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Autor: Losert, V.

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Some properties of groups without the property P_1

V. LOSERT

Let G be a locally compact group with a left Haar measure dx. The group is said to have the property P_1 if for every $\epsilon>0$ and every compact subset K of G there exists $s\in L^1(G)$ with $\|s\|_1=1$ and $\sup_{x\in K}\int_G|s(xy)-s(y)|\,dy<\epsilon$. (see [1] p. 168).

In [1] A. Derighetti introduced the quantity ρ_1, which is defined as the infimum of all non-negative real numbers λ such that for every compact subset K of G there exists $s\geq 0$ with $\|s\|_1=1$ and $\sup_{x\in K}\int_G|s(xy)-s(y)|\,dy<\lambda$. He proved that $\rho_1<1$ implies property P_1.

In this paper we are able to show that $\rho_1<2$ already implies property P_1. It follows that ρ_1 can assume only two values, $\rho_1=0$ if G has property P_1 and $\rho_1=2$ if not. Analogous relations are proved for the constants ρ_p, which are defined in the same manner as ρ_1, with $L^1(G)$ replaced by $L^p(G)$ $(1\leq p<\infty)$. The same constants are obtained if the system of compact subsets of G is replaced by that of finite subsets.

More generally one can consider the case of a locally compact space X on which G acts continuously and which admits a quasi invariant measure μ (see [5]). This defines a representation] of G on $L^2(X,\mu)$. One can define constants ρ_p as above and it turns out that again only two values for each ρ_p are possible, depending whether π weakly contains the one dimensional identity representation of G are not.

In the last section we show that the Fourier algebra $A(G)$ (or more generally $A_p(G)$) factorizes iff the group has property P_1. This generalizes a result proved in [7] 2.3 for the free group of two generators.

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Notations

Let G be a locally compact group with unit element e, X a locally compact topological space on which G acts continuously and which admits a quasi invariant Radon measure μ with modular function χ (cp. [5]).

If f is a function on X and $a \in G$, we define $af(x) = f(ax)$ for $x \in X$. $\mathcal{F}$ (resp. $\mathfrak{F}$) shall denote the family of all finite (resp. compact) subsets of G.

We have a representation $\pi_p(x)s = \gamma(x^{-1}, .)^{1/p}x^{-1s}$ for $x \in G$, $s \in L^p(X, \mu)$. (We will write π instead of π_2.)

We define $\rho(\pi_p) = \sup_{K \in \mathcal{K}} \inf\{\sup_{x \in K} \|\pi(x)s - s\|_p : s \in L^p(X, \mu),\ s \geq 0\ \|s\|_p = 1\}$ $(1 \leq p < \infty)$

$\rho^*(\pi_p)$ is defined in the same manner with $\mathcal{K}$ replaced by $\mathcal{F}$.

$$d(\pi) = \sup_{K \in \mathcal{K}} \inf\{\sup_{x \in K} |(\pi(x)s, s) - 1| : s \in L^2(X, \mu)\}$$

$d^*(\pi)$ with $\mathcal{F}$ instead of $\mathcal{K}$.

$\rho(\pi_2)$ coincides with the quantity $\rho(\pi)$ of [1]. For $X = G$, μ Haar measure $\rho(\pi_1)$ coincides with ρ_1.

If M is a mean on $L^\infty(X, \mu)$ (a positive linear functional with $M(1) = 1$) we put $\alpha(M) = \sup\{|M({}_x\phi) - M(\phi)| : x \in G,\ \phi \in L^\infty(X, \mu),\ \|\phi\|_\infty \leq 1\}$.

If $x \in G$, ϵ_x shall denote the point measure of mass one concentrated in x. We write $L^1_{\mathbf{R}}(G)$ for the space of *real*-valued integrable functions on G and $L^0_{\mathbf{R}}(G) = \{f \in L^1_{\mathbf{R}}(G) : \int f(x)\, dx = 0\}$.

