Zeitschrift: Commentarii Mathematici Helvetici
Herausgeber: Schweizerische Mathematische Gesellschaft
Band: 56 (1981)

Artikel: Aspherical four-manifolds and the centres of two-knot groups.
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DOI: https://doi.org/10.5169/seals-43254

Comment. Math. Helvetici **56** (1981) 465–473

0010-2571/81/003465-9$01.50+0.20/0

Aspherical four-manifolds and the centres of two-knot groups

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§1. Introduction

In [6], Hausmann and Kervaire have shown that any finitely generated abelian group is the centre of the group of an n-knot, for each $n \geq 3$. This result is in sharp contrast to the classical case ($n = 1$) when the centre is either infinite cyclic or trivial. In this note we shall consider the remaining case $n = 2$, our principal result being that if the centre of the group of a 2-knot has rank greater than 1 then the closed 4-manifold obtained by surgery on the knot is aspherical, and the centre is $\mathbf{Z}^2$. We use equivariant Poincaré duality to show that the homology of the universal cover of such a 4-manifold is a stably free module concentrated in degree 2, followed by an Euler characteristic counting argument to show that this module has rank 0, and so must be trivial.

In §2 we shall invoke a result of Kaplansky to show that nonzero stably free modules over certain rings have well defined, strictly positive rank, thus preparing the way for our counting argument. This is first used in §3 to verify a conjecture of Murasugi, for groups with torsion free centre, and to sketch a new proof of a theorem of Gottlieb. In §4 we derive our main result. Twistspinning the trefoil knot in several ways gives fibred 2-knots whose groups have centre $\mathbf{Z}$, $\mathbf{Z} \oplus \mathbf{Z}/2\mathbf{Z}$ or $\mathbf{Z}$ and we conjecture that no other nontrivial group may be the centre of a 2-knot group. We show also that the centre of the group of a 2-link with more than one component must be a torsion group, and we determine the 2-knot groups which contain an abelian subgroup of finite index. Finally, in §5 we give a necessary and sufficient condition for a closed 4-manifold to be aspherical, which is applied to the results of surgery on certain 2-knots.

I would like to thank the referee for his comments on the first version of this paper. This work was done under a grant from the Science Research Council at the University of Durham.

§2. Hopfian rings

A ring R is *prehopfian* if whenever m and n are integers such that $0 < m < n$ there is no epimorphism of R^m onto R^n; it is *hopfian* if every onto endomorphism

of a finitely generated free R-module is an automorphism. If M is a stably free module over a prehopfian ring R, so that $M \oplus R^b = R^a$ for some a, b then R-rank $M = a - b$ is well defined and non-negative; if R is hopfian and $M \neq 0$ then R-rank $M > 0$.

LEMMA. *Let S be the multiplicative system in a ring R generated by a central element s which is not a divisor of zero. If M is a stably free R-module and n an integer such that the localization M_S maps onto R_S^n with nontrivial kernel, then M maps onto R^n with nontrivial kernel. Hence R (pre)hopfian implies that R_S is (pre)hopfian.*

Proof. We shall prove the first assertion by induction on n. It is trivially true if $n = 0$. Suppose that $\phi : M_S \to R_S^n$ is an epimorphism with kernel K. On multiplying ϕ by a suitable power of s, we may assume that $\phi = \psi_S$ for some map $\psi : M \to R^n$ and that the map $\psi' : M \to R$ obtained by composing ψ with projection onto the k^{th} factor is onto, for some $k \leq n$. Let $N = \ker \psi'$ and let $\theta : N \to R^{n-1}$ be the composition of $\psi \mid N$ with the projection onto the complementary free summand of R^n. Then $M \approx N \oplus R$, so N is stably free, and θ_S is onto with kernel $\ker \theta_S = (\ker \theta)_S = (\ker \psi)_S$ nontrivial. By the hypothesis of induction there is an epimorphism $\pi : N \to R^{n-1}$ with nontrivial kernel. The map $\pi \oplus \mathrm{id}_R : N \oplus R$ $(\approx M) \to R^n$ gives an epimorphism of M onto R^n with nontrivial kernel. The second assertion follows immediately.

