

Zeitschrift: Commentarii Mathematici Helvetici
Herausgeber: Schweizerische Mathematische Gesellschaft
Band: 54 (1979)

Artikel: Some properties of groups without the property P1.
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DOI: <https://doi.org/10.5169/seals-41565>

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Some properties of groups without the property P_1

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Let G be a locally compact group with a left Haar measure dx . The group is said to have the property P_1 if for every $\epsilon > 0$ and every compact subset K of G there exists $s \in L^1(G)$ with $\|s\|_1 = 1$ and $\sup_{x \in K} \int_G |s(xy) - s(y)| dy < \epsilon$. (see [1] p. 168).

In [1] A. Derighetti introduced the quantity ρ_1 , which is defined as the infimum of all non-negative real numbers λ such that for every compact subset K of G there exists $s \geq 0$ with $\|s\|_1 = 1$ and $\sup_{x \in K} \int_G |s(xy) - s(y)| dy < \lambda$. He proved that $\rho_1 < 1$ implies property P_1 .

In this paper we are able to show that $\rho_1 < 2$ already implies property P_1 . It follows that ρ_1 can assume only two values, $\rho_1 = 0$ if G has property P_1 and $\rho_1 = 2$ if not. Analogous relations are proved for the constants ρ_p , which are defined in the same manner as ρ_1 , with $L^1(G)$ replaced by $L^p(G)$ ($1 \leq p < \infty$). The same constants are obtained if the system of compact subsets of G is replaced by that of finite subsets.

More generally one can consider the case of a locally compact space X on which G acts continuously and which admits a quasi invariant measure μ (see [5]). This defines a representation π of G on $L^2(X, \mu)$. One can define constants ρ_p as above and it turns out that again only two values for each ρ_p are possible, depending whether π weakly contains the one dimensional identity representation of G or not.

In the last section we show that the Fourier algebra $A(G)$ (or more generally $A_p(G)$) factorizes iff the group has property P_1 . This generalizes a result proved in [7] 2.3 for the free group of two generators.

Acknowledgement I want to thank H. Rindler with whom I had a number of valuable discussions on the subject of this paper.

Notations

Let G be a locally compact group with unit element e , X a locally compact topological space on which G acts continuously and which admits a quasi invariant Radon measure μ with modular function χ (cp. [5]).

If f is a function on X and $a \in G$, we define $af(x) = f(ax)$ for $x \in X$. \mathcal{F} (resp. \mathcal{K}) shall denote the family of all finite (resp. compact) subsets of G .

We have a representation $\pi_p(x)s = \gamma(x^{-1}, \cdot)^{1/p} x^{-1s}$ for $x \in G$, $s \in L^p(X, \mu)$. (We will write π instead of π_2 .)

We define $\rho(\pi_p) = \sup_{K \in \mathcal{K}} \inf \{ \sup_{x \in K} \|\pi(x)s - s\|_p : s \in L^p(X, \mu), s \geq 0, \|s\|_p = 1 \}$ ($1 \leq p < \infty$)

$\rho^*(\pi_p)$ is defined in the same manner with \mathcal{K} replaced by \mathcal{F} .

$$d(\pi) = \sup_{K \in \mathcal{K}} \inf_{x \in K} \{ \sup_{s \in L^2(X, \mu)} |(\pi(x)s, s) - 1| : s \in L^2(X, \mu) \}$$

$d^*(\pi)$ with \mathcal{F} instead of \mathcal{K} .

$\rho(\pi_2)$ coincides with the quantity $\rho(\pi)$ of [1]. For $X = G$, μ Haar measure $\rho(\pi_1)$ coincides with ρ_1 .

If M is a mean on $L^\infty(X, \mu)$ (a positive linear functional with $M(1) = 1$) we put $\alpha(M) = \sup \{ |M(\phi) - M(\phi)| : x \in G, \phi \in L^\infty(X, \mu), \|\phi\|_\infty \leq 1 \}$.

