Zeitschrift: Bulletin de la Société Vaudoise des Sciences Naturelles
Herausgeber: Société Vaudoise des Sciences Naturelles
Band: 24 (1888)
Heft: 99

Artikel: Fonctions abéliennes du genre 3 : un cas particulier
Autor: Amstein, H.
Kapitel: Le problème de Riemann
DOI: https://doi.org/10.5169/seals-261785

Le problème de Riemann.

Au sujet de ce problème fondamental, M. Weber (p. 159 à 168), indique dans tous leurs détails les calculs nécessaires. En les suivant pas à pas, on arrivera sans difficulté aux résultats désirés. Au lieu d'une traduction à peu près littérale de cette partie de l'ouvrage de M. W., il sera plus utile de donner ici une application des formules trouvées à des cas particuliers en n'insistant que sur le commencement de la solution.

I. Les deux caractéristiques (k) et (k′) sont paires.

Soit par exemple

$$(k)=\binom{010}{100},\quad k'=\binom{001}{100},\quad (k)+(k')=(\sqrt{x_1})+(\sqrt{x_2}).$$

En formant les deux groupes

$$\begin{aligned}(k)+(k')=\binom{011}{000}&=\underset{x_1}{\underline{\binom{100}{100}}}+\underset{x_2}{\underline{\binom{111}{100}}}=\underset{\xi_2}{\underline{\binom{100}{111}}}+\underset{\xi_1}{\underline{\binom{111}{111}}}=\underset{\gamma''_1}{\underline{\binom{010}{011}}}+\underset{\gamma''_2}{\underline{\binom{001}{011}}}=\\ &=\underset{\gamma^0_2}{\underline{\binom{010}{111}}}+\underset{\gamma^0_1}{\underline{\binom{001}{111}}}=\underset{\gamma_2}{\underline{\binom{101}{100}}}+\underset{\gamma_1}{\underline{\binom{110}{100}}}=\underset{\gamma'_2}{\underline{\binom{101}{011}}}+\underset{\gamma'_1}{\underline{\binom{110}{011}}},\end{aligned}$$

$$\begin{aligned}(k)+(\sqrt{x_1})=\binom{110}{000}&=\underset{\gamma^0_3}{\underline{\binom{100}{110}}}+\underset{\xi_3}{\underline{\binom{010}{110}}}=\underset{\xi_2}{\underline{\binom{100}{111}}}+\underset{\gamma^0_2}{\underline{\binom{010}{111}}}=\underset{x_6}{\underline{\binom{001}{001}}}+\underset{g''}{\underline{\binom{111}{001}}}=\\ &=\underset{\gamma^0_1}{\underline{\binom{001}{111}}}+\underset{\xi_1}{\underline{\binom{111}{111}}}=\underset{g'}{\underline{\binom{011}{001}}}+\underset{x_5}{\underline{\binom{101}{001}}}=\underset{g}{\underline{\binom{011}{110}}}+\underset{x_4}{\underline{\binom{101}{110}}}\end{aligned}$$

on remarque qu'ils possèdent les 4 caractéristiques communes

$$(\sqrt{y_1})=(\sqrt{\xi_1})=\binom{111}{111},\qquad (\sqrt{y_2})=(\sqrt{\xi_2})=\binom{100}{111},$$
$$(\sqrt{z_1})=(\sqrt{\gamma^0_1})=\binom{001}{111},\qquad (\sqrt{z_2})=(\sqrt{\gamma^0_2})=\binom{010}{111}.$$

Posant, en conséquence,

$$\sqrt{x_1}=\sqrt{s-1},\qquad \sqrt{x_2}=\sqrt{z+1},\qquad \sqrt{y_1}=\sqrt{s+1},$$
$$\sqrt{y_2}=\sqrt{z-1},\qquad \sqrt{z_1}=\sqrt{s-z+\varepsilon},\qquad \sqrt{z_2}=\sqrt{s-z+\varepsilon'},$$

on trouve aisément la formule

$$\sqrt{\frac{\chi_{\binom{010}{100}}}{\chi_{\binom{001}{100}}}} = \frac{\vartheta_{\binom{001}{100}}}{\vartheta_{\binom{010}{100}}} \cdot \frac{\vartheta_{\binom{010}{100}}(v_1, v_2, v_3)}{\vartheta_{\binom{001}{100}}(v_1, v_2, v_3)} = \sqrt[4]{-1} \cdot \frac{\vartheta_{\binom{010}{100}}(v_1, v_2, v_3)}{\vartheta_{\binom{001}{100}}(v_1, v_2, v_3)}$$

Désignant par $x_i^{(\nu)}$, $y_i^{(\nu)}$, $z_i^{(\nu)}$ les valeurs que prennent les fonctions x_i, y_i, z_i pour $\zeta = \zeta_\nu$, soit $s = s_\nu$, $z = z_\nu$, on a ici

$$\sqrt{\chi_{\binom{010}{100}}} =$$

$$= \Sigma \pm \sqrt{x_1 y_1 z_1}, \sqrt{x_1^{(1)} y_2^{(1)} z_2^{(1)}}, \sqrt{x_2^{(2)} y_1^{(2)} z_2^{(2)}}, \sqrt{x_2^{(3)} y_2^{(3)} z_1^{(3)}},$$

$$\sqrt{\chi_{\binom{001}{100}}} =$$

$$= \Sigma \pm \sqrt{x_2 y_2 z_2}, \sqrt{x_2^{(1)} y_1^{(1)} z_1^{(1)}}, \sqrt{x_1^{(2)} y_2^{(2)} z_1^{(2)}}, \sqrt{x_1^{(3)} y_1^{(3)} z_2^{(3)}},$$

et les arguments v_1, v_2, v_3 sont déterminés par la congruence

$$(v_1, v_2, v_3) \equiv (\overset{3}{\underset{1}{h}} (\int_\alpha^{\zeta} du_h + \int_\beta^{\zeta_1} du_h + \int_\alpha^{\zeta_2} du_h + \int_\beta^{\zeta_3} du_h)),$$

où α et β signifient les zéros d'une fonction abélienne quelconque.

