The Interaction of Imperfection Wavelength and Buckling Mode in Plated Structures

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Summary

This short presentation is concerned with the interaction of geometric imperfection forms and buckling modes. It highlights the weakening effects of imperfections sympathetic to the natural buckling form and the strengthening effects to be gained from non-sympathetic modes.

It is concluded that the multitude of modes present in fabricated structures tend to remove the worst of this available imperfection sensitivity and produce strengths in between analytical extremes. The problem remains as to how to specify suitable inputs so that powerful analytical tools now available can be used with confidence for the production of design data without unnecessary conservatism or without an undue risk of structural failure.

Possibilities are discussed for choice of imperfection modes but until such time as these concepts are validated and widely accepted many design rules will continue to be based on potentially inappropriate empirical results.

Interaction of Buckling and Imperfection Modes in Flat Plates

Perhaps one of the simplest interaction problems to understand is related to the effect of two non-sympathetic load types, in many ways an analogous problem to that of non-sympathetic imperfection forms.

A plate panel, such as that found in a longitudinally stiffened girder adjacent to the compression flange, is loaded by combined shear and compression. If the panel is relatively slender the effect of a modest level of shear is negligible or can even enhance the basic compression capacity of the panel (see Figure 1) [1]. This phenomenon is easily understood as the buckling mode of a square panel in shear is of diagonal form and in compression is a squarer sine wave. The interaction of these modes leads to the possibility of enhancement of strength in cases where buckling action rather than yield dominates the ultimate load behaviour. It would be difficult to allow for this effect in design as it is likely that a combination of design forces exists within the maximum envelope where the maximum of one force component acts with a significantly reduced level of the second.

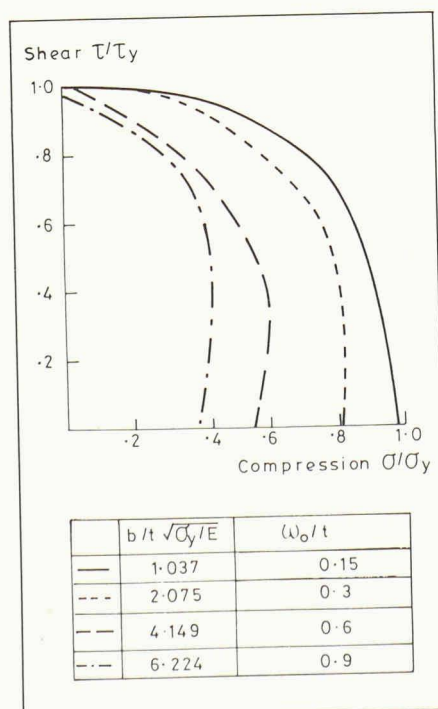An almost equivalent situation occurs due to the presence of non-sympathetic imperfection forms. In reality imperfections present in structures are multimode and this has the effect of reducing the enhancement or weakening resulting from individual waveforms. There is usually, however, one significantly larger imperfection waveform present and this is often in a mode coincident with or close to the preferred buckling mode. This is particularly true in the case of compressive loading and the reason can be readily understood. Civil Engineering plates and shells are generally relatively crudely fabricated by welding individual sections or sub-assemblies together. The result of this welding is to induce compressive residual stresses across a large proportion of the plate (or shell) width. If the structure is moderately slender with small imperfections inevitably already present the effect of this compressive stress is to increase the imperfections in the natural buckling mode. As a result the largest imperfection mode-form will often reflect the critical buckling waveform, particularly if the panel dimensions and critical buckling half wavelengths coincide. If the panel is significantly longer in one direction then the imperfections induced will tend to be a combination of a single half wave over the panel length combined with a smaller level of the critical mode-form.

A good example of the effect of this imperfection interaction is the behaviour of a three to one plate loaded in its long direction by in-plane compression. The action of welding the stiffeners to the four edges will be predominantly to introduce 3×1 and 1×1 half waveforms. The overall wave tends to be larger partly because of the low resistance to transverse compression due to the low critical stress of the plate in the transverse direction and because of the racking of the welds. Large deformations in this mode, however, involve appreciable stretching energy and the critical buckling mode of the plate loaded longitudinally corresponds with the three half wave buckle. Recent elasto-plastic work on 3×1 plates under biaxial compression [2] recognised this imperfection relationship and took the square wave imperfection amplitude to be one half of the full wave. More recent work on shells suggests that a proportion of one-third might have been more realistic [3]. The true ratio depends on the relative lengths of the panel com-

[1] See references at the end.

Figure 1. — Interaction of compression and shear on square plate panels.