If we restate the definition of "prehopfian" in terms of matrices, we see immediately that any ring which maps nontrivially to a field is prehopfian, and hence integral group rings are prehopfian. Similarly any subring of a (pre)hopfian ring is (pre)hopfian. As Kaplansky has shown that for any group G the complex group algebra $\mathbf{C}[G]$ is hopfian (for a proof see [10]), integral group rings are in fact hopfian. (For the remarks in this paragraph I am indebted to Professor P. M. Cohn. See also [4]).

§3. Centres and Euler characteristic

On the evidence of his work on 1-relator groups, Murasugi conjectured that the centre of a finitely presentable group other than $\mathbf{Z}^2$ of deficiency ≥ 1 is infinite cyclic or trivial, and is trivial if the group has deficiency ≥ 2 [11]. In that paper he showed that this is true for the groups of links in S^3, which all have deficiency ≥ 1. (The knots and links in S^3 whose groups have nontrivial centre has been determined by Burde, Zieschang and Murasugi [2, 3]). Here we shall show that it

holds for a much larger class of (finitely presentable) groups, including all those with torsion free centre.

The deficiency of a finite presentation $P = \langle x_1, \ldots x_a \mid r_1, \ldots r_b \rangle$ of a group G is def $P = a - b$; the deficiency of a finitely presentable group G is def $G = \max\{\text{def } P \mid P \text{ presents } G\}$. Such a finite presentation P determines a 2-dimensional cell complex X_P with 1 0-cell, a 1-cells and b 2-cells, which has fundamental group G, and Euler characteristic $\chi(X_P) = 1 - a + b = 1 - \text{def } P$. We shall let $\zeta(G)$ and Γ denote the centre of the group G and the integral group ring $\mathbf{Z}[G]$ respectively.

THEOREM 1. *Let X be a finite 2-dimensional cell complex with fundamental group G such that $\zeta(G)$ contains an element of infinite order. Then $\chi(X) \geq 0$, and $\chi(X) = 0$ if and only if X is aspherical.*

Proof. Let $p : \tilde{X} \to X$ be the universal cover of X. The cell structure of X may be lifted to a cell structure for $\tilde{X}$ which is invariant under the action of G via covering transformations, and the cellular chain complex of $\tilde{X}$ may then be regarded as a finite chain complex of free left Γ-modules

$$C_* : 0 \to C_2 \to C_1 \to C_0 \to 0$$

where $c_i = \Gamma\text{-rank } C_i$ is the number of i-cells of X. Since $\tilde{X}$ is simply connected, $H_0(C_*) = \mathbf{Z}$ and $H_1(X_*) = 0$, while $H_2(C_*) = H_2(\tilde{X}) \approx \pi_2(X)$ is a submodule of C_2.

Let z be an element of $\zeta(G)$ of infinite order, and let S be the central multiplicative system generated by $z - 1$. Then S contains no divisors of zero, so Γ embeds in the localization Γ_S, which by the lemma of §2 is hopfian. Since the augmentation module $\mathbf{Z}$ is annihilated by $g - 1$ for all g in G, localizing the chain complex C_* leads to an exact sequence

$$0 \to H_2(C_*)_S \to C_{2S} \to C_{1S} \to C_{0S} \to 0,$$

from which it follows that $H_2(C_*)_S$ is a stably free Γ_S-module of rank $c_0 - c_1 + c_2 = \chi(X)$, which must therefore be nonnegative. As $H_2(C_*)$ is a submodule of C_2, which embeds in C_{2S}, it is 0 if and only if $H_2(C_*)_S = 0$.
The theorem follows readily.

Remark. A similar argument gives an easy proof of Gottlieb's theorem that an aspherical finite cell complex whose fundamental group has a nontrivial centre has Euler characteristic 0 [5]. For the group must be torsion free [1; page 63] and we may localize the cellular chain complex of the universal covering space to get an exact sequence of finitely generated free modules over a hopfian ring.

