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**Autor:** Sommese, Andrew J.  
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## Non-smoothable varieties

ANDREW J. SOMMESE

In this article I will give new examples of projective manifolds  $A$  in  $\mathbf{P}_{\mathbf{C}}^N$  such that the cone  $CA$  on  $A$  from a point  $x \in \mathbf{P}_{\mathbf{C}}^{N+1} - \mathbf{P}_{\mathbf{C}}^N$  is not smoothable [cf. §1 for precise definitions]. A sample result is:

**PROPOSITION.** *Let  $A$  be a projective manifold in  $\mathbf{P}_{\mathbf{C}}^N$ . The cone  $CA$  on  $A$  in  $\mathbf{P}_{\mathbf{C}}^{N+1}$  is not smoothable if the first Betti number of  $A$  is zero and  $A$  is a product,  $\prod_{i=1}^k A_i$ , of projective manifolds  $A_i$ , such that either  $k > 2$  or  $\dim_{\mathbf{C}} A_i \geq 2$  for each  $i$ .*

Examples of non-smoothable manifolds are not new [cf. 11, 8, 4], but the above example differs from the usual examples. Unlike Schlessinger's examples [11], one cannot compute the  $T_i$ . Unlike Hartshorne's examples [4], the manifold  $A$  can have very high codimension in  $\mathbf{P}_{\mathbf{C}}^N$ , indeed  $N/\dim_{\mathbf{C}} A$  can be as large as we please.

The above examples are based on my earlier work [13] on manifolds that cannot be hyperplane sections in any projective manifold. The connection, Lemma (1.3), between this work and smoothability is that, if  $CA$  can be smoothed then some small deformation of  $A$  in  $\mathbf{P}_{\mathbf{C}}^N$  is a hyperplane section of a projective manifold. In §1 I give definitions and background material. In §2 I present my examples. In §3 I ask a question and make some closing remarks.

This article was inspired by a reading of [10].

### §1. Preliminaries

(1.1) **DEFINITIONS.** Let  $Y$  be an analytic subspace of a projective manifold  $X$ . A **deformation of  $Y$  in  $X$**  is a triple  $(\tilde{Y}, \Delta, \pi)$  where  $\Delta$  is the open unit disc in  $\mathbf{C}$  and:

- (a)  $\tilde{Y}$  is an analytic subspace of  $X \times \Delta$ ,
- (b) the restriction  $\pi: \tilde{Y} \rightarrow \Delta$ , of the product projection  $p: X \times \Delta \rightarrow \Delta$  is a proper, flat, surjection, and
- (c)  $\pi^{-1}(0) = Y$  where 0 is the origin of  $\Delta$ .  **$Y$  is smoothable in  $X$**  if there exists a deformation as above with  $\pi^{-1}(t)$  a submanifold of  $X \times \{t\}$  for each  $t \in \Delta - \{0\}$ .

(1.2) *Remark.* It is easy to see by considering the Hilbert scheme of  $X$ , that the above definition of smoothing is equivalent to Hartshorne's definition [4, p. 241] over  $\mathbf{C}$ .

The results in this paper are stated for varieties with isolated singularities. The reader should specialize them to the case of a cone,  $CA$ , in  $\mathbf{P}_{\mathbf{C}}^{N+1}$  on a submanifold,  $A$ , of  $\mathbf{P}_{\mathbf{C}}^N$ . Note in this case  $A$  is a hyperplane section of  $CA$ , i.e.  $(CA) \cap \mathbf{P}_{\mathbf{C}}^N$ .

(1.3) *LEMMA.* *Let  $Y$  be a subvariety of  $\mathbf{P}_{\mathbf{C}}^N$ . Let  $\mathcal{H}$  be a smooth hypersurface of  $\mathbf{P}_{\mathbf{C}}^N$  which intersects  $Y$  transversely in a submanifold  $H$  which doesn't contain any singularities of  $Y$ . Assume that given any deformation  $(\tilde{H}, \Delta, \pi)$  of  $H$  in  $\mathcal{H}$ , there is a neighborhood  $U$  of 0 in  $\Delta$  such that for each  $t \in U$ ,  $\pi^{-1}(t)$ , considered as a submanifold of  $\mathbf{P}_{\mathbf{C}}^N$ , is not the transverse intersection of  $\mathcal{H}$  with any projective submanifold of  $X$  in  $\mathbf{P}_{\mathbf{C}}^N$ . Then  $Y$  has isolated singularities and is not smoothable in  $\mathbf{P}_{\mathbf{C}}^N$ .*

*Proof.* First note that  $Y$  must have singularities, since otherwise we could take the trivial deformation  $(\tilde{H}, \Delta, \pi)$  with  $\tilde{H} = H \times \Delta$ , and for each  $t \in S$  use  $X = Y$  to contradict the hypothesis. Further  $Y$  has at most isolated singularities since otherwise the singularities would be a positive dimensional subvariety of  $\mathbf{P}_{\mathbf{C}}^N$  that was disjoint from a hypersurface  $\mathcal{H}$ .