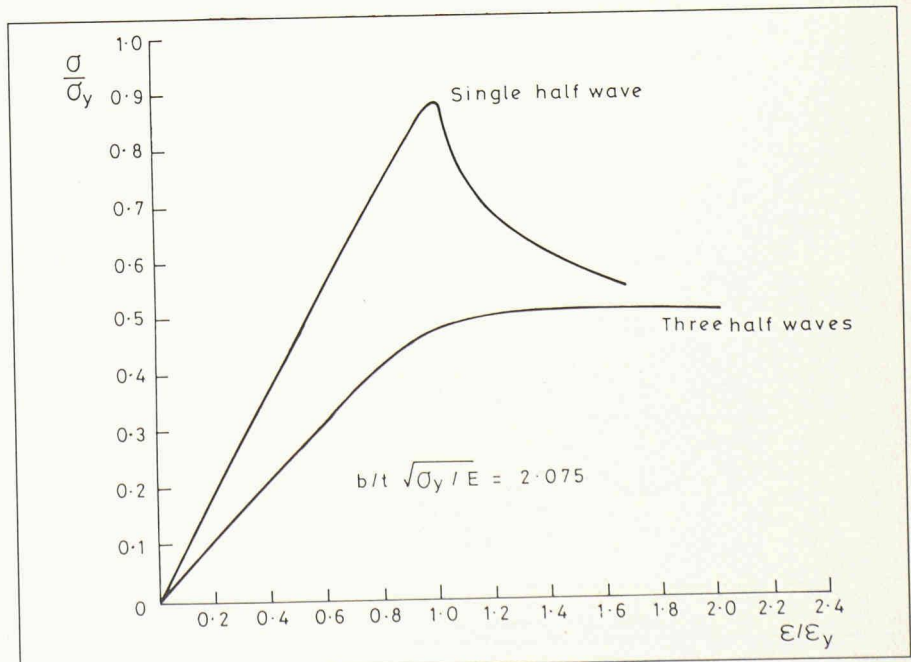| $b/t \sqrt{\sigma_y/E}$	ω_0/t
1·037	0·15
2·075	0·3
4·149	0·6
6·224	0·9

Figure 2. — Effect of imperfection form on compressive strength.

pared to the critical buckling wavelength. For example a panel with a five to one aspect ratio would have significant 1 × 1, 3 × 1 and 5 × 1 imperfection modes but it is doubtful that the amplitudes would be in an exact inverse ratio. It would be expected that the 3 × 1 mode would be reduced in relation to the others because of its lack of relationship with either the longitudinal or transverse buckling modes. The above representation, however, might well be a reasonable design premise.

The result of the two imperfection modes of the preceeding case can be seen in Figure 2 [4]. The non-sympathetic imperfection produces a peakier characteristic with rapid unloading because of a snap through at ultimate load, while the sympathetic imperfection produces a lower ultimate load with a softer characteristic. The true design strength (for an appropriate characteristic level of imperfection) will tend to lie between the extremes and the difficult decision when putting forward design data is what proportion of the individual modes can safely be taken as being present in the analyses. There is no replacement for field measurement and the use of probabilastic data in this context but such data is generally missing or inconclusive and analysis work normally resorts to experimental data often relating to small scale models which are inevitably made by a different, albeit possibly similar, process to real fabrication.

Interaction of Buckling and Imperfection Modes in Cylinders

The problems outlined above become even more severe in the case of shells because of their inherent increased imperfection sensitivity. Figure 3 shows the variation of axial compressive strength that can be obtained for ring stiffened cylinders with a given amplitude of
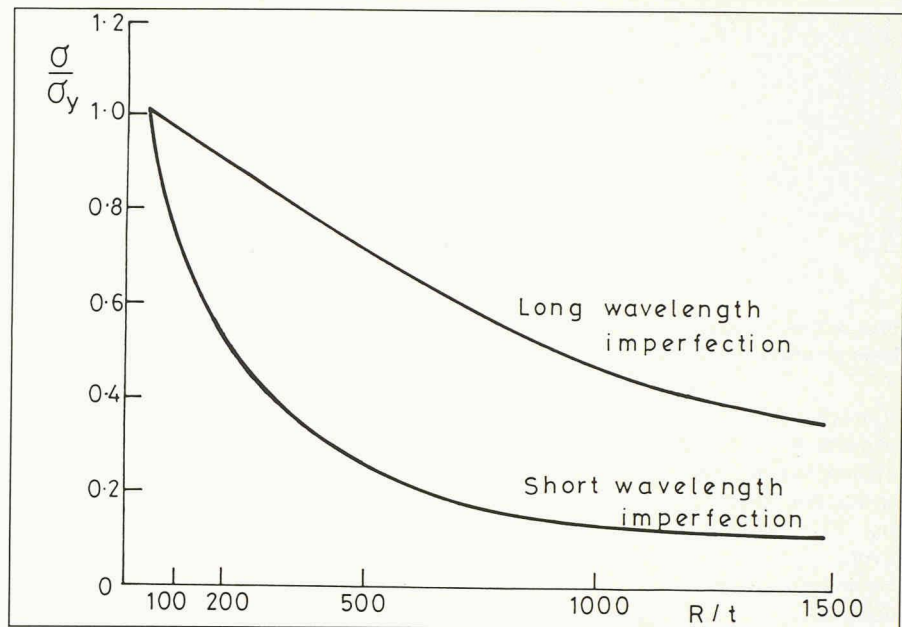
Figure 3. — Effect of longitudinal imperfection wavelength on strength.