II. Les caractéristiques (k) et (k′) sont impaires.

Soit

$$(k) = (\sqrt{z_1}) = (\sqrt{\gamma^\circ_1}) = \binom{001}{111}, \qquad (k') = (\sqrt{z_2}) = (\sqrt{\gamma^\circ_2}) = \binom{010}{111}.$$

Alors on a

$$(\sqrt{x_1 x_2}) = (\sqrt{y_1 y_2}) = (\sqrt{z_1 z_2}) = (k) + (k') = \binom{011}{000}$$

et les caractéristiques

$$(\sqrt{x_1 y_1 z_1}) = (k) + (\sqrt{x_1 y_1}) = (k_1) = \binom{001}{111} + \binom{100}{100} + \binom{111}{111} = \binom{010}{100},$$

$$(\sqrt{x_1 y_1 z_2}) = (k') + (\sqrt{x_1 y_1}) = (k'_1) = \binom{010}{111} + \binom{100}{100} + \binom{111}{111} = \binom{001}{100},$$

sont paires. En admettant encore, comme dans le cas précédent,

$$\sqrt{x_1} = \sqrt{s-1}, \ \sqrt{x_2} = \sqrt{z+1}, \ \sqrt{y_1} = \sqrt{\xi_1} = \sqrt{s+1},$$

$$\sqrt{y_2} = \sqrt{\xi_2} = \sqrt{z-1}, \ \sqrt{z_1} = \sqrt{\gamma^\circ_1} = \sqrt{s-z+\varepsilon},$$

$$\sqrt{z_2} = \sqrt{\gamma^\circ_2} = \sqrt{s-z+\varepsilon'},$$

Fig. 1.

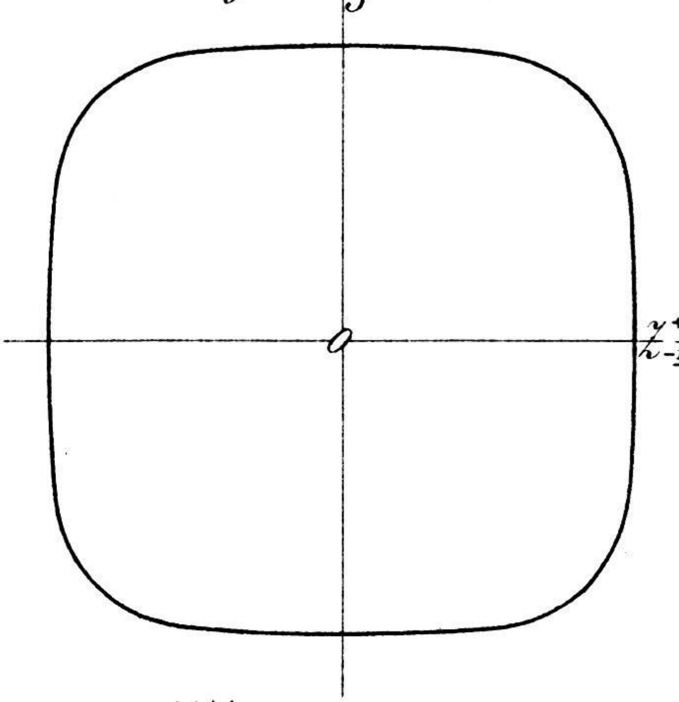


Fig. 6.

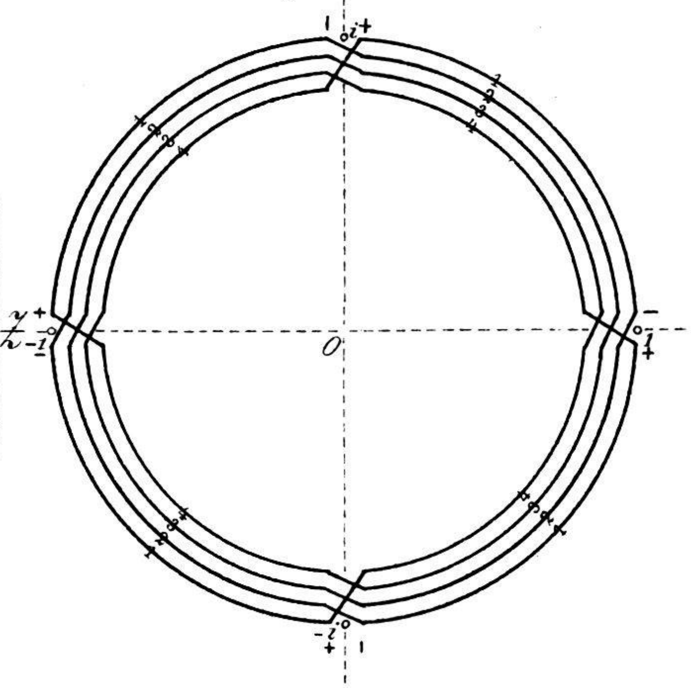


Fig. 4.

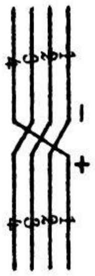


Fig. 5.

