

*Remark.* All of the proofs above can be refined from  $Q$  to  $Z[\frac{1}{2}]$  so the invariant  $\eta$  can be made to keep track of odd torsion. If one is willing (as we have been) to throw away the odd torsion then [Wa 1] or [TW] can be used to prove that  $\text{Im} [\sum M : G/\text{Top}] \otimes Q \subset \text{Im } I \otimes Q$ .

Now, by definition  $\eta$  is natural with respect to inclusions. Therefore if  $h : M' \rightarrow M^4$  is CP by  $(N, g, H)$   $\eta(h) = i_* \eta(H : N \times I \text{ rel } \partial \rightarrow N \times I \text{ rel } \partial)$  which vanishes for  $\pi_1 M$  very large by Theorem 2.  $\square$

*Proof of Theorem 1.* First we calculate  $G(Z \times Z_n)$ .

$$L_1^s(Z \times Z_n) \approx L_0^h(Z_n), \quad \text{Im } I = \text{Im } L_0(0)$$

so that  $G(Z \times Z_n) \rightarrow \tilde{L}_0^h(Z_n) \otimes Q$  is an isomorphism, so that a codimension one multisignature (modulo multiples of the regular representation) detects  $G(Z \times Z_n)$ . Let  $h: S^1 \times L_1 \rightarrow S^1 \times L_2$ ,  $\eta(h)$  is the (reduced) multisignature of a normal cobordism between  $L_1$  and  $L_2$  which is nonzero when  $L_1 \neq L_2$ , by [AB].  $\square$

## 5. Concluding remarks

1. The following conjecture seems plausible.

CONJECTURE. *If  $h: M' \rightarrow M^4$  is CP,  $h$  is homotopic to a homeomorphism.*

The results of §§3, 4 verify this modulo two-torsion and up to  $s$ -cobordism if  $\pi_1 M$  is very large. If the strong Novikov conjecture were true, then the same would hold independently of  $\pi_1 M$ . Of course the whole conjecture would follow from “homotopy implies isotopy for homeomorphisms of three-manifolds”.

An interesting question is whether the fake  $RP^4$  of [CS 2] is CP. Neither of the techniques of this paper yield any interesting information.

2. It may perhaps be interesting to compare the approaches of §2 and that of §§3, 4. Philosophically, both are a reduction of the general situation to one where only irreducible sufficiently large 3-manifolds occur. Geometrically this is accomplished by [La] and algebraically by [C1]. (Interestingly, the heart of the geometry of §1 deals with showing that the nonsufficiently large summands contribute nothing. Algebraically, they trivially cause no difficulty.) The sufficiently large cases are dealt with either by [Wd 1] [La] or [C2]. The analogy between [Wd 1] and [C2] is well known (cf. [Wd 2]).

3. The application of the Novikov conjecture given here is a “global” one, in that to get a result about even a single homotopy equivalence requires the verification of the conjecture for infinitely many groups. For another geometric application of the strong Novikov conjecture, see [FH 3], but that application only requires knowledge of the conjecture for  $\pi_1 M$ , and is a “local” application.

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