LEMMA 1 *Put* $B = \{\epsilon_x * f - \epsilon_y * f : x, y \in G,\ f \in L^1_{\mathbf{R}}(G),\ \|f\|_1 \leq 1\}$. *Then the closed absolutely convex hull of* B *in* $L^1_{\mathbf{R}}(G)$ *coincides with* $\{f \in L^0_{\mathbf{R}}(G) : \|f\|_1 \leq 2\}$.

Proof. We use the bipolar theorem [10] Th. 4 p. 35:

$$\begin{aligned} B^0 &= \{\phi \in L^\infty(G) : |\int g(x)\phi(x)\, dx| \leq 1 \quad \forall g \in B\} \\ &= \{\phi \in L^\infty_{\mathbf{R}}(G) : \|{}_x\phi - {}_y\phi\|_\infty \leq 1 \quad \forall x, y \in G\}. \end{aligned}$$

If $\phi \in B^0$ is continuous this means that $|\phi(x) - \phi(y)| \leq 1$ for all $x, y \in G$. It follows that $\sup_{x \in G} \phi(x) - \inf_{x \in G} \phi(x) \leq 1$ (ϕ is real valued) and so there exists some $c \in \mathbf{R}$ such that $\|\phi - c\|_\infty \leq \frac{1}{2}$. B^0 is by definition closed with respect to the topology $\sigma(L^\infty, L^1)$, it is convex and left-translation invariant. It follows that $\phi \in B^0$, $f \in L^1(G)$, $f \geq 0$, $\int f\, dx = 1$ implies $f * \phi \in B^0$.

If $\phi \in B^0$ is arbitrary, we choose an appropriate approximate identity (u_i) in $L^1(G)$ such that $u_i * \phi \in B^0$ and $u_i * \phi$ converges to ϕ in the topology $\sigma(L^\infty, L^1)$. Since $u_i * \phi$ is continuous we can find $c_i \in \mathbf{R}$ such that $\|u_i * \phi - c_i\| \leq \frac{1}{2}$. Let $c \in \mathbf{R}$ be a cluster point of the c_i, then some subset of $u_i * \phi - c_i$ converges to $\phi - c$ (for $\sigma(L^\infty, L^1)$) and it follows that $\|\phi - c\|_\infty \leq \sup \|u_i * \phi - c_i\|_\infty \leq \frac{1}{2}$. This means that $B^0 = \{\phi \in L^\infty_{\mathbf{R}}(G) : \exists c \in \mathbf{R} : \|\phi - c\|_\infty \leq \frac{1}{2}\}$ (the converse inclusion is trivial).

If $f \in L^1_{\mathbf{R}}(G)$, $\int f\, dx = 0$, $\|f\|_1 \leq 2$, $\phi \in B^0$, $\|\phi - c\|_\infty < \frac{1}{2}$ then $|\int f(x)\phi(x)\, dx| = |\int f(x)\phi(x) - c)\, dx| \leq 1$.

Consequently $f \in B^{00}$ and it is again trivial that any f in the closed absolutely convex hull of B satisfies $\int f\, dx = 0$ and $\|f\|_1 \leq 2$.

Remark. The conclusion of Lemma 1 is not valid in the case of complex valued functions:

Let $\alpha_1, \alpha_2, \alpha_3 \in \mathbf{C}$ be the roots of $z^3=1$ and assume that $\phi \in L^\infty(G)$ takes only the values $3^{-1/2}\ \alpha_i\ (i=1,2,3)$ each on a set A_i of positive measure. Then ϕ belongs to B^0 but there does not exist a constant $c \in \mathbf{C}$ such that $\|\phi-c\|_\infty \le \frac{1}{2}$. Now one can easily construct a function $f \in L^0(G)$ such that $\|f\|_1=2$ and $\arg f(x)=\bar{\alpha}_i$ for $x \in A_i$. Consequently $\int f(x)\phi(x)\,dx = 3^{-1/2}\int |f(x)|\,dx = 2.3^{-1/2} > 1$ and so f does not belong to the closed, absolutely convex hull of B.