COROLLARY. *Let G be a finitely presentable group with a central element of infinite order. Then* def $G \leqslant 1$. *If* def $G = 1$ *and G is neither $\mathbf{Z}$ nor $\mathbf{Z}^2$, then G has cohomological dimension 2, $\zeta(G) = \mathbf{Z}$ and the commutator subgroup G' is free.*

Proof. The theorem together with the paragraph preceding it imply directly all but the last two assertions, which then follow from Theorem 8.8 of [1] which asserts that if G has finite cohomological dimension, then c.d. $\zeta(G) \leqslant$ c.d. $G - 1$, with equality only if G' is free.

As the groups of links in S^3 are all torsion free and have deficiency $\geqslant 1$, this corollary implies immediately the above-mentioned results of Murasugi [11].

§4. Centres of two-knot groups

A 2-knot is a locally flat *PL* embedding of the (oriented) 2-sphere S^2 in an oriented homotopy 4-sphere Σ^4. The exterior of such a knot $K: S^2 \to \Sigma^4$ is the compact bounded 4-manifold $X_K = \Sigma^4 - N$, where N is an open regular neighbourhood of the image of K, and the group of K is $G = \pi_1(X_K)$. The group G is also the fundamental group of the closed orientable 4-manifold $Y_K = X_K \cup S^1 \times D^3$ obtained from Σ^4 by surgery on K.

THEOREM 2. *Let M be a closed PL 4-manifold with fundamental group G, and suppose*

(i) *$\zeta(G)$ contains an element z of infinite order;*

(ii) *$H^s(G; \Gamma) = 0$ for $s \leqslant 2$.*

Then M is aspherical if and only if $\chi(M) = 0$.

Proof. Let $p: \tilde{M} \to M$ be the universal cover of M, and let C_* be the cellular chain complex of $\tilde{M}$ with respect to a cell structure lifted through p, as in Theorem 1. Let C^* denote the dual complex $\mathrm{Hom}_\Gamma(C_*, \Gamma)$, a cochain complex of free right Γ-modules. Then there are Poincaré duality isomorphisms $\bar{H}^p(C^*) \to H_{4-p}(C_*)$ (where $\bar{A}$ denotes the left Γ-module deduced from a right Γ-module A via the anti-involution of the group ring: $g \cdot a = w_1(g) a \cdot g^{-1}$ for all a in A and g in G) [12; page 23].

Since $\tilde{M}$ is simply connected $H_0(C_*) = \mathbf{Z}$ and $H_1(C_*) = 0$, while since G is infinite $\tilde{M}$ is an open 4-manifold and so $H_4(C_*) = 0$. Since $\mathrm{Ext}^s_\Gamma(\mathbf{Z}, \Gamma) = H^s(G; \Gamma) = 0$ for $s \leqslant 2$, it follows from the universal coefficient spectral sequence

$$\mathrm{Ext}^q_\Gamma(H_p(C_*), \Gamma) \Rightarrow H^{p+q}(C^*)$$

that $H^1(C^*)=0$ and that $H^2(C^*)\approx \mathrm{Hom}_\Gamma(H_2(C_*), \Gamma)$. Poincaré duality then implies that $H=H_2(C_*)$ is isomorphic to $\overline{H^*}$, where $H^*=\mathrm{Hom}_\Gamma(H, \Gamma)$, and $H_3(C_*)=0$. Thus to prove that $\tilde{M}$ is contractible it shall suffice to show that $H^*=0$.

Let S be the central multiplicative system generated by $z-1$. Then as in Theorem 1, Γ embeds in Γ_S, while the only nonzero homology of C_{*S} is H_S in degree 2. If B is any Γ_S-module $H^3(\mathrm{Hom}_{\Gamma_S}(C_{*S}, B))=H_1(C_{*S}\otimes B)=0$ by Poincaré duality and the Künneth theorem. Hence as in [12; page 26] H_S is stably free and we may split the boundary maps of the complex C_{*S} to obtain an isomorphism $H_S\oplus C_{1S}\oplus C_{3S}\approx C_{0S}\oplus C_{2S}\oplus C_{4S}$, so Γ_S-rank $H_S=\chi(M)$.