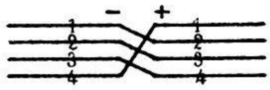
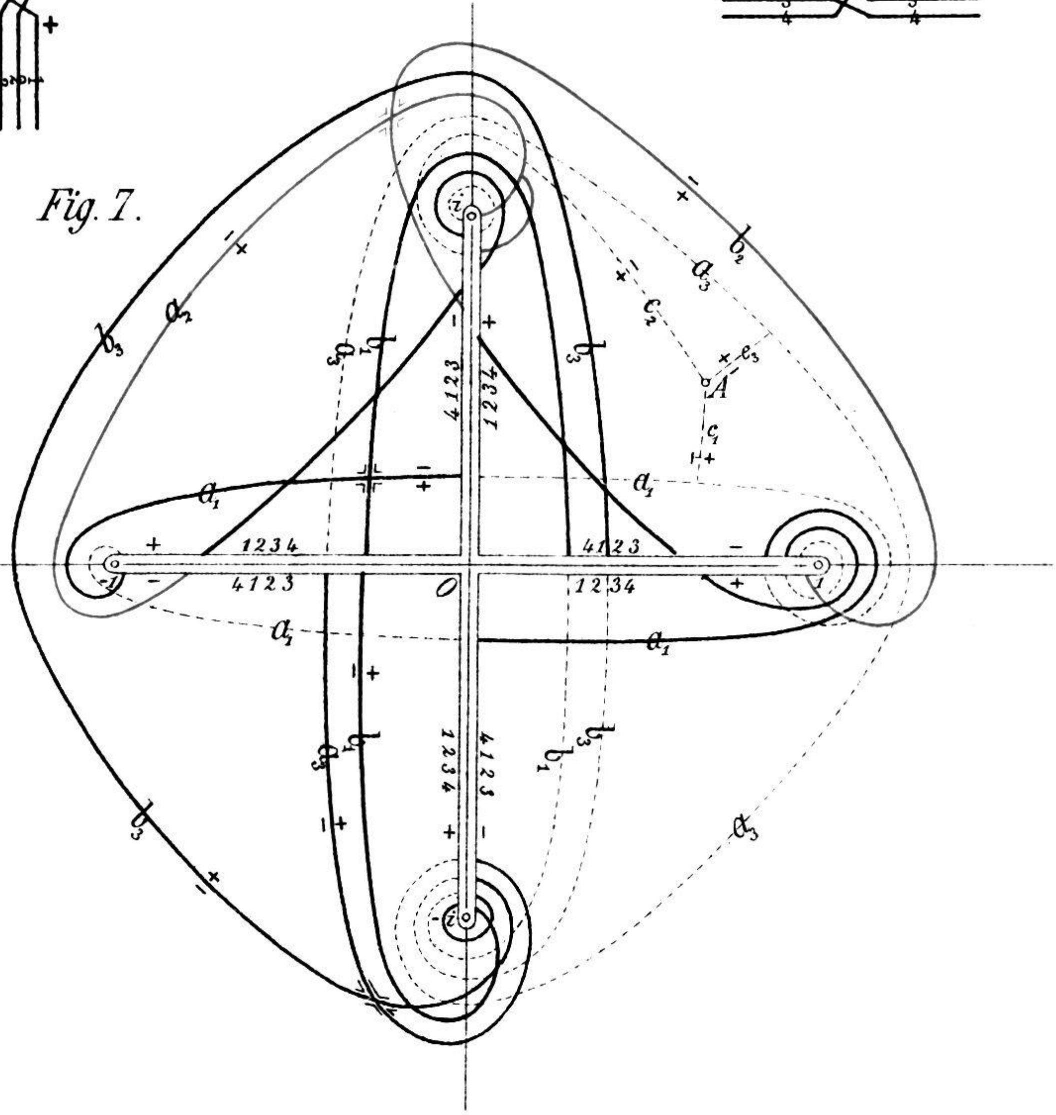


Fig. 7.



H. Amstein.

Lith. J. Chappuis, Lausanne.

Fig. 2.