LEMMA 2. *If M is a mean on $L^\infty(X,\mu)$, $\phi \in L^\infty(X,\mu)$, $f \in L^0_{\mathbf{R}}(G)$, we have* $|M(f*\phi)| \le 2^{-1}\ \alpha(M)\|f\|_1\ \|\phi\|_\infty$.

Proof. For fixed $\phi \in L^\infty(X,\mu)$ the map $f \to f*\phi$ is continuous from $L^1(G)$ to $L^\infty(X,\mu)$ (for the norm topology). For $x, y \in G$, $f \in L^1(G)$ we have by the definition of $\alpha(M)$:

$$|M(\epsilon_x * f * \phi - \epsilon_y * f * \phi)| \le \alpha(M)\,\|f*\phi\|_\infty \le \alpha(M)\|f\|_1\,\|\phi\|_\infty.$$

The set $B_1=\{g \in L^1_{\mathbf{R}}(G)\colon\ |M(g*\phi)| \le \alpha(M)\|\phi\|_\infty\}$ is a closed convex subset of $L^1_{\mathbf{R}}(G)$ which contains the set $0.B=\{\epsilon_x * f - \epsilon_y * f\colon\ x, y \in G,\ f \in L^1_{\mathbf{R}}(G),\ \|f\|_1 \le 1\}$. By Lemma 1 B_1 contains all functions $g \in L^0_{\mathbf{R}}(G)$ with $\|g\| \le 2$ and so the result follows.

LEMMA 3. *If M is a mean on $L^\infty(X,\mu)$, C a compact subset of G, $\epsilon>0$, then there exists $f \in L^1(X,\mu)$, $f \ge 0$ with $\int f(x)\,d\mu(x)=1$ such that $\|\pi_1(x)f-f\| \le \alpha(M)+\epsilon$ for all $x \in C$.*

Proof. Let h be an arbitrary function in $L^1(G)$ with $h \ge 0$, $\int h(x)\,dx=1$. Then there exists some neighbourhood U of the unit element e in G such that $\|\epsilon_x * h - h\|_1 \le \epsilon/2$ for $x \in U$. Let $F=\{x_1,\ldots,x_n\}$ be a finite subset of G such that $C \subseteq FU$.

Now we consider the space Y which is defined as a product of $n+1$-copies of $L^1_{\mathbf{R}}(X)$ with the norm $\|(f_i)\|=\sup\|f_i\|_1$. Put $\alpha=\alpha(M)^{-1}$ (if $\alpha(M)=0$ the same proof works if α is sufficiently large) and define a map $u\colon L^1_{\mathbf{R}}(X,\mu) \to Y$ by

$$u(f)=(\alpha\pi_1(\epsilon_{x_1}*h-h)f,\ldots,\alpha\pi_1(\epsilon_{x_n}*h-h)f, f).$$

(We write also π_1 for the extension of the representation to $L^1(G)$.) u is clearly linear and satisfies $\|f\|_1 \le \|u(f)\| \le 2\alpha\|f\|_1$.

The dual space Y' of Y can be identified with the sum of $(n+1)$-copies of $L^\infty_{\mathbf{R}}(X,\mu)$, equipped with the norm $\|(\phi_i)\|=\sum_{i=1}^n \|\phi_i\|_\infty$ and the dual map u'