If $x \in G$, ϵ_x shall denote the point measure of mass one concentrated in x . We write $L_{\mathbf{R}}^1(G)$ for the space of *real*-valued integrable functions on G and $L_{\mathbf{R}}^0(G) = \{ f \in L_{\mathbf{R}}^1(G) : \int f(x) dx = 0 \}$.

LEMMA 1 Put $B = \{ \epsilon_x * f - \epsilon_y * f : x, y \in G, f \in L_{\mathbf{R}}^1(G), \|f\|_1 \leq 1 \}$. Then the closed absolutely convex hull of B in $L_{\mathbf{R}}^1(G)$ coincides with $\{ f \in L_{\mathbf{R}}^0(G) : \|f\|_1 \leq 2 \}$.

Proof. We use the bipolar theorem [10] Th. 4 p. 35:

$$\begin{aligned} B^0 &= \{ \phi \in L^\infty(G) : |\int g(x)\phi(x) dx| \leq 1 \quad \forall g \in B \\ &= \{ \phi \in L_{\mathbf{R}}^\infty(G) : \|\phi - \phi\|_\infty \leq 1 \quad \forall x, y \in G \}. \end{aligned}$$

If $\phi \in B^0$ is continuous this means that $|\phi(x) - \phi(y)| \leq 1$ for all $x, y \in G$. It follows that $\sup_{x \in G} \phi(x) - \inf_{x \in G} \phi(x) \leq 1$ (ϕ is real valued) and so there exists some $c \in \mathbf{R}$ such that $\|\phi - c\|_\infty \leq \frac{1}{2}$. B^0 is by definition closed with respect to the topology $\sigma(L^\infty, L^1)$, it is convex and left-translation invariant. It follows that $\phi \in B^0$, $f \in L^1(G)$, $f \geq 0$, $\int f dx = 1$ implies $f * \phi \in B^0$.

If $\phi \in B^0$ is arbitrary, we choose an appropriate approximate identity (u_i) in $L^1(G)$ such that $u_i * \phi \in B^0$ and $u_i * \phi$ converges to ϕ in the topology $\sigma(L^\infty, L^1)$. Since $u_i * \phi$ is continuous we can find $c_i \in \mathbf{R}$ such that $\|u_i * \phi - c_i\|_\infty \leq \frac{1}{2}$. Let $c \in \mathbf{R}$ be a cluster point of the c_i , then some subset of $u_i * \phi - c_i$ converges to $\phi - c$ (for $\sigma(L^\infty, L^1)$) and it follows that $\|\phi - c\|_\infty \leq \sup \|u_i * \phi - c_i\|_\infty \leq \frac{1}{2}$. This means that $B^0 = \{ \phi \in L_{\mathbf{R}}^\infty(G) : \exists c \in \mathbf{R} : \|\phi - c\|_\infty \leq \frac{1}{2} \}$ (the converse inclusion is trivial).

If $f \in L_{\mathbf{R}}^1(G)$, $\int f dx = 0$, $\|f\|_1 \leq 2$, $\phi \in B^0$, $\|\phi - c\|_\infty < \frac{1}{2}$ then $|\int f(x)\phi(x) dx| = |\int f(x)(\phi(x) - c) dx| \leq 1$.

Consequently $f \in B^{00}$ and it is again trivial that any f in the closed absolutely convex hull of B satisfies $\int f dx = 0$ and $\|f\|_1 \leq 2$.

Remark. The conclusion of Lemma 1 is not valid in the case of complex valued functions:

Let $\alpha_1, \alpha_2, \alpha_3 \in \mathbf{C}$ be the roots of $z^3 = 1$ and assume that $\phi \in L^\infty(G)$ takes only the values $3^{-1/2} \alpha_i$ ($i = 1, 2, 3$) each on a set A_i of positive measure. Then ϕ belongs to B^0 but there does not exist a constant $c \in \mathbf{C}$ such that $\|\phi - c\|_\infty \leq \frac{1}{2}$. Now one can easily construct a function $f \in L^0(G)$ such that $\|f\|_1 = 2$ and $\arg f(x) = \bar{\alpha}_i$ for $x \in A_i$. Consequently $\int f(x)\phi(x) dx = 3^{-1/2} \int |f(x)| dx = 2 \cdot 3^{-1/2} > 1$ and so f does not belong to the closed, absolutely convex hull of B .