Now to see that  $Y$  can't be smoothed, assume that it could be. Let  $(\tilde{Y}, \Delta, \pi)$  be a smoothing. Then for  $t \neq 0$  in a small enough subdisc,  $\Delta'$ , we have that  $\pi^{-1}(t)$  is a submanifold of  $\mathbf{P}_{\mathbf{C}}^N$  transverse to  $\mathcal{H}$ . Thus  $(Z, \Delta', \pi|_Z)$  with  $Z = \tilde{Y} \cap (\mathcal{H} \times \Delta')$  is a deformation of  $H$  in  $\mathcal{H}$  of the sort we've hypothesized didn't exist.

The next two lemmas, which will be used in §2, show that certain properties are invariant under small deformations. The first is folk-lore, while the second is due to Kodaira [5].

(1.4) *LEMMA.* *Let  $A$  be a projective (Kaehler) manifold with the integral cohomology ring of an Abelian variety, e.g.  $A$  is diffeomorphic to an Abelian variety. Then  $A$  is an Abelian variety (a Kaehler torus).*

*Proof.* Let  $\alpha: A \rightarrow ALB(A)$  be the Albanese map of  $A$ . If  $A$  is projective (Kaehler),  $ALB(A)$  is an Abelian variety (a Kaehler torus) of complex dimension equal to half of  $\dim_{\mathbf{C}} H^1(A, \mathbf{C})$ . Now a basic property of  $ALB(A)$  is that  $\alpha$  induces an isomorphism of  $H^1(A, \mathbf{Z})$  with  $H^1(ALB(A), \mathbf{Z})$ . This combined with the hypothesis about the integral cohomology ring of  $A$  immediately implies that  $\alpha$  induces an isomorphism of  $H^*(ALB(A), \mathbf{Z})$  with  $H^*(A, \mathbf{Z})$ . This implies that  $\alpha$  is onto. Further the fibres of  $\alpha$  are zero dimensional. If they weren't and  $F$  was a

positive dimensional fibre then we conclude that the restriction of  $H^2(A, \mathbf{Z})$  to  $F$  is 0 since it equals the pullback of  $H^2(ALB(A), \mathbf{Z})$  by  $\alpha|_F$  which is a constant map. This is absurd as a simple consideration of the Kaehler class of  $A$  shows, i.e. raise the Kaehler class to the  $\dim_{\mathbf{C}} F$  power and restrict it to  $F$ . Now  $\alpha$  is one to one, since otherwise  $\alpha$  could not pull a generator of  $H^{2a}(ALB(A), \mathbf{Z})$  back to a generator of  $H^{2a}(A, \mathbf{Z})$  where  $a = \dim_{\mathbf{C}} A$ . Finally note that any one to one and onto map between complex manifolds is a biholomorphism [9, p. 86, Theorem 5].

(1.5) LEMMA (Kodaira). *Let  $p: X \rightarrow \Delta$  be a proper, holomorphic surjection, of maximal rank from a connected complex manifold  $X$  onto the unit disc. Assume there is a maximal rank, holomorphic surjection,  $q: p^{-1}(0) \rightarrow Y$  for some complex manifold  $Y$ . Assume that for each  $y \in Y$ ,  $H^1(q^{-1}(y), \mathcal{O}_{q^{-1}(y)}) = 0$  where  $\mathcal{O}_{q^{-1}(y)}$  is the holomorphic structure sheaf of  $q^{-1}(y)$ . Then there exists a subdisc  $\Delta'$  of  $\Delta$ , a complex manifold  $\tilde{Y}$ , and holomorphic maximal rank surjections  $\tilde{q}: p^{-1}(\Delta') \rightarrow \tilde{Y}$  and  $\phi: \tilde{Y} \rightarrow \Delta'$  with  $\phi \cdot \tilde{q} = p|_{p^{-1}(\Delta')}$ ,  $\phi^{-1}(0) = Y$ , and  $\tilde{q}|_{p^{-1}(0)} = q$ .*

*Proof.* See [3, p. 87, §2]. The idea of the proof is simply that for each  $y \in Y$ , the normal bundle  $N_{q^{-1}(y)}$  of  $q^{-1}(y)$  in  $X$  is a direct sum of some number of copies of  $\mathcal{O}_{q^{-1}(y)}$ . Thus the condition  $H^1(q^{-1}(y), \mathcal{O}_{q^{-1}(y)}) = 0$  implies that  $H^1(q^{-1}(y), N_{q^{-1}(y)}) = 0$ , and thus there are no obstructions to deformation.