of u is then given by $u((\phi_i)) = \sum_{i=1}^{n+1} \alpha[(\epsilon_{x_i} * h)^* * \phi_i - h^* * \phi_i] + \phi_{n+1}.(h * (x) = h(x^{-1})\wedge_G(x^{-1}))$. Let K be the preimage of the unit ball of Y by u, then the polar K^0 is the image of the unit ball of Y' by u'. If $\phi \in K^0$, it follows that there exist $(\phi_i)_{i=1}^{n+1} \in Y'$ with $\sum_{i=1}^{n+1} \|\phi_i\|_\infty \leq 1$ such that $\phi = \sum_{i=1}^{n} \alpha[(\epsilon_{x_i} * h)^* * \phi_i - h^* * \phi_i] + \phi_{n+1}$. By Lemma 2 $|M(\phi)| \leq 1$ and this means that M belongs to the bipolar K^{00} of K in $L^1_{\mathbf{R}}(X, \mu)''$. By the bipolar theorem K is dense in K^{00} for the topology $\sigma(L^1_{\mathbf{R}}(X, \mu)'', L^\infty_{\mathbf{R}}(X, \mu))$. Since $M(1) = 1$ it follows that there exists $f_0 \in K$ such that $|\int_x f_0(x)\, d\mu(x) - 1| < \epsilon/8$. $f_0 \in K$ means that $\|f_0\| \leq 1$ and $\|\pi_1(\epsilon_{x_i} * h - h)\, f_0\|_1 \leq \alpha(M)$ for $i = 1, \ldots, n$.

We decompose f_0 into its positive and negative part: $f_0 = f_0^+ - f_0^-$. From $\|f_0^+\|_1 + \|f_0^-\|_1 \leq 1$, $|1 - \|f_0^+\|_1 + \|f_0^-\|_1| < \epsilon/8$ it follows that $\|f_0 - f_0^+\|_1 = \|f_0^-\|_1 \leq \epsilon/16$ and $\|f_0^+\|_1 \geq 1 - \epsilon/8$. Put $f = f_0^+/\|f_0^+\|$. Then $\|\pi_1(\epsilon_{x_i} * h - h)f\|_1 \leq (\alpha(M) + \epsilon/8)\ (1 - \epsilon/8)^{-1} \leq \alpha(M) + \epsilon/2$. (We assume $\epsilon < 1$.)

Finally if $x \in U$ then $\|\pi_1(\epsilon_{x_i x} * h - h)f\|_1 \leq \epsilon/2 + \|\pi_1(\epsilon_{x_i} * h - h)f\|_1 \leq \alpha(M) + \epsilon$. The following lemma, as well as its proof, is due to H. Rindler:

LEMMA 4. $\rho(\pi_1) = 2$ *if and only if* $\rho(\pi_2) = 2^{1/2}$ $\rho^*(\pi_1) = 2$ *if and only if* $\rho^*(\pi_2) = 2^{1/2}$.

Proof. Assume that $x \in G$, $f \in L^1(X, \mu)$, $f \geq 0$, $\|f\|_1 = 1$, $\|\pi_1(x)f - f\|_1 \geq 2 - \epsilon$. Put $g = f^{1/2}$.

Then

$$2 - \epsilon \leq \int_X |(\pi_2(x)g)^2 - g^2|\, d\mu$$

$$= \int_X |\pi_2(x)g - g|\, |\pi_2(x)g + g|\, d\mu$$

$$\leq \|\pi_2(x)g - g\|_2\, \|\pi_2(x)g + g\|_2$$

$$= (2\langle g, g\rangle - 2\langle \pi_2(x)g, g\rangle)^{1/2}(2\langle g, g\rangle$$

$$+ 8\langle \pi_2(x)g, g\rangle)^{1/2}$$

$$= 2(1 - \langle \pi_2(x)g, g\rangle^2)^{1/2}.$$

It follows that $\langle \pi_2(x)g, g\rangle \leq (1 - (1 - \epsilon/2)^2)^{1/2} \leq \epsilon^{1/2}$ and

$$\|\pi_2(x)g - g\|_2^2 = 2 - 2\langle \pi_2(x)g, g\rangle \geq 2.\ (1 - \epsilon^{1/2}).$$

Conversely if $\|\pi_2(x)g - g\|_2^2 \geq 2 - \epsilon$ ist follows from the equation $|a - b|^2 \leq |a^2 - b^2|$ $(a, b \geq 0)$ that $\|\pi_1(x)f - f\|_1 \geq 2 - \epsilon$.