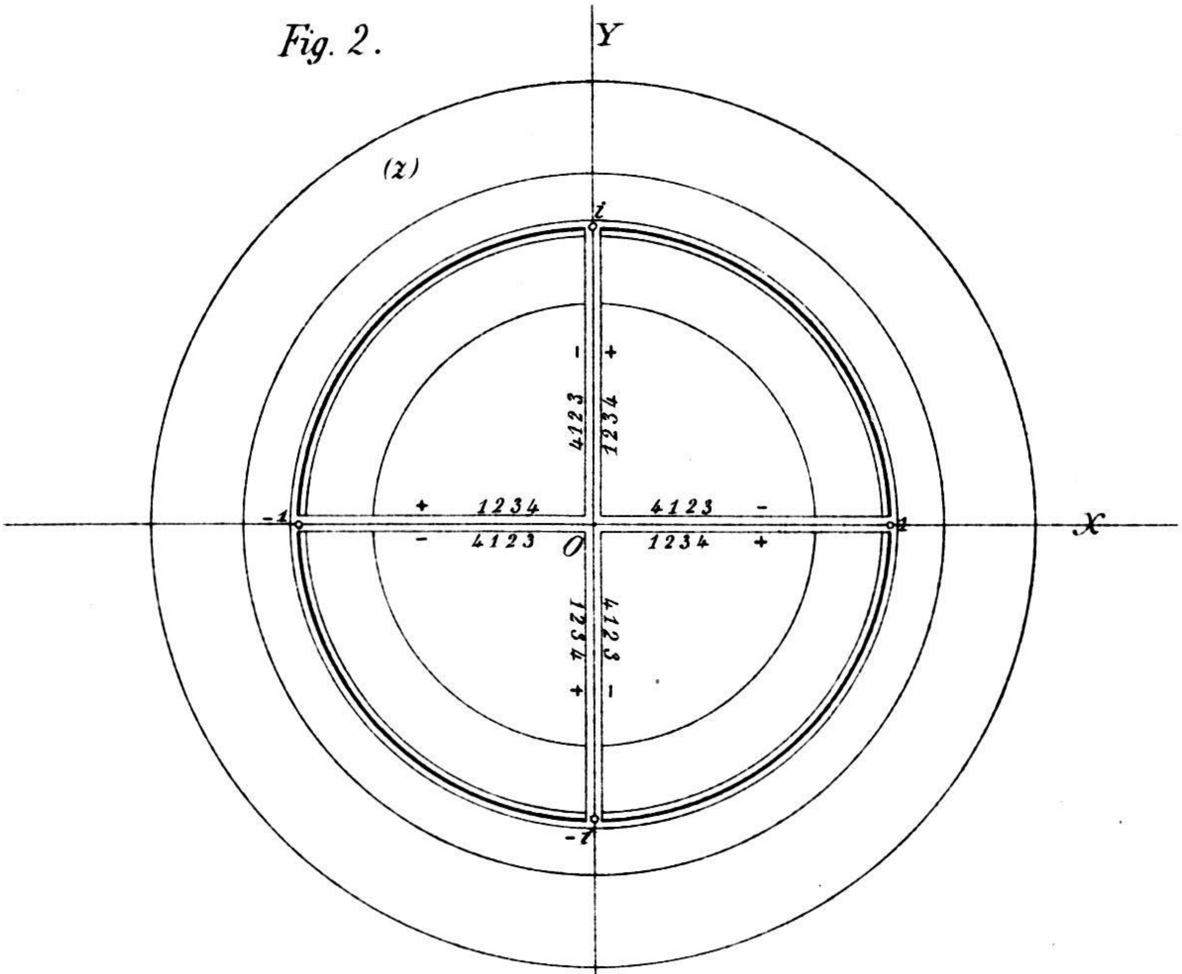
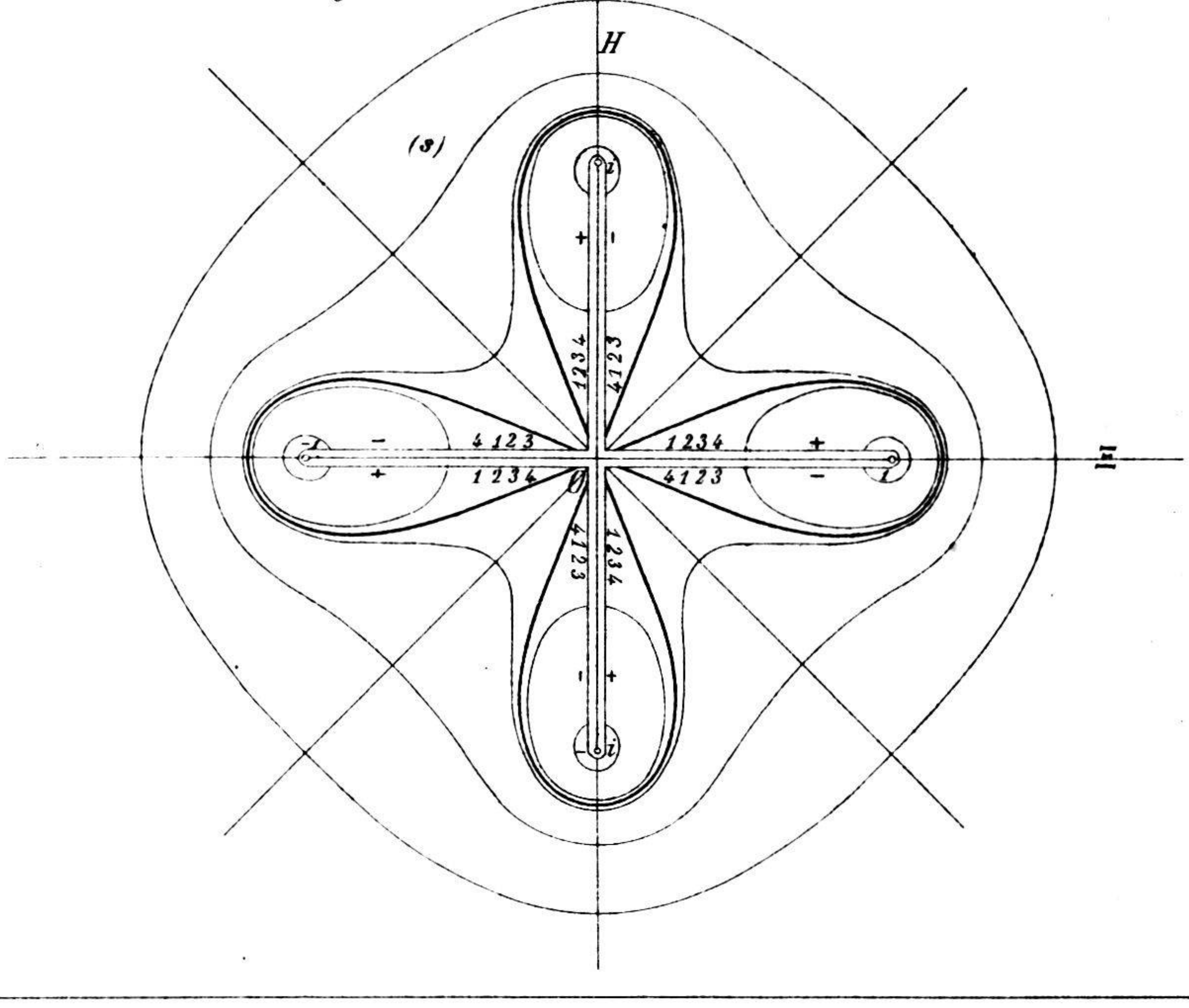


Fig. 3.



