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# On the Ends of the Fundamental Groups of 3-Manifolds with Boundary

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## § 1. Introduction

Let  $M$  be any compact 3-manifold, closed<sup>1)</sup> or with boundary, whose components are *orientable* closed surfaces. According to<sup>2)</sup> H. HOPF [2], the number  $e$  of ends of  $\pi_1(M)$  is either 0 or 1 or 2 or  $\infty$ , where  $e = 0$  if and only if  $\pi_1(M)$  is finite. The naturally arising problem is: *When is  $e = 1$  or 2 or  $\infty$ ?* This problem has been solved by E. SPECKER [10], p. 325, Satz VI, in case  $M$  is *closed*. Thus the remaining question is: What is the solution of this problem when  $M$  has non-vacuous boundary? We notice that, if some of the components of the boundary of  $M$  are 2-spheres, then there exists a 3-manifold  $M'$  closed or with boundary, whose components are orientable closed surfaces of positive genus, and such that<sup>3)</sup>  $\pi_1(M) \cong \pi_1(M')$ . Thus the problem may be stated: *Let  $M$  be a compact 3-manifold with boundary, whose components are orientable closed surfaces of positive genus. When is<sup>4)</sup>  $e = 1$  or 2 or  $\infty$ ?* To the best knowledge of this author, some partial results have been obtained by E. SPECKER [10], pp. 326–327, Sätze VII and VIII, and this author [5], p. 296, theorems 1 and 2. In the present paper we solve this problem, and the solution is:

(1) *If  $M$  is aspherical and the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is an isomorphism for every component  $F$  of the boundary of  $M$ , then  $e = 1$ .*

(2) *If  $M$  is aspherical, the boundary  $F$  of  $M$  is connected of genus one, and the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism, then  $e = 2$ .*

(3) *In any other case,  $e = \infty$ .*

These are provided us by the theorem 6 in § 4, which is the main theorem of this paper. The proof of theorem 6 is based on theorems 1, 2 and 3 4, 5. The theorems 3, 4 are lent from authors paper [5], and the theorem 5 is lent from E. SPECKER's paper [10]. The theorems 1 and 2 are explained in § 3, and their proofs are based on the lemmas 1, 2, 3, 4 and 5 of § 2.

In § 5 we give a short proof of theorems 1 and 2, using DEHN's lemma [7], p. 169, [8], p. 1, and theorem 1, [6], p. 281.

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<sup>1)</sup> *Closed* means compact without boundary.

<sup>2)</sup> Numbers in brackets refer to the bibliography at the end of the paper.

<sup>3)</sup>  $\sim$  means isomorphic to.

<sup>4)</sup> According to lemma 3,  $\pi_1(M)$  is infinite, therefore  $e > 0$ .

All 3-manifolds under consideration in this paper will be considered as having a certain fixed triangulation. This is possible according to E. E. MOISE's work [4].

The paper is presented here in a form suggested by Professor H. HOPF to the autor, who would like to express his gratitude to Professor H. HOPF for his suggestions.

## § 2. Five Lemmas

1. In the Nos. 1-2,  $M$  will mean a 3-manifold with boundary, and  $F$  will mean a component of its boundary, but in No. 3,  $F$  will mean an abstract surface.

**Lemma 1.** *Let  $L_1, L_2$  be loops on  $F$  such that<sup>5)</sup>  $s(Z_1, Z_2) \neq 0$ , where  $Z_1, Z_2$  are the 1-cycles corresponding to  $L_1, L_2$ . Then, at most one of  $Z_1, Z_2$  is<sup>6)</sup>  $\sim 0$  in  $M$ .*

**Proof.** Let us suppose that  $Z_1 \sim 0$  in  $M$ . Then there exists a 2-chain  $C$  in  $M$ , such that  $\partial C = Z_1$ . Let  $M^* = M \cup M'$  be the duplication<sup>7)</sup> of  $M$ , and let  $C'$  be the copy of  $-C$  in  $M'$ , where  $M'$  is the second copy of  $M$ . Then  $Z^* = C + C'$  is a 2-cycle in  $M^*$ , and<sup>5)</sup><sup>8)</sup>

$$s^*(Z^*, Z_2) = \pm s(Z_1, Z_2) \neq 0.$$

Thus  $Z_2 \not\sim 0$  in  $M^*$ , and hence  $Z_2 \not\sim 0$  in  $M$ . This proves lemma 1.