THEOREM 1. *The following statements are equivalent*

(i) π *does not contain weakly* i_G

(ii) $\rho(\pi_p)=2^{1/p}$ *for* $1\leq p<\infty$

(iii) $\rho^*(\pi_p)=2^{1/p}$ *for* $1\leq p<\infty$

(iv) $d(\pi)=1$

(v) $d^*(\pi)=1$

Proof. If π contains i_G weakly then all the quantities in (ii)–(v) are zero and so (ii)–(v) implies (i).

If $\rho^*(\pi_2)<2^{1/2}$ then by Lemma 4 $\rho^*(\pi_1)<2$. The same argument as in [1] Prop. 1 shows that there exists a mean M on $L^\infty(X, \mu)$ with $\alpha(M)<2$. By Lemma 3 $\rho(\pi_1)<2$ and by Lemma 4 $\rho(\pi_2)<2^{1/2}$. If $\|\pi_2(x)s-s\|_2^2<\alpha<2$ then $\langle\pi_2(x)s, s\rangle\gg 1-\alpha/2$. It follows that $d(\pi)<1$ and by [1] Cor. 14 π contains i_G weakly. This shows that (i) implies (ii), (iii) for $p=1$, 2 and (iv).

If $d^*(\pi)=1$ a simple computation shows that $\rho^*(\pi_2)=2^{1/2}$. It remains to show that $\rho(\pi_2)^2=2$ implies $\rho(\pi_p)=2^{1/p}$ for $1\leq p<\infty$ (analogously for $\rho^*(\pi_p)$). By the inequality $|a-b|^p\leq a^p+b^p$ for a, $b\geq 0$, $p\geq 1$ it follows that $\rho(\pi_p)\leq 2^{1/p}$. Assume that $\|\pi_2(x)s-s\|_2^2\geq 2-2\epsilon^2$ i.e. $\langle\pi_2(x)s, s\rangle\leq\epsilon^2$ and that $s\geq 0$, $\|s\|_2=1$. Put $t=\pi_2(x)s$, $A_1=\{x\in X: s(x)\leq\epsilon t(x)\}$, $A_2=\{x\in X:\ t(x)\leq\epsilon s(x)\}$ (we assume $\epsilon<1$). Then it follows that $\epsilon\int_{X/A_2}s(x)^2\,d\mu(x)\leq\int_{X/A_2}s(x)t(x)\,d\mu(x)\leq\epsilon^2$.

Consequently $\int_{X\setminus A_2}s(x)^2\,d\mu(x)\leq\epsilon$ and similarly $\int_{X\setminus A_1}t(x)^2\,d\mu(x)\leq\epsilon$. For $1\leq p<\infty$ we get:

$$\left(\int_{A_1}|s(x)^{2/p}-t(x)^{2/p}|^p\,d\mu(x)\right)^{1/p}$$
$$\geq\left(\int_{A_1}t(x)^2\,d\mu(x)\right)^{1/p}-\left(\int_{A_1}s(x)^2\,d\mu(x)\right)^{1/p}\geq(1-\epsilon)^{1/p}-\epsilon^{1/p}.$$

The same estimate holds for $(\int_{A_2}|s(x)^{1/p}-t(x)^{2/p}|^p\,d\mu(x))/p$. This gives combined $\int_X|s(x)^{2/p}-t(x)^{2/p}|\,d\mu(x)\geq 2-\delta(\epsilon)$ where $\delta(\epsilon)\to 0$ for $\epsilon\to 0$. If one puts $s_1=s^{2/p}$ then $\|\pi_p(x)s_1-s_1\|_p^p\geq 2-\delta(\epsilon)$ and it follows that $\rho(\pi_p)\geq 2^{1/p}$.