(1.6) Remarks. If  $p^{-1}(0)$  is projective, then  $H^1(q^{-1}(y), \mathbf{C}) = 0$  implies  $H^1(q^{-1}(y), \mathcal{O}_{q^{-1}(y)}) = 0$  for  $y \in Y$  by the Hodge decomposition theorem [6].

If  $p^{-1}(0)$  was a product,  $\prod_{i=1}^k Y_i$  with projections  $q_i: p^{-1}(0) \rightarrow Y_i$  and if  $H^1(q_i^{-1}(y), \mathcal{O}_{q_i^{-1}(y)}) = 0$  for all  $y \in Y_i$  for all  $i$ , then (1.5) implies that  $p^{-1}(\Delta')$  for some subdisc is a fibre product of holomorphic maximal rank surjections  $\tilde{\phi}_i: \tilde{Y}_i \rightarrow \Delta'$  with  $\tilde{\phi}_i(0) = Y_i$ . By the last paragraph and the Kunneth theorem the vanishing first cohomology condition is satisfied for all  $q_i^{-1}(y)$  if  $H^1(p^{-1}(0), \mathbf{C}) = 0$  and  $p^{-1}(0)$  is projective.

Finally one last lemma:

(1.7) LEMMA. *Let  $A$  be a connected projective manifold. Let  $p: A \rightarrow Y$  be a maximal rank holomorphic surjection onto a compact complex manifold,  $Y$ . Then  $Y$  is projective.*

*Proof.* We assume  $\dim_{\mathbf{C}} Y \geq 1$  or there is nothing to prove. Let  $\omega$  be a closed, positive, integral  $(1, 1)$  form  $A$ , i.e. the Chern curvature form of a positive metric on a holomorphic line bundle on  $A$ . These exist since  $A$  is projective [cf. 6]. Let  $f = \dim_{\mathbf{C}} A - \dim_{\mathbf{C}} Y$ . We fibre integrate  $\omega^{f+1}$  to get a closed, positive, integral  $(1, 1)$  form on  $Y$ ; [7] is a good reference for the basic facts about fibre integration. Now  $Y$  is projective by the Kodaira embedding theorem [6].



## §2. The examples

(2.1) PROPOSITION. *Let  $Y$  be an algebraic subvariety of  $\mathbf{P}_{\mathbf{C}}^N$ . Assume there is an hypersurface  $\mathcal{H}$  of  $\mathbf{P}_{\mathbf{C}}^N$  that intersects  $Y$  transversely in a submanifold  $H$  that does not meet the singular set of  $Y$ . Then  $Y$  has isolated singularities and is nonsmoothable in  $\mathbf{P}_{\mathbf{C}}^N$  if  $H$  is any of the following:*

(2.1.1)  *$H$  is an Abelian variety of dimension greater than 1,*

(2.1.2) *the first Betti number of  $H$  is zero and  $H$  is a product  $\prod_{i=1}^k H_i$  of projective manifolds  $H_i$  with either  $k \geq 3$  or  $\dim_{\mathbf{C}} H_i \geq 2$  for each  $i$ ,*

(2.1.3) *there exists a surjective, maximal rank, holomorphic map  $f: H \rightarrow Z$  where  $Z$  is a projective manifold, any fibre of  $f$  has its first Betti number zero, and either:*

$$(a) \quad 2 + \dim_{\mathbf{C}} Z \leq \dim_{\mathbf{C}} H \leq 2 \dim_{\mathbf{C}} Z - 2$$

*or,*

(b)  *$\dim_{\mathbf{C}} H = 2 \dim_{\mathbf{C}} Z - 1$  or  $2 \dim_{\mathbf{C}} Z$  and a fibre of  $f$  doesn't have the Betti number's of projective space.*

*Proof.* Assume  $Y$  is smoothable. Then by (1.3) there exists a deformation  $(H, \Delta, \pi)$  of  $H$  in  $\mathcal{H}$ , such that for each neighborhood  $U$  of 0 in  $\Delta$ , there is a  $t \in U$  such that  $\pi^{-1}(t)$ , considered as a submanifold of  $\mathbf{P}_{\mathbf{C}}^N$  is the transverse intersection of  $\mathcal{H}$  with a projective manifold  $X$  of  $\mathbf{P}_{\mathbf{C}}^N$ .