imperfection but with varying longitudinal wavelength [5].

The dependence of strength on the relationship between the critical buckling wavelength and the longitudinal wavelength of the imperfection can be clearly demonstrated in the context of unstiffened or ring stiffened cylinders of moderately stocky construction. Figure 4 shows the effect, for two values of radius to thickness, of varying the wavelength l of the longitudinal imperfection mode and how the behaviour depends on the proximity of this length to the critical length [6]. For these results it has been assumed that either non-deflecting (clamped or simply supported) or discrete (stiffened) rings form boundaries at the single half wave end locations. It can be seen that when the ring spacing and therefore the imperfection wavelength correspond to the critical wavelength for the simply supported case a minimum

strength is obtained. For the clamped and discretely stiffened cases the effect of clamping increases the ring spacing corresponding to the critical imperfection. It is perhaps a little surprising that the elastic critical wavelength has such a controlling influence in an ultimate load situation when very appreciable plasticity is present.

These results would lead to the conclusion that designers should avoid the critical wavelength area for ring spacing, and this may indeed be sound advice, but it should be remenbered that the multitude of imperfections present in real structures tends to alleviate this effect. There has been experimental evidence, however, to support the possibility that closing up the ring spacing and thereby inducing critical mode imperfections can actually weaken the strength [7].

The comments regarding clamping also bear consideration from a design view-
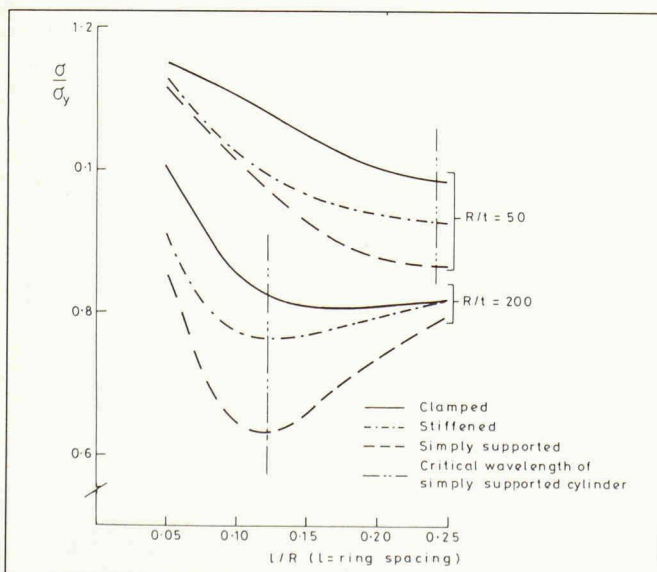
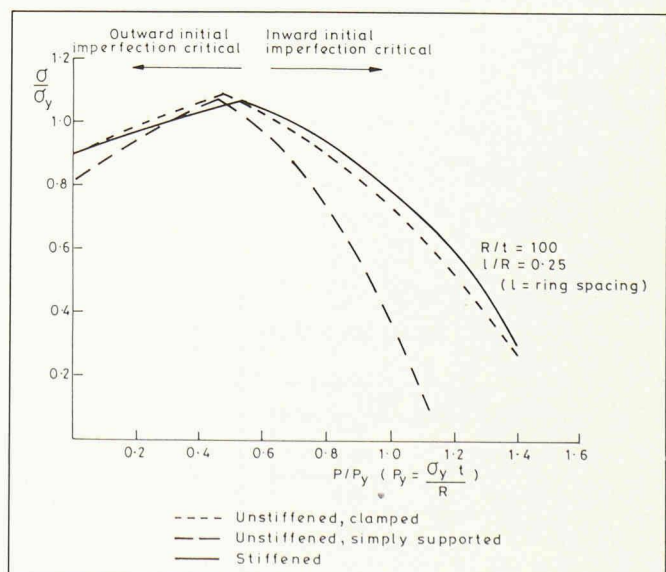Figure 4. — Effects of ring spacing on strength under axial compression.