**Lemma 2.** *If  $M$  is simply connected, then  $F$  is homeomorphic to a region of a 2-sphere.*

**Proof.** By lemma 1, any simple<sup>9)</sup> loop on  $F$  decomposes  $F$ , i. e.  $F$  is "schlichtartig" [3], p. 140. Therefore  $F$  can be imbedded in a 2-sphere, according to [3], p. 165. This proves lemma 2.

**Lemma 3.** *Let  $M$  be a compact 3-manifold with boundary, where some one of its components, say  $F$ , is a closed orientable surface of positive genus  $g(F)$ . Then  $\pi_1(M)$  is infinite.*

**Proof.** Let us suppose that  $\pi_1(M)$  is finite. Let  $p: \tilde{M} \rightarrow M$  be the universal covering of  $M$ , and let  $\tilde{F}$  be a boundary surface of  $\tilde{M}$  lying over  $F$ .

<sup>5)</sup>  $s$  means intersection numbers on  $F$ .

<sup>6)</sup>  $\sim$  or  $\sim_R$  means homologous to  $\dots$ , over the integers or rationals respectively.

<sup>7)</sup> *Duplication* = Verdoppelung [9], pp. 129, 223. Actually there the duplication is defined for a *solid torus* (= Henkelkörper), but the generalization to any 3-manifold with boundary is immediate.

<sup>8)</sup>  $s^*$  means intersection numbers in  $M^*$ .

<sup>9)</sup> *Simple* means without multiple points.

Then  $\tilde{F}$  is closed, because  $\tilde{M}$  is compact, and therefore  $g(\tilde{F}) > 0$ . But by lemma 2,  $g(\tilde{F}) = 0$ . We arrived at a contradiction. This proves lemma 3.

2. Let us start this No. with some remarks about the ends of  $F$  and those of  $M$ , where  $M$  is supposed to be simply connected. By lemma 2,  $F$  may be thought of as a region of a 2-sphere  $S$ . The ends of  $F$  are precisely the components of  $S - F$ , and  $\bar{F} = F \cup (\text{ends of } F) = S$ , see [2], p. 86, No. 4. An open subset of  $S$  is a neighbourhood of any of its points in  $\bar{F}$ . According to [2], p. 87, No. 6, to any end  $\varepsilon$  of  $F$  there corresponds a unique end  $\eta$  of  $M$ .

**Lemma 4.** *Let  $M$  be simply connected,  $\varepsilon_1, \varepsilon_2$  be two ends of  $F$ , such that  $\varepsilon_1 \neq \varepsilon_2$ , and let  $\eta_1, \eta_2$  be the ends of  $M$  corresponding to  $\varepsilon_1, \varepsilon_2$ . Then  $\eta_1 \neq \eta_2$ .*

**Proof.** Let  $L$  be a simple loop on  $F$  separating  $\varepsilon_1, \varepsilon_2$ , i. e. there exist open connected sets on  $\bar{F}$ , say  $U_1, U_2$ , which are disjoint from  $L$ , neighbourhoods of  $\varepsilon_1, \varepsilon_2$ , and such that the intersection number of  $L$  with any path on  $F$  connecting  $U_1$  and  $U_2$  is  $\pm 1$ . There exists a 2-cell  $D$  with self-intersections, such that<sup>10)</sup>  $\text{bd} D = L$ , and  $D - L \subset \text{int } M$ , because  $M$  is simply connected. Let  $C$  be the 2-chain corresponding to the oriented  $D$ , and let  $Z$  be the 1-cycle corresponding to  $L$ . Then  $\partial C = Z$ . By [2], p. 84, Nos. 2–3, the lemma 3 will be proved if we have shown, that each path  $P$  in  $M$  with initial point  $p_1 \in U_1$  and final point  $p_2 \in U_2$  meets the compact set  $D$ .