Since Γ_S is hopfian, by the lemma of §2, $\chi(M)=0$ implies that $H_S^*=H_S=0$, and since Γ embeds in Γ_S, H^* embeds in H_S^* and so must also be 0. Hence $H=0$ and $\tilde{M}$ is contractible, so M is aspherical. As the converse is a particular case of Gottlieb's theorem ([5]–see the Remark in §3) the theorem is proven.

COROLLARY 1. *If K is a 2-knot whose group G contains a central subgroup C isomorphic to $\mathbf{Z}^2$ then Y_K is aspherical and $\zeta(G)=\mathbf{Z}^2$.*

Proof. The quotient G/C is infinite, for otherwise the commutator subgroup G' would be finite [7; page 102] and so C would map monomorphically to $G/G'=\mathbf{Z}$. A spectral sequence argument (as in [1; page 158]) then shows that $H^s(G;\Gamma)=0$ for $s\leqslant 2$ and so by the theorem Y_K is aspherical and G has cohomological dimension 4. Therefore G is torsion free and c.d. $\zeta(G)\leqslant 3$, by Theorem 8.8 of [1]. The same theorem proves that c.d. $\zeta(G)=3$ only if G' is free, which cannot be the case, for otherwise c.d. $G\leqslant$ c.d. $G'+$c.d. $G/G'=2$. Since an abelian group of cohomological dimension $\leqslant 2$ must be either $\mathbf{Z}^2$ or a subgroup of the additive rational numbers [1; pages 101–110], the corollary follows.

The group of the 0-twist spun trefoil knot has centre $\mathbf{Z}$ but does not satisfy the second hypothesis of the theorem, and the result of surgery on this knot is not aspherical. The centres of the groups of the 1-, 3- and 6-twist spun trefoil knots are $\mathbf{Z}$, $\mathbf{Z}\oplus\mathbf{Z}/2\mathbf{Z}$ and $\mathbf{Z}^2$ respectively [14]. We conjecture that no other nontrivial group can be the centre of a 2-knot group.

We shall now consider briefly the centres of link groups. In the classical case the centre must be 1, $\mathbf{Z}$ or $\mathbf{Z}^2$, the latter being realised only by the (2-component) abelian link [2]. The argument of Hausmann and Kervaire may be readily modified to show that any finitely generated abelian group is the centre of the group of some μ-component boundary n-link, for each $\mu\geqslant 1$ and $n\geqslant 3$. If $n=2$ and $\mu\geqslant 2$ the centre must have rank 0 for otherwise an argument as in Theorem 2 leads to the contradictory conclusion that the localization H_S of the homology of

the universal cover of the manifold obtained by surgery on the link is a stably free Γ_S-module of Γ_S-rank $2-2\mu<0$. (To obtain the contradiction we need only know that Γ_S is prehopfian, and that $H^t(G;\Gamma)=0$ for $t\leqslant 1$).

We shall say that a group G just contains a subgroup A if the index of A in G finite.

COROLLARY 2. *Let M be a closed 4-manifold whose fundamental group Π just contains an abelian subgroup A. Then $\chi(M)\geqslant 0$, and if $\chi(M)=0$ the subgroup A has rank 1, 2 or 4.*

Proof. By passing to a finite covering space we may assume that M is orientable and that Π is a free abelian group. We may clearly assume that $\beta=\text{rank }\Pi$ is greater than 1. Let $\tilde{M}$ be the universal covering space of M. If S is as in the theorem the localized spectral sequence collapses (even if $\beta=1$ or 2) and Poincaré duality then implies that $H_2(\tilde{M})_S$ is the only nonzero localised homology module, and its $\mathbf{Z}[\Pi]$-rank is $\chi(M)$ which therefore must be nonnegative. If $\beta>2$ and $\chi(M)=0$ the theorem implies that M is aspherical and hence that $\beta=4$.

