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Erratum: Addendum to the Paper Entitled: "Structural Theorems for Topological Actions of Z_2 -Tori on Real, Complex and Quaternionic Projective Spaces".

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**Addendum to the Paper Entitled:
 “Structural Theorems for Topological Actions of
 Z_2 -Torion Real, Complex and Quaternionic Projective Spaces”**

WU YI HSIANG

In the statement of Theorem 3, there is a missing case that all connected components of $F(G, X)$ are of RP -type. The condition for case 2 should be: $Sq^2(\zeta) = \gamma \cdot \zeta \neq 0$ and $Sq^1(\gamma) = 0$; and there should be the following third case, namely,

(c) *Case 3, $Sq^2(\zeta) = \gamma \cdot \zeta \neq 0$ and $Sq^1(\gamma) \neq 0$; then all connected components of $F(G, X)$ are of RP -type, i.e., $F^j \sim RP^{(k_j-1)}$ and there exist $w_j, v_1, v_2 \in H^1(B_G)$ such that*

$$\gamma = v_1^2 + v_1 v_2 + v_2^2, \quad \alpha_j = w_j (w_j + v_1) (w_j + v_1) (w_j + v_1 + v_2)$$

and

$$i_j^*(\zeta) = \zeta_j^4 + \gamma \zeta_j^2 + (Sq^1 \gamma) \zeta_j + \alpha_j, \quad i_j^*: H_G^*(X) \rightarrow H_G^*(F^j).$$

And furthermore, the system of local weights at F^j, Ω_j , is given by

$$\Omega_j = \{(w_i + w_j), (v_1 + w_i + w_j), (v_2 + w_i + w_j), (v_1 + v_2 + w_i + w_j) \text{ multi. } k_i \ (i \neq j); \\ v_1, v_2, (v_1 + v_2), 0 \text{ multi. } (k_j - 1)\},$$

and

$$F(x) \text{ is of } \begin{cases} RP\text{-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly independent,} \\ CP\text{-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly dependent but not all} \\ & \text{zero,} \\ QP\text{-type} & \text{if } v_1 \mid G_x = v_2 \mid G_x = 0. \end{cases}$$

Correspondingly, one should add the following proof to the proof of Theorem 3 for the above case 3:

In the case $Sq^2(\zeta) = \gamma \cdot \zeta \neq 0$ and $Sq^1(\gamma) = \delta \neq 0$, it is not difficult to show that all connected components of $F(G, X)$ are of RP -type. Let ζ_j be the generator of $H^*(F^j)$, i_j^* be the restriction homomorphism of $H_G^*(X)$ to $H_G^*(F^j)$. Then, it follows from the following equations:

$$Sq^1(i_j^* \zeta) = i_j^*(Sq^1 \zeta) = 0 \quad \text{and} \quad Sq^2(i_j^* \zeta) = i_j^*(Sq^2 \zeta) = \gamma \cdot (i_j^* \zeta)$$

that

$$\begin{aligned} i_j^* \zeta &= \xi_j^4 + \gamma \cdot \xi_j^2 + \delta \xi_j + \alpha_j; \\ Sq^1 \delta &= Sq^1 \alpha_j = 0, \quad Sq^2 \delta = \gamma \delta, \quad Sq^2 \alpha_j = \gamma \alpha_j. \end{aligned}$$

Let $f_j = (0, \dots, 0, \xi_j^{(k_j-1)}, 0, \dots, 0)$ be the fundamental cohomology class of F_j . Then, simple computation will show that the ideal $I_X(f_j)$ is generated by $a(f_j) = \delta^{(k_j-1)} \cdot \prod_{i \neq j} (\alpha_i + \alpha_j)$, (cf. the proof of Theorem 1). For simplicity, we may assume without loss of generality that $\alpha_1 = 0$.

(i) *Suppose that at least one $k_j > 1$* : Then it follows from Theorem B that δ splits into product of linear factors, say $\delta = v_1 \cdot v_2 \cdot v_3$. Notice that

$$Sq^1 \delta = (v_1 + v_2 + v_3) \cdot \delta = 0 \quad \text{implies} \quad v_1 + v_2 + v_3 = 0,$$

i.e.,

$$\delta = v_1 v_2 (v_1 + v_2),$$

and

$$Sq^2 \delta = \gamma \cdot \delta \quad \text{implies} \quad \gamma = v_1^2 + v_1 v_2 + v_2^2.$$

Again, it follows from Theorem B that $a(f_1) = \delta^{(k_1-1)} \cdot \prod_{j \neq 1} \alpha_j^{k_j}$ splits, and hence all $\alpha_j, j \neq 1$, split into product of linear factors, say

$$\alpha_j = w_j \cdot (w_j + \mu_{j,1}) (w_j + \mu_{j,2}) \cdot (w_j + \mu_{j,3}).$$

On the other hand,

$$\begin{aligned} Sq^1 \alpha_j &= (\mu_{j,1} + \mu_{j,2} + \mu_{j,3}) \cdot \alpha_j = 0 \quad \text{implies that} \quad \mu_{j1} + \mu_{j2} + \mu_{j3} = 0 \\ Sq^2 \alpha_j &= (\mu_{j1} \mu_{j2} + \mu_{j2} \mu_{j3} + \mu_{j3} \mu_{j1}) \cdot \alpha_j = \gamma \cdot \alpha_j \end{aligned}$$

implies that

$$\{\mu_{j1}, \mu_{j2}, \mu_{j3}\} = \{v_1, v_2, v_1 + v_2\}$$

as a set, namely

$$\alpha_j = w_j (w_j + v_1) (w_j + v_2) (w_j + v_1 + v_2).$$

(ii) *Suppose all $k_j = 1$* : Then $s = (n+1) > 1$, $a(f_1) = \prod_{j \neq 1} \alpha_j$. Hence, α_j again splits and the above proof will show that

$$\alpha_j = w_j (w_j + \mu_{j1}) (w_j + \mu_{j2}) (w_j + \mu_{j3}), \quad \mu_{j1} + \mu_{j2} + \mu_{j3} = 0,$$

and

$$(\mu_{j1}\mu_{j2} + \mu_{j2}\mu_{j3} + \mu_{j3}\mu_{j1}) = \gamma \quad \text{for all } j > 1,$$

and the assertion follows easily.

Remark. All actions of Case 3 are cohomologically modelled after the following type of linear actions on $QP^n = Sp(n+1)/Sp(1) \times Sp(n)$.

EXAMPLE 3c. Notice that the effective group of isometry on the symmetric space QP^n is $Sp(n+1)/\{\pm 1\}$ and $SO(3) = Sp(1)/\{\pm 1\}$ sits in $Sp(n+1)/\{\pm 1\}$ as diagonal unit quaternions. Let G be a \mathbf{Z}_2 -torus of $Sp(n+1)/\{\pm 1\}$ containing a maximal \mathbf{Z}_2 -torus of $SO(3)$. Then the restricted G -action on QP^n is of the type of Case 3.

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