LEMMA 2. If M is a mean on $L^\infty(X, \mu)$, $\phi \in L^\infty(X, \mu)$, $f \in L^0_{\mathbf{R}}(G)$, we have $|M(f * \phi)| \leq 2^{-1} \alpha(M) \|f\|_1 \|\phi\|_\infty$.

Proof. For fixed $\phi \in L^\infty(X, \mu)$ the map $f \rightarrow f * \phi$ is continuous from $L^1(G)$ to $L^\infty(X, \mu)$ (for the norm topology). For $x, y \in G$, $f \in L^1(G)$ we have by the definition of $\alpha(M)$:

$$|M(\epsilon_x * f * \phi - \epsilon_y * f * \phi)| \leq \alpha(M) \|f * \phi\|_\infty \leq \alpha(M) \|f\|_1 \|\phi\|_\infty.$$

The set $B_1 = \{g \in L^1_{\mathbf{R}}(G) : |M(g * \phi)| \leq \alpha(M) \|\phi\|_\infty\}$ is a closed convex subset of $L^1_{\mathbf{R}}(G)$ which contains the set $0.B = \{\epsilon_x * f - \epsilon_y * f : x, y \in G, f \in L^1_{\mathbf{R}}(G), \|f\|_1 \leq 1\}$. By Lemma 1 B_1 contains all functions $g \in L^0_{\mathbf{R}}(G)$ with $\|g\| \leq 2$ and so the result follows.

LEMMA 3. If M is a mean on $L^\infty(X, \mu)$, C a compact subset of G , $\epsilon > 0$, then there exists $f \in L^1(X, \mu)$, $f \geq 0$ with $\int f(x) d\mu(x) = 1$ such that $\|\pi_1(x)f - f\| \leq \alpha(M) + \epsilon$ for all $x \in C$.

Proof. Let h be an arbitrary function in $L^1(G)$ with $h \geq 0$, $\int h(x) dx = 1$. Then there exists some neighbourhood U of the unit element e in G such that $\|\epsilon_x * h - h\|_1 \leq \epsilon/2$ for $x \in U$. Let $F = \{x_1, \dots, x_n\}$ be a finite subset of G such that $C \subseteq FU$.

Now we consider the space Y which is defined as a product of $n+1$ -copies of $L^1_{\mathbf{R}}(X)$ with the norm $\|(f_i)\| = \sup \|f_i\|_1$. Put $\alpha = \alpha(M)^{-1}$ (if $\alpha(M) = 0$ the same proof works if α is sufficiently large) and define a map $u : L^1_{\mathbf{R}}(X, \mu) \rightarrow Y$ by

$$u(f) = (\alpha \pi_1(\epsilon_{x_1} * h - h)f, \dots, \alpha \pi_1(\epsilon_{x_n} * h - h)f, f).$$

(We write also π_1 for the extension of the representation to $L^1(G)$.) u is clearly linear and satisfies $\|f\|_1 \leq \|u(f)\| \leq 2\alpha \|f\|_1$.

The dual space Y' of Y can be identified with the sum of $(n+1)$ -copies of $L^\infty_{\mathbf{R}}(X, \mu)$, equipped with the norm $\|(\phi_i)\| = \sum_{i=1}^n \|\phi_i\|_\infty$ and the dual map u'