H. Amstein.

Lith. J. Chappuis, Lausanne.

Fig. 8.a

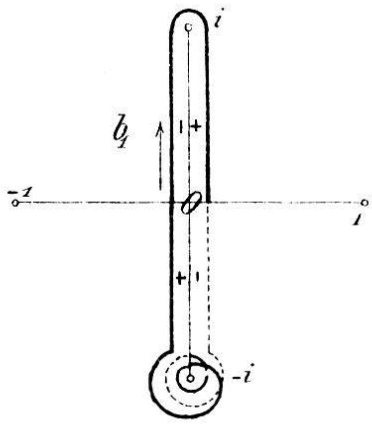


Fig. 8.b

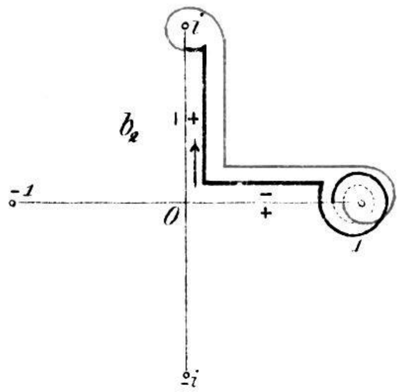


Fig. 8.c

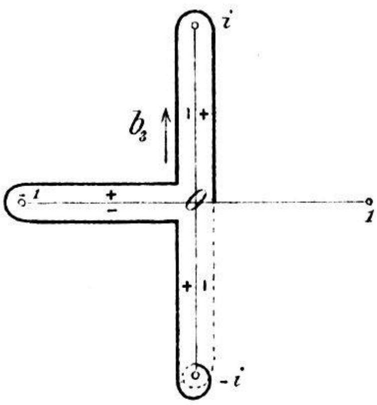


Fig. 8.d

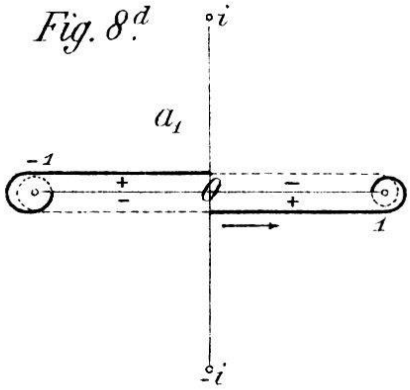


Fig. 8.e

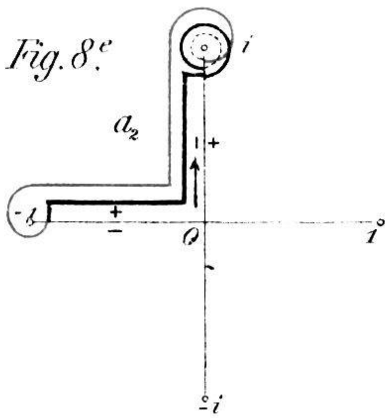


Fig. 8.f

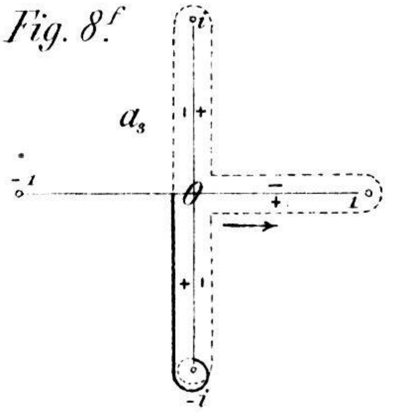


Fig. 9.a

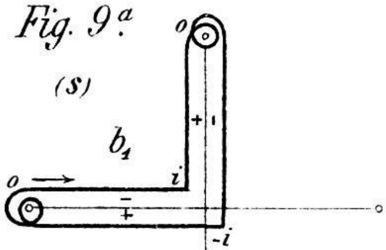


Fig. 9.b

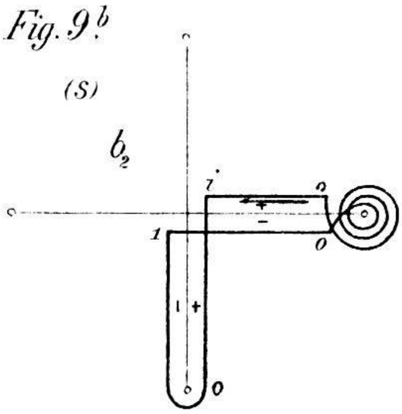


Fig. 9.c

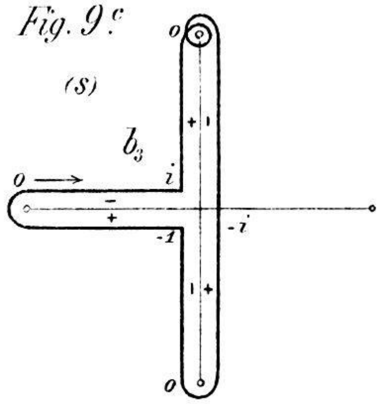


Fig. 9.d

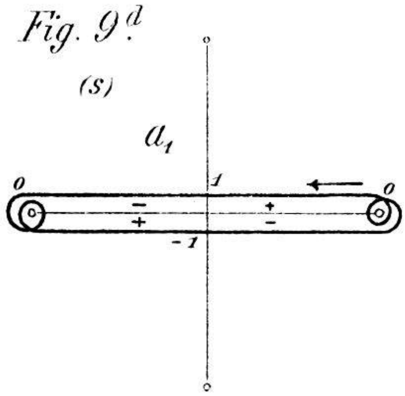


Fig. 9.e

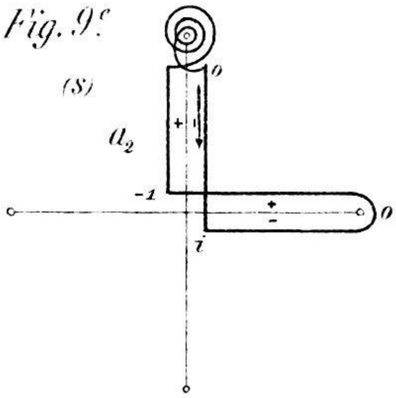
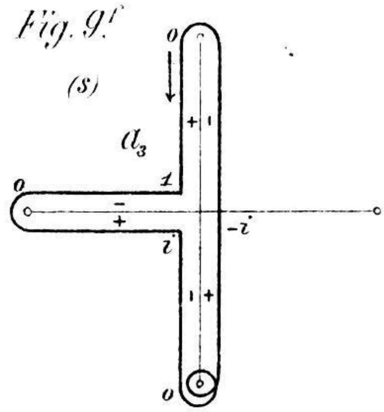


Fig. 9.f



H. Amstein.

Lith. J. Chappuis, Lausanne.