For $f\in L^1(G)$, $x\in G$ put $A_x f=A_G(x)f_x$. If H is a closed normal subgroup we write T_H for the canonical map $L^1(G)\to L^1(G/H)$ and $\mathscr{A}$ for the convex hull of $\{A_x: x\in H\}$. It is well known (see [8] p. 174) that if H has the property P_1 then

$$\inf_{A\in\mathscr{A}}\|Af\|_1=\|T_Hf\|_1 \text{ for all } f\in L^1(G).$$

The next proposition shows that a converse similar to Th. 1 holds for this characterization:

PROPOSITION 1. *If H does not have the property* P_1, *then for any* $\lambda<1$ *there exists* $f\in L^1(G)$ *such that* $\|f\|_1=1$, $T_Hf=0$ *and* $\inf_{A\in\mathscr{A}}\|Af\|>\lambda$.

Proof. If $\lambda<1$ and for any $f\in L^1(G)$ with $T_H f=0$ there exists $A\in\mathcal{A}$ such that $\|Af\|\le\lambda\|f\|$, one can iterate this procedure to get $A\in\mathcal{A}$ such that $\|Af\|\le\lambda^n\|f\|$. An analogous argument as in [8] Ch. 8, 4.5 p. 176 shows that for $f_1,\ldots,f_m\in L^1(G)$ with $T_H f_i=0$ and $\epsilon>0$ there exists $A\in\mathcal{A}$ such that $\|Af_i\|_i<\epsilon$ for $i=1,\ldots,m$. Let $\{x_1,\ldots,x_n\}$ be a finite subset of H, $\epsilon>0$. If $f\in L^1(G)$ with $f\ge 0$, $\|f\|_1=1$ is arbitrary put $f_i=\epsilon_{x_i}*f-f$. Then $T_H f_i=0$ and consequently there exists $A\in\mathcal{A}$ such that $\|Af_i\|<\epsilon$. Then $g=Af$ satisfies $g\ge 0$, $\|g\|_1=1$ and $\|\epsilon_{x_i}*g-g\|_1<\epsilon$ for $i=1,\ldots,n$. This means that (G,H) has the property $P_*(G,H)$ (see [9] §3 p. 12) and by Prop. 1 p. 13 of [9], H has the property P_1.

2. In the last section we consider some function algebras on a locally compact group, whose factorization properties depend on the property P_1. The proof of this result was obtained independently by A. Derighetti.

Let $A_p(G)$ be the algebra of all functions $\sum \check{f}_n * g_n$, where $f_n\in L^p(G)$, $g_n\in L^q(G)$, $1/p+1/q=1$, $\check{g}_n(x)=g(x^{-1})$ (see [6]).—for $p=2$ one gets the ordinary Fourier algebra of G. Let $B_\rho(G)$ be the algebra gernerated by those positive definite functions which are subordinate to the left regular representation of G on $L^2(G)$. [2] An algebra A is said to *factorize weakly*, if it coincides with the linear span of A.A.

PROPOSITION 2. *If one of the algebras $A_p(G)$ $(1\le p<\infty)$ or $B_\rho(G)$ factorizes weakly, then G has the property* P_1.

Remark. If G has the property P_1 then $B_\rho(G)$ has a unit element and each $A_p(G)$ has a bounded approximate identity (see e.g. [6] p. 121).

Proof. If a function f belongs to one of these algebras, then its complex conjugate does also and has the same norm. By Th. 2.3 of [3] there exists a constant $K_1>0$ such that for each compact subset C of G there exists $f\in A$ such that $f\ge 1$ on C, $f\ge 0$ on G and $\|f\|\le K_1$ (A denotes of the algebras cited above.) In the case of $A_p(G)$ one can now use the same argument as given in [6] Th. 6 to see that G has property P_1. An analogous argument holds for $B_\rho(G)$ since this is the dual of the C^*-algebra on $L^2(G)$ generated by left convolution operators from $L^1(G)$ ([2] p. 192).

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Viktor Losert
Mathematisches Institut
der Universität Wien
A-1900 *Wien,*
Strudlhofgasse 4
AUSTRIA

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