Let  $Q$  be a path on  $F$  with initial point  $p_1$  and final point  $p_2$ , and let us consider the loop  $L_0 = PQ^{-1}$ . Let  $V, W$  and  $Z_0$  be the 1-chains and 1-cycle corresponding to  $P, Q$  and  $L_0$ . Then  $V - W = Z_0 \sim 0$  in  $M$ , because  $M$  is simply connected. Let  $M^* = M \cup M'$  be the duplication<sup>7)</sup> of  $M$ , and let  $C'$  be the copy of  $-C$  in  $M'$ , where  $M'$  is the second copy of  $M$ . Then  $Z^* = C + C'$  is a 2-cycle in  $M^*$ , and<sup>5)</sup><sup>8)</sup>

$$0 = s^*(Z_0, Z^*) = s^*(V, Z^*) - s^*(W, Z^*)$$

$$s^*(W, Z^*) = \pm s(W, Z) = \pm 1.$$

Thus  $s^*(V, Z^*) = \pm 1 \neq 0$ . Hence  $P \cap D \neq \emptyset$ . This completes the proof of lemma 4.

3. In the preceding Nos. 1–2,  $F$  was a component of the boundary of the 3-manifold  $M$ . In the present one  $F$  will be an abstract surface.

**Lemma 5.** *Let  $q: \tilde{F} \rightarrow F$  be a regular<sup>11)</sup> covering, where  $F$  is a closed orientable surface, and  $\tilde{F}$  is homeomorphic to a cylinder. Then the genus  $g(F) = 1$ .*

<sup>10)</sup>  $\text{int}$  = interior,  $\text{cl}$  = closure,  $\text{bd}$  = boundary.

<sup>11)</sup> [9], § 57, p. 195.



**Proof.** Let  $G = \pi_1(F)$  and<sup>3)</sup>  $K = \pi_1(\tilde{F}) \cong \mathbb{Z}$ . Then  $K$  is a normal subgroup of  $G$ , because the covering  $q: \tilde{F} \rightarrow F$  is regular, and  $H = G/K$  is the group of covering translations.

According to [2], p. 96, No. 16<sup>12)</sup>,  $e(H) = e(\tilde{F}) = 2$ , because  $\tilde{F}$  is a cylinder. By [2], p. 97, Satz V, the group  $H$  has an infinite cyclic subgroup  $B$  with finite index in  $H$ . Let<sup>13)</sup>

$$G' = G/[G, G], \quad H' = H/[H, H], \quad K' = K/(K \cap [G, G]) .$$

Then  $G'/K' \cong H'$ , and because  $G', H', K'$  are abelian we have by [1], p. 573, Satz<sup>14)</sup>

$$r(G') = r(K') + r(H') .$$

$r(K') \leq 1$ , because  $K \cong \mathbb{Z}$ . Abelianizing  $H$  we obtain the group  $H'$  which has an infinite cyclic subgroup  $B'$  with finite index in  $H'$ , where  $B'$  is obtained from the subgroup  $B$  of  $H$ . Thus

$$r(H') = r(B') + r(H'/B') = 1 .$$

Hence  $r(G') \leq 2$ . But  $G'$  is the 1-homology group, and  $r(G')$  is the 1-Betti number of  $F$ . Thus  $g(F) \leq 1$ . On the other hand  $g(F) > 0$ , because  $\tilde{F}$  is infinite. Hence  $g(F) = 1$ . This proves lemma 5.

### § 3. Two Theorems

4. The conjecture in [5], p. 298, § 5, is a special case of the following

**Theorem 1.** *Let  $M$  be a compact 3-manifold with boundary, and let  $F$  be a component of its boundary, where  $F$  is an orientable surface of genus  $g(F) > 1$ , and the injection  $j: \pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism. Then  $\pi_1(M)$  has infinitely many ends.*

**Proof.** Let  $p: \tilde{M} \rightarrow M$  be the universal covering of  $M$ , where  $\tilde{M}$  has the induced triangulation, and let  $\tilde{F}$  be a component of  $p^{-1}(F)$ . Then  $\tilde{F}$  may be considered as a region of a 2-sphere  $S$ , by lemma 2. The number<sup>12)</sup>  $e(\tilde{F})$  is equal to the number of the components of  $S - \tilde{F}$ , by No. 2. It is easily seen that

$$\pi_1(\tilde{F}) \cong j^{-1}(1) \neq 1$$

where the isomorphism is induced by the projection map  $p$ . Thus  $\tilde{F}$  is not

<sup>12)</sup>  $e$  means the number of ends of a group or a space.