Figure 5. — Interaction of axial compression and pressure on panel collapse.

point. The presence of imperfections of similar form in adjacent panels can lead to an effective clamping action, especially prior to extensive plasticity, and this effect can be enhanced by the presence of lateral pressure. This clamping can provide modest strength increases in the case of axial loading and more significant increases for lateral loading. The difficulty is in deciding what proportion of this effect can be allowed for in design. The effect of imperfection choice is even more marked in the case of the interaction of axial compression and lateral pressure. Figure 5 shows the effect of a single half wave imperfection between ring stiffeners on the interactive capacity of an inter-ring shell panel. The normally dominant mode in fabricated cylinders of this geometrical range is a single outward bulge caused by weld shrinkage at the rings. If this mode could be relied on to occur stronger results would be obtained to the right of the transition peak. Again, the presence of multi-mode imperfections will reduce this apparent strength enhancement and furthermore the presence of a significant component of the critical wavelength imperfection, rather shorter than the ring spacing, could reduce the strengths shown in the figure. It is unlikely, however, for the amplitude of the critical mode imperfection to approach that of the 0.25 R wavelength mode assumed in the figure because of the effect of the welding.
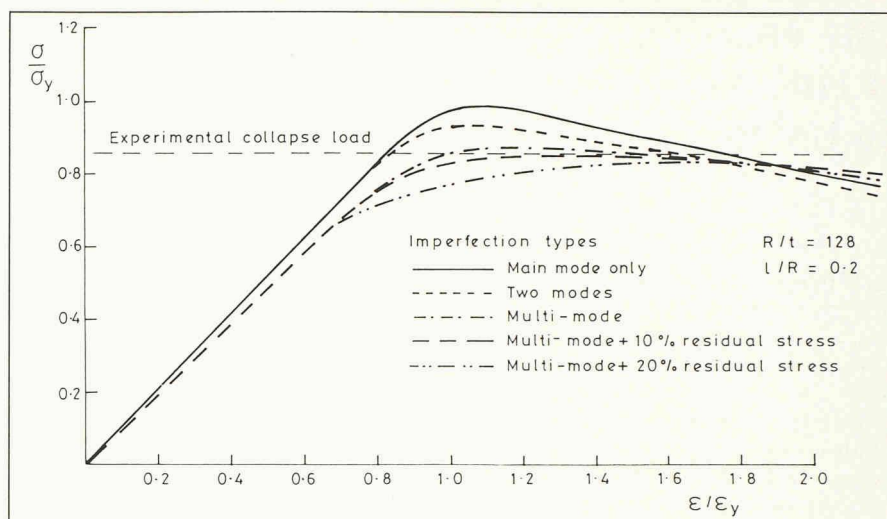

Figure 6. — *Correlation of analytical and experimental collapse loads.*

It is particularly interesting that the transition region in this case corresponds to the position where a shell with zero imperfections would have a negligible tendency to buckle inwards or outwards. The pressure loading is exactly counteracting the bulging of the panel due to Poisson expansion. Reference 8 has shown that this behaviour induces extreme imperfecion sensitivity in this transition area with the possibility of very low ultimate strengths if critical imperfection modes are present. Again the practical relevance of this will depend on imperfection combinations present in practice.

Choice of Imperfection Modes

Correlation with experiments demonstrates the importance of full modal representation [5]. Figure 6 shows that using only the largest and second largest imperfection modes in correlation exercises can generate higher strengths and that a large proportion of the imperfection mode forms have to be included before an accurate representation of the experimental result can be obtained. Reference 2 suggests that including all significant mode forms with amplitudes inversely proportional to their wavelengths provides a realistic representa-

tion of measured values and this approach may certainly prove viable for the generation of design data. The amplitudes still need relating to the design tolerance level normally specified in a single mode.

The alternative approach, which has also been suggested, is to use the design tolerance level in the critical mode, but there seems little doubt that this must lead to over-conservatism in many instances. The complexity of imperfections has the effect of reducing sensitivity and in general raising the strength above the critical imperfection lower bound.

Until more field measurements are carried out the production of design data using numerical analysis techniques must inevitably be tempered by reference to experimental results. It must never, however, be forgotten that experiments are prone to modelling and loading deflects and may not be directly representative of true structural behaviour.

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