Now note that for small enough  $t$ ,  $\pi^{-1}(t)$  is diffeomorphic to  $\pi^{-1}(0)$ . This follows since the fact that  $\pi^{-1}(0)$  is a manifold, and the fact that  $\pi$  is flat, imply that  $\pi$  is of maximal rank in a neighborhood of  $\pi^{-1}(0)$ . Thus the result will be shown if we show that for all  $t$  near 0,  $\pi^{-1}(t)$  cannot be a hyperplane section of any projective manifold  $X$ . This will follow if we show that  $\pi^{-1}(t)$  satisfies the same properties (2.1, 1–3) as  $\pi^{-1}(0)$  for  $t$  near 0 and if we show that no projective manifold satisfying any of (2.1, 1–3) can be a hyperplane section of any projective manifold  $X$ . Now the latter follows from [13, Corollary I-A] for (2.1.1), from [13, Proposition IV] for (2.1.2), from [13, Proposition V] for (2.1.3). The former are an immediate consequence of (1.4), (1.5), and (1.6).

(2.2) PROPOSITION. *Let  $Y$  be an algebraic subvariety of  $\mathbf{P}_{\mathbf{C}}^N$ . Assume there is an hypersurface of  $\mathbf{P}_{\mathbf{C}}^N$  that intersects  $Y$  transversely in a submanifold  $H$  of complex dimension at least two that doesn't meet the singular set of  $Y$ . Let  $c_1(N_H)$  and  $c_1(T_H)$  be the first Chern classes of the holomorphic normal bundle of  $H$  in  $Y$  and the holomorphic tangent bundle of  $H$  respectively. Assume  $c_1(N_H) = \lambda g$  and  $c_1(T_H) = \mu g$  where  $\lambda$  is a positive integer and  $g \in H^2(H, \mathbf{Z})$ . Then  $Y$  has isolated singularities and is not smoothable if  $\mu + \lambda > \dim_{\mathbf{C}} H + 2$ .*

*Proof.* If  $Y$  is smoothable then by (1.3) there exists a projective manifold  $X$  in  $\mathbf{P}_{\mathbf{C}}^N$  which  $\mathcal{H}$  is transverse to and such that  $H' = \mathcal{H} \cap X$  satisfies the same relations as  $H$ , i.e.  $c_1(N_{H'}) = \lambda g$  and  $c_1(T_{H'}) = \mu g$  where  $\lambda > 0$  and  $g \in H^2(H', \mathbf{Z})$ . This is because the conditions are topological and are easily seen to be preserved by the deformation from  $H$  to  $H'$ .

Now there exists an element  $g'$  of  $H^2(X, \mathbf{Z})$  that restricts to  $g$ . To see this first note that  $[H']$ , the holomorphic line bundle on  $X$  associated to the divisor  $H'$ , restricts to  $N_{H'}$  on  $H'$ . Thus  $\lambda g = c_1(N_{H'}) = c_1([H'])|_{H'}$  extends to  $X$ ; the extension being  $c_1([H'])$ . Next note that since  $\dim_{\mathbf{C}} H' \geq 2$ , the first Lefschetz theorem [1, 2] says that the cokernel of the image under restriction of  $H^2(X, \mathbf{Z})$  in  $H^2(H', \mathbf{Z})$  has no torsion.

Now by the adjunction formula we have  $(K_X|_{H'}) \otimes_{\mathbf{C}} [H'] = K_{H'}$ . Thus since  $K_{H'} = \det T_{H'}^*$ , we have that  $c_1(K_X) = (\lambda + \mu)g'$ . Now let  $L$  be a holomorphic line bundle on  $X$  such that  $L^{-(\lambda + \mu)} = K_X$ . This is possible as we see from the Kummer sequence:

$$0 \rightarrow \mathbf{Z}_{\lambda + \mu} \rightarrow \mathcal{O}_X^* \rightarrow \mathcal{O}_X^* \rightarrow 0.$$

Now  $L$  has a Hermitian metric whose curvature form is positive. To see this note that  $c_1(L^{\lambda}) = \lambda g' = c_1([H'])$ . Now  $[H']$  is the restriction of a power of the hyperplane bundle on projective space and thus  $c_1([H'])$  is represented by a closed positive (1,1) form. Thus [3, p. 17, Lemma (1.13)],  $L^{\lambda}$  possesses a Hermitian metric whose curvature form is positive. By taking the  $\lambda^{\text{th}}$  root we get the desired metric on  $L$ .