<sup>13)</sup>  $[ , ]$  means the commutator subgroup of.

<sup>14)</sup>  $r$  means the rank of an abelian group.

simply connected, and therefore  $e(\tilde{F}) > 1$ . The covering  $q: \tilde{F} \rightarrow F$  is regular, where  $q = p|_{\tilde{F}}$ , because  $j^{-1}(1)$  is a normal subgroup of  $\pi_1(F)$ , [9], p. 195. Therefore, by lemma 5 and because  $g(F) > 1$ ,  $\tilde{F}$  is not homeomorphic to a cylinder. Thus  $e(\tilde{F}) > 2$ . Hence  $e(\tilde{F}) = \infty$ , by [2], p. 93, Satz II. Thus  $e(\tilde{M}) = \infty$ , by lemma 4. Hence  $e(\pi_1(M)) = e(\tilde{M}) = \infty$ , by [2], p. 96, No. 16. This proves theorem 1.

**Theorem 2.** *Let  $M$  be a 3-manifold with boundary, which is not connected if  $M$  is compact, and let  $F$  be a component of its boundary, where  $F$  is an orientable closed surface of genus  $g(F) = 1$ , and the injection  $j: \pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism. Then  $\pi_2(M) \neq 0$ .*

**Proof.** Let us suppose that  $\pi_2(M) = 0$ . We are going to prove that<sup>6)</sup>  $\Phi \sim_R 0$ , where  $\Phi$  is the basic 2-cycle of  $F$ .

There exists on  $F$  a loop  $L$  which is<sup>15)</sup>  $\simeq 0$  in  $M$  but  $\not\simeq 0$  on  $F$ , because  $j$  is not an isomorphism. Let  $X, Y$  be the 1-cycles of two simple loops  $A, B$  on  $F$ , having only one point in common, and such that  $X, Y$  form a generating system of<sup>16)</sup>  $H_1(F)$ . Then there exist integers  $a, b, t$  such that<sup>17)</sup>

$$V \sim t(aX + bY) \text{ on } F, \quad (|a|, |b|) = 1, \quad t \neq 0, \quad (1)$$

where  $V$  is the 1-cycle of  $L$ . Thus there exist integers  $c, d$  such that

$$ad - bc = 1, \quad (|c|, |d|) = 1.$$

Hence the 1-cycles

$$X_1 = aX + bY, \quad Y_1 = cX + dY \quad (2)$$

from a generating system of  $H_1(F)$ , and moreover there exist on  $F$  two simple loops  $A_1, B_1$ , having only one point in common, and such that their 1-cycles are  $X_1, Y_1$  respectively.

Let  $F_0$  be a torus and let  $X_0, Y_0$  be the 1-cycles of two simple loops  $A_0, B_0$  on  $F_0$ , having only one point in common, and such that  $X_0, Y_0$  form a generating system of  $H_1(F_0)$ . Let  $f: F_0 \rightarrow F$  be a  $t$ -sheeted covering, such that  $f(A_0) = A_1^t, f(B_0) = B_1$ .

Let now  $C$  be a 2-cell such that  $\text{bd}C = C \cap F_0 = X_0$ . Then the map  $f$  can be extended to a map  $f': C \cup F_0 \rightarrow M$ , because  $f(A_0) \simeq 0$  in  $M$ , by  $L \simeq 0$  in  $M$ , and

$$f(A_0) = A_1^t \simeq L \text{ on } F,$$

where the last relations hold by (1) and (2).

<sup>15)</sup>  $\simeq$  means homotopic to.

<sup>16)</sup>  $H_1$  means 1-homology group.

<sup>17)</sup>  $(, )$  means greatest common divisor.