of u is then given by $u((\phi_i)) = \sum_{i=1}^{n+1} \alpha[(\epsilon_{x_i} * h)^* * \phi_i - h^* * \phi_i] + \phi_{n+1} \cdot (h * (x) = h(x^{-1}) \wedge_G(x^{-1}))$. Let K be the preimage of the unit ball of Y by u , then the polar K^0 is the image of the unit ball of Y' by u' . If $\phi \in K^0$, it follows that there exist $(\phi_i)_{i=1}^{n+1} \in Y'$ with $\sum_{i=1}^{n+1} \|\phi_i\|_\infty \leq 1$ such that $\phi = \sum_{i=1}^n \alpha[(\epsilon_{x_i} * h)^* * \phi_i - h^* * \phi_i] + \phi_{n+1}$. By Lemma 2 $|M(\phi)| \leq 1$ and this means that M belongs to the bipolar K^{00} of K in $L^1_{\mathbf{R}}(X, \mu)''$. By the bipolar theorem K is dense in K^{00} for the topology $\sigma(L^1_{\mathbf{R}}(X, \mu)'', L^\infty_{\mathbf{R}}(X, \mu))$. Since $M(1) = 1$ it follows that there exists $f_0 \in K$ such that $|\int_X f_0(x) d\mu(x) - 1| < \epsilon/8$. $f_0 \in K$ means that $\|f_0\| \leq 1$ and $\|\pi_1(\epsilon_{x_i} * h - h) f_0\|_1 \leq \alpha(M)$ for $i = 1, \dots, n$.

We decompose f_0 into its positive and negative part: $f_0 = f_0^+ - f_0^-$. From $\|f_0^+\|_1 + \|f_0^-\|_1 \leq 1$, $|1 - \|f_0^+\|_1 + \|f_0^-\|_1| < \epsilon/8$ it follows that $\|f_0 - f_0^+\|_1 = \|f_0^-\|_1 \leq \epsilon/16$ and $\|f_0^+\|_1 \geq 1 - \epsilon/8$. Put $f = f_0^+ / \|f_0^+\|$. Then $\|\pi_1(\epsilon_{x_i} * h - h)f\|_1 \leq (\alpha(M) + \epsilon/8) (1 - \epsilon/8)^{-1} \leq \alpha(M) + \epsilon/2$. (We assume $\epsilon < 1$.)

Finally if $x \in U$ then $\|\pi_1(\epsilon_{x,x} * h - h)f\|_1 \leq \epsilon/2 + \|\pi_1(\epsilon_{x_i} * h - h)f\|_1 \leq \alpha(M) + \epsilon$. The following lemma, as well as its proof, is due to H. Rindler:

LEMMA 4. $\rho(\pi_1) = 2$ if and only if $\rho(\pi_2) = 2^{1/2}$ $\rho^*(\pi_1) = 2$ if and only if $\rho^*(\pi_2) = 2^{1/2}$.

Proof. Assume that $x \in G$, $f \in L^1(X, \mu)$, $f \geq 0$, $\|f\|_1 = 1$, $\|\pi_1(x)f - f\|_1 \geq 2 - \epsilon$. Put $g = f^{1/2}$.

Then

$$\begin{aligned} 2 - \epsilon &\leq \int_X |(\pi_2(x)g)^2 - g^2| d\mu \\ &= \int_X |\pi_2(x)g - g| |\pi_2(x)g + g| d\mu \\ &\leq \|\pi_2(x)g - g\|_2 \|\pi_2(x)g + g\|_2 \\ &= (2\langle g, g \rangle - 2\langle \pi_2(x)g, g \rangle)^{1/2} (2\langle g, g \rangle \\ &\quad + 8\langle \pi_2(x)g, g \rangle)^{1/2} \\ &= 2(1 - \langle \pi_2(x)g, g \rangle)^{1/2}. \end{aligned}$$

It follows that $\langle \pi_2(x)g, g \rangle \leq (1 - (1 - \epsilon/2)^2)^{1/2} \leq \epsilon^{1/2}$ and

$$\|\pi_2(x)g - g\|_2^2 = 2 - 2\langle \pi_2(x)g, g \rangle \geq 2 \cdot (1 - \epsilon^{1/2}).$$

Conversely if $\|\pi_2(x)g - g\|_2^2 \geq 2 - \epsilon$ it follows from the equation $|a - b|^2 \leq |a^2 - b^2|$ ($a, b \geq 0$) that $\|\pi_1(x)f - f\|_1 \geq 2 - \epsilon$.