The manifolds $S^1\times S^3$, $S^1\times S^1\times S^2$, and $S^1\times S^1\times S^1\times S^1$ have Euler characteristic 0 and fundamental group $\mathbf{Z}$, $\mathbf{Z}^2$ and $\mathbf{Z}^4$ respectively. (It may be shown that they are determined up to homotopy type among orientable 4-manifolds by these properties).

COROLLARY 3. *Let K be a 2-knot whose group just contains an abelian subgroup A. Then either Y is aspherical (rank $A=4$) or the commutator subgroup G' is finite (rank $A=1$).*

Proof. It is readily verified that a group with abelianization $\mathbf{Z}$ just contains an infinite cyclic subgroup if and only if its commutator subgroup is finite, while no such group can just contain a subgroup isomorphic to $\mathbf{Z}^2$ (or $\mathbf{Z}^3$). The assertions now follow on applying Corollary 2 to the manifold Y.

Necessary conditions for a group with finite commutator subgroup to be a 2-knot group were given in [8], and these conditions were shown to be sufficient by Yoshikawa [14].

If G is a 2-knot group which just contains $\mathbf{Z}^4$, then the commutator subgroup G' just contains $\mathbf{Z}^3$ and is torsion free, by the corollary. Therefore it is the fundamental group of a flat Riemannian 3-manifold [13; page 103], which must be orientable [9]. Of the six possible such groups listed on page 117 of [13], only $\mathbf{Z}^3$ and the group G_6 admit meridional [8] automorphisms; moreover no such automorphism of $\mathbf{Z}^3$ can have finite order. The group G_6 is presented by

$\{\alpha, \gamma \mid \alpha\gamma^2\alpha^{-1}\gamma^2 = \gamma\alpha^2\gamma^{-1}\alpha^2 = 1\}$ and the automorphism $\Phi: G_6 \to G_6$ sending α to $\alpha\gamma$ and γ to α is meridional and of order 6. Since the group $G = G_6 *_\Phi$ presented by $\{G_6, t \mid tgt^{-1} = \Phi(g)$ for all g in $G_6\}$ is torsion free and just contains $\mathbf{Z}^4$, there is a flat 4-manifold M with fundamental group G [13; page 103]. As in [8] we may then obtain a 2-knot with group G by surgery on M. (In fact we may assume that M is the mapping torus of a diffeomorphism of a flat 3-manifold with fundamental group G_6, and hence that the knot is fibred.) Which high dimensional knot groups just contain abelian groups?

§5. Aspherical four-manifolds

If $f: M \to N$ is an $(n-1)$-connected degree 1 map between closed orientable $2n$-manifolds with fundamental group Π, the only obstruction to its being a homotopy equivalence is $H_n(f) = \ker(f_*: \pi_n(M) \to \pi_n(N))$, which is a stably free $\mathbf{Z}[\Pi]$-module, by lemma 2.3 of [12]. Arguing as in Theorem 2 we may show that $\mathbf{Z}[\Pi]$-rank $H_n(f) = (-1)^n(\chi(M) - \chi(N))$ and so f is a homotopy equivalence if it is an integral homology equivalence. In the nonorientable case an $(n-1)$-connected map f is a homotopy equivalence if it preserves the orientation character and induces an integral homology equivalence on the orientation covers.

In this section we shall adapt the argument outlined above to a case in which it is not known a priori that the map has degree 1.