Fig. 10.a

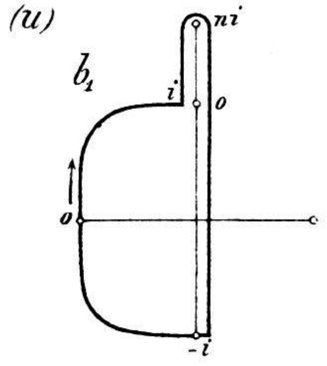


Fig. 10.b

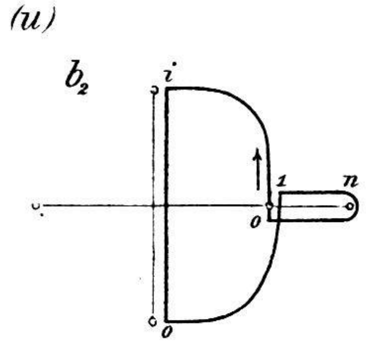


Fig. 10.c

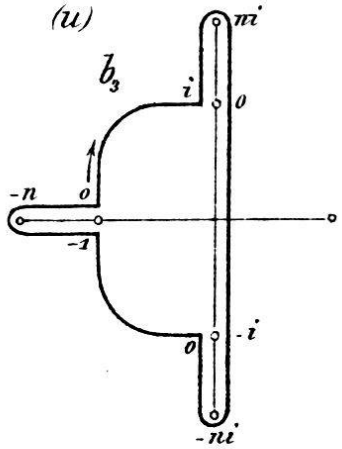


Fig. 10.d

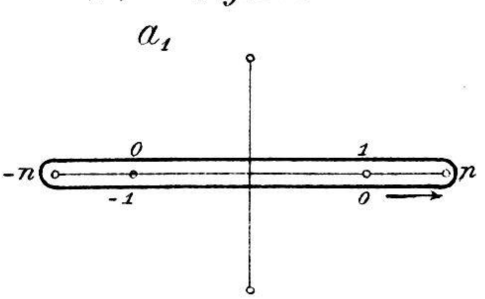


Fig. 10.e

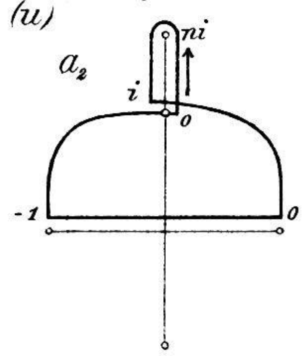


Fig. 10.f

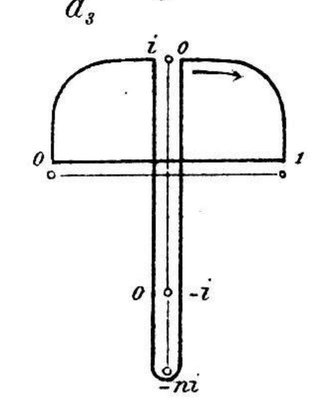


Fig. 11.a

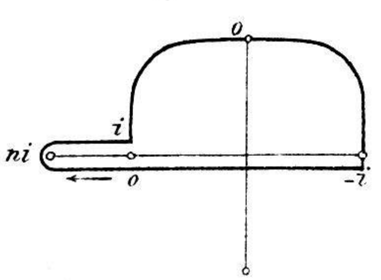


Fig. 11.b

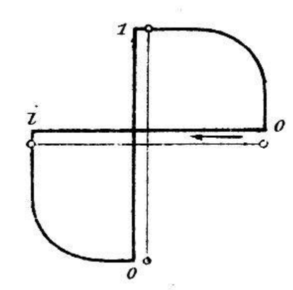


Fig. 11.c

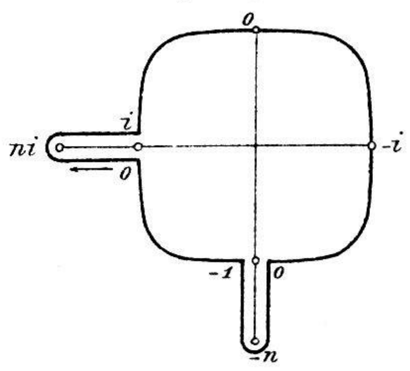


Fig. 11.d

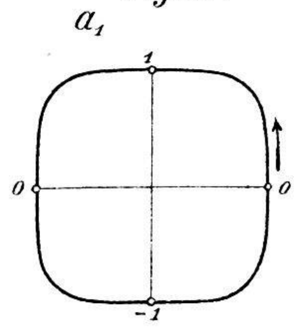


Fig. 11.e

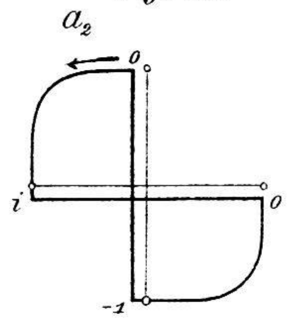


Fig. 11.f

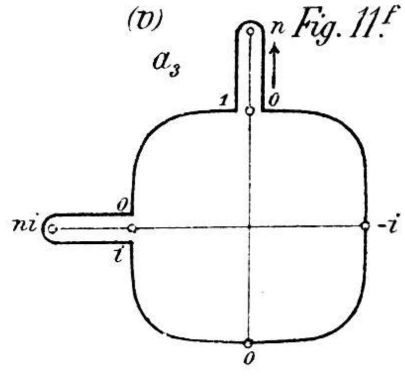


Fig. 12.a

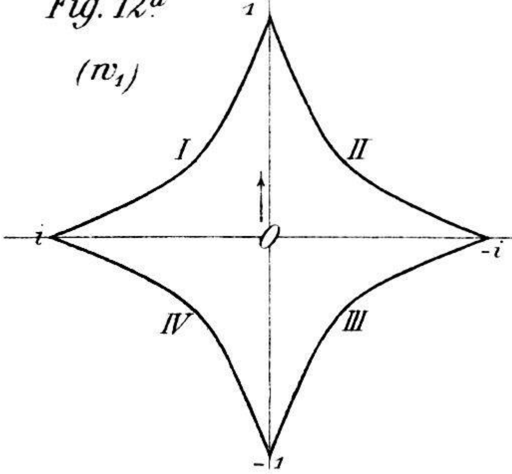


Fig. 12.b

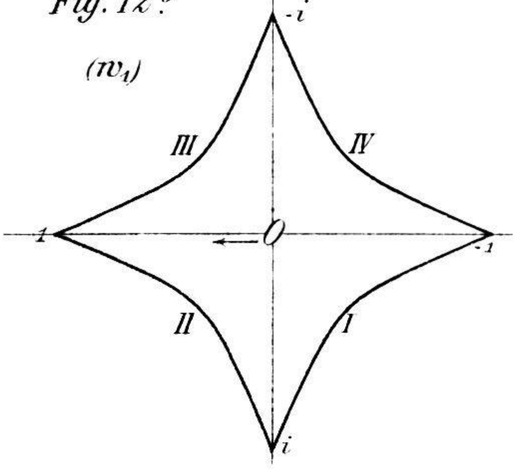


Fig. 12.c

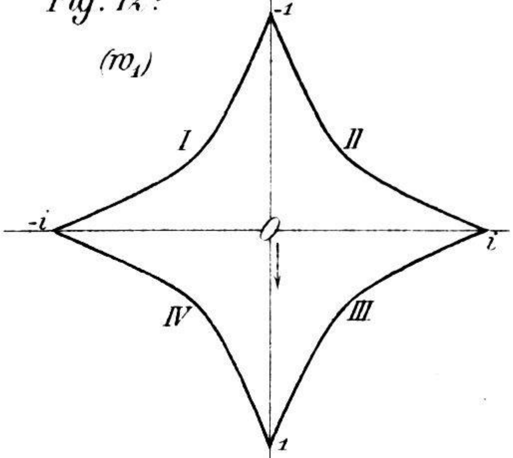


Fig. 12.d

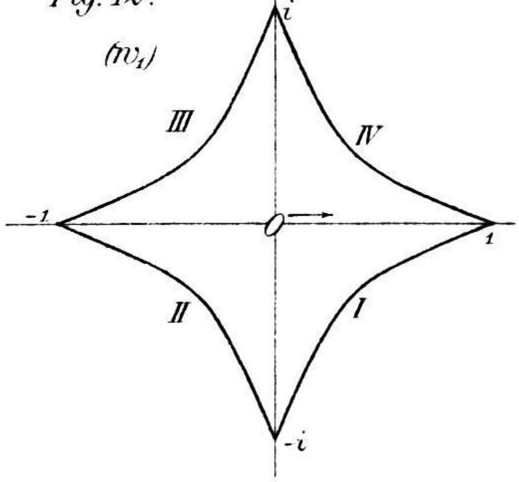


Fig. 12.e

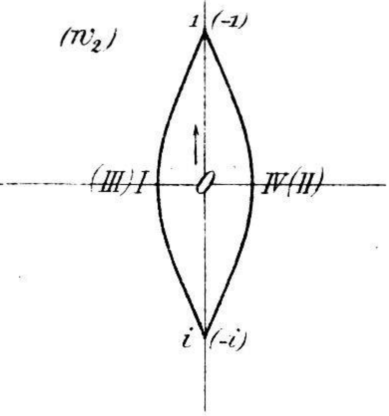