THEOREM 1. *The following statements are equivalent*

- (i) π does not contain weakly i_G
- (ii) $\rho(\pi_p) = 2^{1/p}$ for $1 \leq p < \infty$
- (iii) $\rho^*(\pi_p) = 2^{1/p}$ for $1 \leq p < \infty$
- (iv) $d(\pi) = 1$
- (v) $d^*(\pi) = 1$

Proof. If π contains i_G weakly then all the quantities in (ii)–(v) are zero and so (ii)–(v) implies (i).

If $\rho^*(\pi_2) < 2^{1/2}$ then by Lemma 4 $\rho^*(\pi_1) < 2$. The same argument as in [1] Prop. 1 shows that there exists a mean M on $L^\infty(X, \mu)$ with $\alpha(M) < 2$. By Lemma 3 $\rho(\pi_1) < 2$ and by Lemma 4 $\rho(\pi_2) < 2^{1/2}$. If $\|\pi_2(x)s - s\|_2^2 < \alpha < 2$ then $\pi_2(x)s, s \gg 1 - \alpha/2$. It follows that $d(\pi) < 1$ and by [1] Cor. 14 π contains i_G weakly. This shows that (i) implies (ii), (iii) for $p = 1, 2$ and (iv).

If $d^*(\pi) = 1$ a simple computation shows that $\rho^*(\pi_2) = 2^{1/2}$. It remains to show that $\rho(\pi_2)^2 = 2$ implies $\rho(\pi_p) = 2^{1/p}$ for $1 \leq p < \infty$ (analogously for $\rho^*(\pi_p)$). By the inequality $|a - b|^p \leq a^p + b^p$ for $a, b \geq 0, p \geq 1$ it follows that $\rho(\pi_p) \leq 2^{1/p}$. Assume that $\|\pi_2(x)s - s\|_2^2 \geq 2 - 2\epsilon^2$ i.e. $\langle \pi_2(x)s, s \rangle \leq \epsilon^2$ and that $s \geq 0, \|s\|_2 = 1$. Put $t = \pi_2(x)s, A_1 = \{x \in X : s(x) \leq \epsilon t(x)\}, A_2 = \{x \in X : t(x) \leq \epsilon s(x)\}$ (we assume $\epsilon < 1$). Then it follows that $\epsilon \int_{X/A_2} s(x)^2 d\mu(x) \leq \int_{X/A_2} s(x)t(x) d\mu(x) \leq \epsilon^2$.

Consequently $\int_{X \setminus A_2} s(x)^2 d\mu(x) \leq \epsilon$ and similarly $\int_{X \setminus A_1} t(x)^2 d\mu(x) \leq \epsilon$. For $1 \leq p < \infty$ we get:

$$\begin{aligned} & \left(\int_{A_1} |s(x)^{2/p} - t(x)^{2/p}|^p d\mu(x) \right)^{1/p} \\ & \geq \left(\int_{A_1} t(x)^2 d\mu(x) \right)^{1/p} - \left(\int_{A_1} s(x)^2 d\mu(x) \right)^{1/p} \geq (1 - \epsilon)^{1/p} - \epsilon^{1/p}. \end{aligned}$$

The same estimate holds for $(\int_{A_2} |s(x)^{1/p} - t(x)^{2/p}|^p d\mu(x))/p$. This gives combined $\int_X |s(x)^{2/p} - t(x)^{2/p}| d\mu(x) \geq 2 - \delta(\epsilon)$ where $\delta(\epsilon) \rightarrow 0$ for $\epsilon \rightarrow 0$. If one puts $s_1 = s^{2/p}$ then $\|\pi_p(x)s_1 - s_1\|_p^p \geq 2 - \delta(\epsilon)$ and it follows that $\rho(\pi_p) \geq 2^{1/p}$.