THEOREM 3. *Let M be a closed 4-manifold with fundamental group F. Then the classifying map $f: M \to K(F, 1)$ is a homotopy equivalence if and only if F is a Poincaré duality group of formal dimension 4 and orientation character $w = w_1(M)$, and f induces an equivalence on homology with w-twisted rational coefficients.*

Proof. As these conditions are clearly necessary, we need only show that they are sufficient. By passing to the covering spaces associated with $\ker w$ we may suppose that M and F are orientable. Up to homotopy type we may suppose also that f is an inclusion and that $K(F, 1)$ is a finite cell complex containing M as a subcomplex. Let C_*, D_* and E_* be the cellular chain complexes of the universal covers $\tilde{M}$, $\tilde{K}(F, 1)$ and $(\tilde{K}(F, 1), \tilde{M})$ respectively, with respect to their natural F-invariant cell structures, and let $\Phi = \mathbf{Z}[F]$. Since F is a 4-dimensional duality group $H^s(F; \Phi) = 0$ for $s < 4$, and Poincaré duality together with the universal coefficient spectral sequence then give an isomorphism $H_2(C_*) \approx \overline{\mathrm{Hom}}_\Phi(H_2(C_*), \Phi)$ as in Theorem 2, while $H_s(C_*) = 0$ if $s \neq 0$ or 2. Since Φ maps monomorphically to $\mathbf{Q}[F]$, $H_2(C_*)$ embeds in $\mathbf{Q} \otimes H_2(C_*) = H_2(\tilde{M}; \mathbf{Q})$. As $\tilde{K}(F, 1)$ is contractible, the only possibly nontrivial homology module of $\mathbf{Q} \otimes E_*$ is

$\mathbf{Q}\otimes H_3(E_*)=\mathbf{Q}\otimes H(E_*)=H_2(\tilde{M};\mathbf{Q})$, which is a stably free $\mathbf{Q}[F]$-module by Lemma 2.3 of [12]. Since f induces a rational homology equivalence the Euler characteristics of M and $K(F,1)$ are equal. As these are also the Euler characteristics of C_* and D_*, and as the sequence

$$0\to C_*\to D_*\to E_*\to 0$$

is exact, the Euler characteristic of E_* is 0. Therefore the stably free $\mathbf{Q}[F]$-module $H_2(\tilde{M};\mathbf{Q})=\mathbf{Q}\otimes H_3(E_*)$ has rank 0, and, by Kaplansky's theorem, must in fact be 0. Thus $H_2(C_*)=0$ and f is a homotopy equivalence.

COROLLARY 4. *Let M be an orientable 4-manifold such that $H=\pi_1(M)$ is an orientable Poincaré duality group of formal dimension 4 and suppose that the cohomology ring $H^*(M;\mathbf{Q})$ is generated by $H^1(M;\mathbf{Q})$. Then M is aspherical.*

Proof. The classifying map from M to $K(H,1)$ is clearly a rational (co)homology equivalence, and so we may apply the theorem.

COROLLARY 2. *Let K be a 2-knot whose group G is an orientable Poincaré duality group of formal dimension 4 such that $G'\neq G''$. Then the closed 4-manifold Y_K obtained by surgery on K is aspherical.*

Proof. Since $G'\neq G''$, there is a field E such that $E\otimes(G'/G'')\neq 0$. An argument as in Theorem 1 of [9] using the Wang sequence and Milnor duality with coefficients E instead of $\mathbf{Z}/2\mathbf{Z}$ shows that the classifying map $c: Y_K\to K(G,1)$ has nonzero degree and hence induces a rational homology equivalence, so the above theorem applies.

Remark. If $(\mathbf{Z}/2\mathbf{Z})\otimes(G'/G'')\neq 0$ the orientability of G follows from the other conditions [9]. Is the corollary still true if $G'=G''$? In [9] we used an argument similar to that of Theorem 1 to show that if a 2-knot group G is a Poincaré duality group with one end (i.e. other than $\mathbf{Z}$) such that $\mathbf{Z}[G]$ embeds nicely in a division ring then the result of surgery on the knot is aspherical. An examination of that argument shows that the first condition can be weakened to $H^s(G;\Gamma)=0$ for $s\leqslant 2$. Can the second condition be relaxed to G being torsion free, or dispensed with entirely?

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Received February 24, June 23 1981