Fig. 12.f

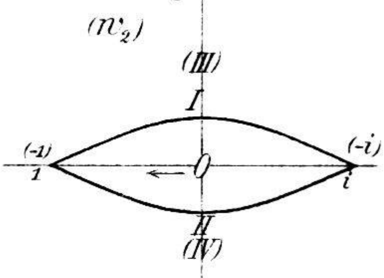


Fig. 12g

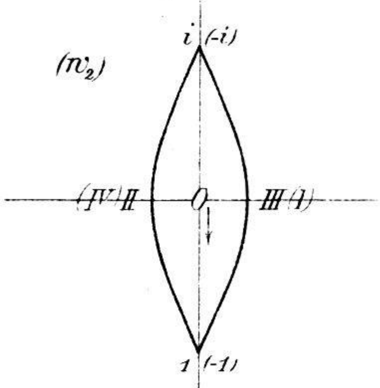


Fig. 12.h

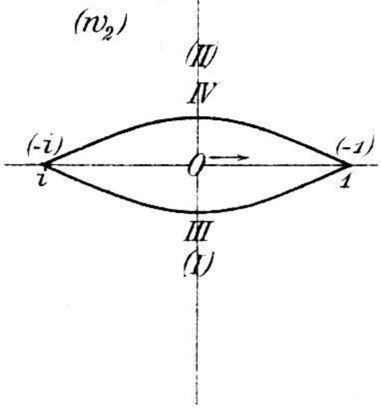


Fig. 12.i

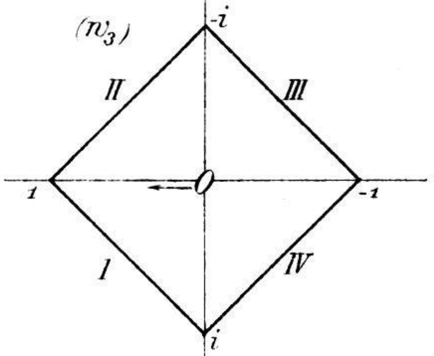


Fig. 12.k

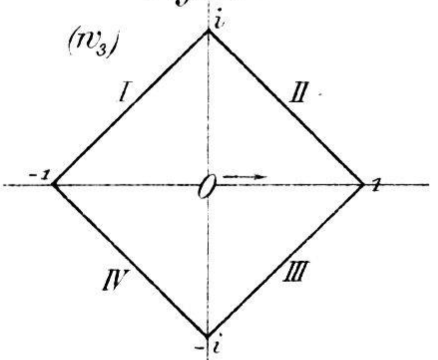


Fig. 12.l

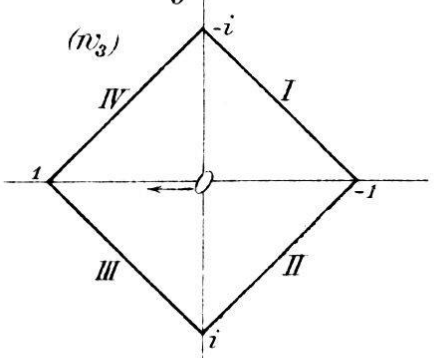
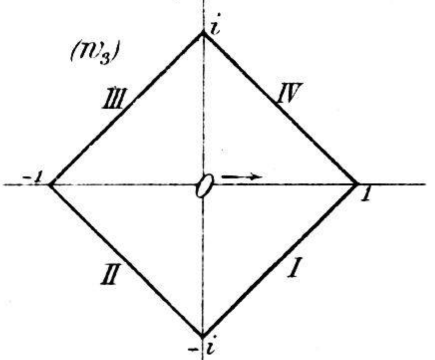


Fig. 12.m



H. Amstein.

Lith. J. Chappuis, Lausanne

on arrive à la formule

$$\sqrt{-\frac{\chi_{\binom{001}{111}}}{\chi_{\binom{010}{111}}}} = -\frac{\vartheta_{\binom{001}{100}}}{\vartheta_{\binom{010}{100}}} \cdot \frac{\vartheta_{\binom{001}{111}}(v_1\, v_2\, v_3)}{\vartheta_{\binom{010}{111}}(v_1\, v_2\, v_3)},$$

dans laquelle

$$\sqrt{\chi_{\binom{001}{111}}} = \Sigma \pm x_1\sqrt{z_1},\ y_1^{(1)}\sqrt{z_1^{(1)}},\ \sqrt{x_1^{(2)}x_2^{(2)}z_2^{(2)}},\ \sqrt{y_1^{(3)}y_2^{(3)}z_2^{(3)}},$$