Let  $h: S \rightarrow C \cup F_0$  be a map, where  $S$  is a 2-sphere defined in the following way: two disjoint 2-cells  $C_1, C_2$  on  $S$  are mapped homeomorphically on  $C$  such that<sup>18)</sup>  $h(C_1) = C^+$ ,  $h(C_2) = C^-$ , and  $S - C_1 - C_2$  is mapped homeomorphically on  $F_0 - X_0$ . So we have the following map  $h' = f'h: S \rightarrow M$ .

Let now  $\Phi_0$  and  $\Psi$  be the basic 2-cycles of  $F_0$  and  $S$ . Then

$$f'(\Phi_0) = f(\Phi_0) = t\Phi, \quad \text{and} \quad h(\Psi) = \Phi_0.$$

Therefore  $h'(\Psi) = t\Phi$ . But  $h' \simeq 0$ , because  $\pi_2(M) = 0$ . Hence

$$t\Phi = h'(\Psi) \sim 0 \quad \text{in } M.$$

Thus  $\Phi \sim_R 0$  in  $M$ , as we asserted. This contradicts the fact that  $M$  is a 3-manifold, which is either infinite, or in case it is finite  $\text{bd}M$  is not connected. Hence  $\pi_2(M) \neq 0$ . This proves theorem 2.

#### § 4. The main Theorem

5. The following three theorems will be used in the proof of the main theorem.

**Theorem 3.** *Let  $M$  be an aspherical compact 3-manifold with boundary, where the components are orientable closed surfaces of positive genus, such that the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is an isomorphism, for any component  $F$  of the boundary. Then  $e(\pi_1(M)) = 1$ .*

**Proof.** According to lemma 3,  $\pi_1(M)$  is infinite. Hence  $e(\pi_1(M)) = 1$ , by [5], p. 296, theorem 1.

**Theorem 4.** *Let  $M$  be an aspherical 3-manifold whose boundary  $F$  is an orientable closed surface of genus one, and suppose that the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism. Then  $M$  is compact and orientable and  $\pi_1(M)$  is free cyclic, therefore  $e(\pi_1(M)) = 2$ .*

See [5], p. 296, theorem 2.

**Theorem 5.** *Let  $M$  be a compact 3-manifold with boundary, where the components are orientable surfaces of positive genus, such that  $e(\pi_1(M)) = 1$  or 2. Then  $M$  is aspherical.*

This is a slight modification of Satz VII, [10], p. 326, and the proof of this is precisely the proof of Satz VII. The following corollary refines Satz VIII, [10], p. 327.

**Corollary 1.** *Let  $M$  be a compact 3-manifold with non-connected boundary,*

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<sup>18)</sup> Here we suppose that  $S$  and  $C$  have an arbitrary orientation, and that  $C_1$  and  $C_2$  have the induced orientation.  $C^+$  and  $C^-$  mean the two orientations of  $C$ .

where the components are orientable surfaces of genus one, then  $e = e(\pi_1(M)) = 1$  or  $\infty$ , i. e. always  $\neq 2$ .

**Proof.** If  $M$  is not aspherical, then  $e = \infty$ , by theorem 5. If  $M$  is aspherical, then the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is an isomorphism for any component  $F$  of  $\text{bd}M$ , by theorem 2. Hence  $e = 1$ , by theorem 3.

6. The following is the main theorem of the paper.

**Theorem 6.** *Let  $M$  be a compact 3-manifold with boundary, where the components are orientable closed surfaces of positive genus. Then<sup>19)</sup>  $e = e(\pi_1(M)) \neq 0$ , and the following hold*

(1) *If  $M$  is aspherical and the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is an isomorphism, for any component  $F$  of the boundary of  $M$ , then  $e = 1$ .*

(2) *If  $M$  is aspherical, its boundary  $F$  is connected and has genus  $g(F) = 1$ , and the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism, then  $e = 2$ .*

(3) *In any other case (i. e. if either  $M$  is not aspherical; or  $M$  is aspherical, and there is a component  $F$  of the boundary, of genus  $g(F) > 1$ , such that the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism)  $e = \infty$ .*

**Proof.** If  $M$  is not aspherical, then  $e = \infty$ , by theorem 5. Let us from now on suppose that  $M$  is aspherical. If the hypotheses of (1) hold, then  $e = 1$ , by theorem 3. Let us from now on suppose that the hypotheses of (1) do not hold, i. e. there is a component  $F$  of the boundary of  $M$ , such that the injection  $\pi_1(F) \rightarrow \pi_1(M)$  is not an isomorphism. We have to consider the following two cases: (i) The boundary of  $M$  is not connected. (ii) The boundary of  $M$  is connected.