For $f \in L^1(G), x \in G$ put $A_x f = A_G(x)f_x$. If H is a closed normal subgroup we write T_H for the canonical map $L^1(G) \rightarrow L^1(G/H)$ and \mathcal{A} for the convex hull of $\{A_x : x \in H\}$. It is well known (see [8] p. 174) that if H has the property P_1 then

$$\inf_{A \in \mathcal{A}} \|Af\|_1 = \|T_H f\|_1 \text{ for all } f \in L^1(G).$$

The next proposition shows that a converse similar to Th. 1 holds for this characterization:

PROPOSITION 1. *If H does not have the property P_1 , then for any $\lambda < 1$ there exists $f \in L^1(G)$ such that $\|f\|_1 = 1, T_H f = 0$ and $\inf_{A \in \mathcal{A}} \|Af\|_1 > \lambda$.*

Proof. If $\lambda < 1$ and for any $f \in L^1(G)$ with $T_H f = 0$ there exists $A \in \mathcal{A}$ such that $\|Af\| \leq \lambda \|f\|$, one can iterate this procedure to get $A \in \mathcal{A}$ such that $\|Af\| \leq \lambda^n \|f\|$. An analogous argument as in [8] Ch. 8, 4.5 p. 176 shows that for $f_1, \dots, f_m \in L^1(G)$ with $T_H f_i = 0$ and $\epsilon > 0$ there exists $A \in \mathcal{A}$ such that $\|Af_i\|_1 < \epsilon$ for $i = 1, \dots, m$. Let $\{x_1, \dots, x_n\}$ be a finite subset of H , $\epsilon > 0$. If $f \in L^1(G)$ with $f \geq 0$, $\|f\|_1 = 1$ is arbitrary put $f_i = \epsilon_{x_i} * f - f$. Then $T_H f_i = 0$ and consequently there exists $A \in \mathcal{A}$ such that $\|Af_i\|_1 < \epsilon$. Then $g = Af$ satisfies $g \geq 0$, $\|g\|_1 = 1$ and $\|\epsilon_{x_i} * g - g\|_1 < \epsilon$ for $i = 1, \dots, n$. This means that (G, H) has the property $P_*(G, H)$ (see [9] §3 p. 12) and by Prop. 1 p. 13 of [9], H has the property P_1 .

2. In the last section we consider some function algebras on a locally compact group, whose factorization properties depend on the property P_1 . The proof of this result was obtained independently by A. Derighetti.

Let $A_p(G)$ be the algebra of all functions $\sum \check{f}_n * g_n$, where $f_n \in L^p(G)$, $g_n \in L^q(G)$, $1/p + 1/q = 1$, $\check{g}_n(x) = g_n(x^{-1})$ (see [6]). – for $p = 2$ one gets the ordinary Fourier algebra of G . Let $B_p(G)$ be the algebra generated by those positive definite functions which are subordinate to the left regular representation of G on $L^2(G)$. [2] An algebra A is said to *factorize weakly*, if it coincides with the linear span of $A \cdot A$.

PROPOSITION 2. *If one of the algebras $A_p(G)$ ($1 \leq p < \infty$) or $B_p(G)$ factorizes weakly, then G has the property P_1 .*

Remark. If G has the property P_1 then $B_p(G)$ has a unit element and each $A_p(G)$ has a bounded approximate identity (see e.g. [6] p. 121).

Proof. If a function f belongs to one of these algebras, then its complex conjugate does also and has the same norm. By Th. 2.3 of [3] there exists a constant $K_1 > 0$ such that for each compact subset C of G there exists $f \in A$ such that $f \geq 1$ on C , $f \geq 0$ on G and $\|f\| \leq K_1$ (A denotes of the algebras cited above.) In the case of $A_p(G)$ one can now use the same argument as given in [6] Th. 6 to see that G has property P_1 . An analogous argument holds for $B_p(G)$ since this is the dual of the C^* -algebra on $L^2(G)$ generated by left convolution operators from $L^1(G)$ ([2] p. 192).

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Received January 9, 1978