By the Kodaira embedding theorem [6], some power  $L^A$  of  $L$  has enough global holomorphic sections to give an embedding of  $X$  into projective space. This implies that the Hilbert polynomial:

$$p(n) = \sum_{j=0}^{\dim_{\mathbf{C}} X} (-1)^j \dim_{\mathbf{C}} H^j(X, L^n)$$

has degree  $\dim_{\mathbf{C}} X$ , and that the coefficient of the term of degree  $\dim_{\mathbf{C}} X$  is non zero.

Note that if  $-(\lambda + \mu) < -\dim_{\mathbf{C}} X - 1$  then  $L^r \otimes_{\mathbf{C}} K_X^{-1} = L^{r + \lambda + \mu}$  has a positive metric for  $r > -(\lambda + \mu)$ , since  $L$  has a positive metric. Thus by the Kodaira vanishing theorem [6],  $H^i(X, L^r) = 0$  for  $r > -(\lambda + \mu)$  and  $i > 0$ . Further  $H^0(X, L^r) = 0$  for  $r < 0$ . To see this use Serre duality to get  $H^0(X, L^r) \approx H^{\dim_{\mathbf{C}} X}(X, K_X \otimes_{\mathbf{C}} L^{-r}) = H^{\dim_{\mathbf{C}} X}(X, L^{-r - \lambda - \mu})$ , note that  $-r - \lambda - \mu > -(\lambda + \mu)$  for  $r < 0$ , and use the Kodaira vanishing theorem again.

Now we have shown that  $p(n)=0$  if  $-(\lambda+\mu)<n<0$ . If  $-(\lambda+\mu)<-\dim_{\mathbb{C}} X-1$  this implies there are at least  $\dim_{\mathbb{C}} X+1$  zeros which is incompatible with  $p(n)$  being a non-zero polynomial of degree equal to  $\dim_{\mathbb{C}} X$ .

(2.3) COROLLARY. *Let  $A$  be a connected projective embedded into  $P_{\mathbb{C}}^N$  by global sections of  $K_A^r$  for some  $r$  where  $K_A$  is the canonical bundle. Assume  $\dim_{\mathbb{C}} A \geq 2$ . Then the cone  $CA$  in  $P_{\mathbb{C}}^{N+1}$ , on  $A$  from a point  $x \in P_{\mathbb{C}}^{N+1} - P_{\mathbb{C}}^N$  cannot be smoothed if  $r > \dim_{\mathbb{C}} A + 3$ .*

*Proof.*  $c_1(K_A^r) = rc_1(K_A) = -rc_1(T_A)$ .

### §3. Closing remarks

(3.1) QUESTION. *Let  $A$  be a submanifold of an Abelian variety. Assume that the holomorphic tangent bundle of  $A$  splits into a direct sum of proper holomorphic subbundles, e.g.  $A$  is a product of submanifolds of Abelian varieties. Let  $A$  be embedded in  $P_{\mathbb{C}}^N$  and let  $CA$  be the cone in  $P_{\mathbb{C}}^{N+1}$  on  $A$  from a point  $x \in P_{\mathbb{C}}^{N+1} - P_{\mathbb{C}}^N$ . Then  $CA$  cannot be smoothed in  $P_{\mathbb{C}}^{N+1}$ .*

This is made plausible since such an  $A$  cannot be a hyperplane section in any projective manifold  $X$  [13, Proposition I]. Unfortunately neither the conditions on  $A$  or the properties that are drawn from the conditions on  $A$  are stable under small deformations.

It should be noted that by considering cones on submanifolds of  $P_{\mathbb{C}}^N$  from  $P_{\mathbb{C}}^k \subseteq P_{\mathbb{C}}^{N+k+1} - P_{\mathbb{C}}^N$ , one can construct subvarieties with  $k$  dimensional singular sets and such that small deformations still have  $k$  dimensional singular sets.

Finally I would like to call attention to [12], where there are some generalizations of the results of [13]. Also I would like to mention that T. Fujita has informed me of some nice progress he has made on a number of questions of [13], in particular the question of blowing down, and [13, Question III-B] which he answered in the negative.

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*Dept. of Mathematics, White Hall*  
*Cornell University*  
*Ithaca, N.Y. 14853*  
*U.S.A.*

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