*Case (i):* By theorem 2, the genus  $g(F) > 1$ , because  $M$  is aspherical. Hence  $e = \infty$ , by theorem 1.

*Case (ii):* In this case  $F$  is the whole boundary of  $M$ . If the genus  $g(F) = 1$ , then  $e = 2$ , by theorem 4. If  $g(F) > 1$ , then  $e = \infty$ , by theorem 1.

From the above we conclude the truth of theorem 6.

## § 5. Applications of DEHN's Lemma

7. We now are going to give a short proof of theorems 1 and 2, using DEHN's lemma [7], p. 169, [8], p. 1, and theorem 1, [6], p. 281, which we shall call the *loop theorem* for convenience.

In both theorems 1 and 2, there exists a loop  $L$  on  $F$  which is  $\simeq 0$  in  $M$ , and  $\not\simeq 0$  on  $F$ , because  $j$  is not an isomorphism. Thus there exists a simple<sup>9)</sup>

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<sup>19)</sup> By lemma 3,  $\pi_1(M)$  is infinite, therefore  $e \neq 0$ .

loop  $L_0$  on  $F$  which is  $\simeq 0$  in  $M$ , and  $\neq 0$  on  $F$ , by the loop theorem. Hence there exists a 2-cell  $D$ , such that<sup>10)</sup>  $\text{bd}D = L_0$ , and  $D - L_0 \subset \text{int}M$ , by DEHN's lemma. Let  $M_0$  be the 3-manifold with boundary we obtain cutting  $M$  along  $D$ , and let  $F_0$  be the component of  $\text{bd}M_0$ , which comes from  $F$ , and on which there lie the two copies  $D'_0$  and  $D''_0$  of  $D$ . Let  $p_0: \tilde{M}_0 \rightarrow M_0$  be the universal covering of  $M_0$ , and let  $D'_{0j}, D''_{0j}$ ,  $j = 1, 2, \dots$ , be the 2-cells on  $\text{bd}\tilde{M}_0$  lying over  $D'_0, D''_0$ . The universal covering  $p: \tilde{M} \rightarrow M$  of  $M$  is composed of a (denumerable) number of copies of  $\tilde{M}_0$ , and is obtained glueing the  $D'_{0j}$  of a copy with the  $D''_{0j}$  of another copy, in such a way that we obtain a simply connected complex. Let us denote by  $\tilde{D}_k$ ,  $k = 1, 2, \dots$ , the 2-cells lying over  $D$ .

Let us now pass to the proof of theorem 1: By lemma 1, there is a simple loop  $L_1$  on  $F_0$ , which is  $\neq 0$  in  $M_0$ , because the genus  $g(F_0) > 0$ . As we can easily see, using the loop theorem,  $L_1^s \neq 0$  in  $M_0$  for any natural number  $s$ <sup>20)</sup>. This implies that  $\pi_1(M_0)$  is infinite, and thus  $\tilde{M}_0$  is not compact. As we can easily see, using the cells  $\tilde{D}_k$ ,  $k = 1, 2, \dots$ ,  $\tilde{M}$  has infinitely many ends. This proves theorem 1.

Let us now pass to the proof of theorem 2:  $F_0$  is a 2-sphere, therefore  $p_0^{-1}(F_0)$  consists of 2-spheres. But  $\tilde{M}_0$  is at least either not compact, or  $\text{bd}\tilde{M}_0 - p_0^{-1}(F_0) \neq \emptyset$ . Thus, if  $\tilde{X}_0$  is a basic 2-cycle of a component  $\tilde{F}_0$  of  $p_0^{-1}(F_0)$ ,  $\tilde{X}_0 \sim 0$  in  $\tilde{M}$ . Hence, by standard Hurewicz theorems,  $\pi_2(M) \neq 0$ . This proves theorem 2.

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<sup>20)</sup> Compare also [6], p. 287, lemma (9.3).