$$\sqrt{\chi_{\binom{010}{111}}} = \Sigma \pm x_1\sqrt{z_2},\ y_1^{(1)}\sqrt{z_2^{(1)}},\ \sqrt{x_1^{(2)}x_2^{(2)}z_1^{(2)}},\ \sqrt{y_1^{(3)}y_2^{(3)}z_1^{(3)}}.$$

Les variables v_1, v_2, v_3 conservent toujours la même signification.

III. La caractéristique (k) est impaire, (k′) paire.

Soit $\quad (k) = (\sqrt{x}) = \binom{001}{101}, \quad (k') = \binom{010}{101}.$

Les deux groupes

$$(\sqrt{x_1x_2}) = \binom{011}{000} = \underset{x_1}{\underline{\binom{100}{100}}} + \underset{x_2}{\underline{\binom{111}{100}}} = \underset{\xi_2}{\underline{\binom{100}{111}}} + \underset{\xi_1}{\underline{\binom{111}{111}}} = \underset{\gamma''_1}{\underline{\binom{010}{011}}} + \underset{\gamma''_2}{\underline{\binom{001}{011}}} =$$

$$= \underset{\gamma^0_2}{\underline{\binom{010}{111}}} + \underset{\gamma^0_1}{\underline{\binom{001}{111}}} = \underset{\gamma_2}{\underline{\binom{101}{100}}} + \underset{\gamma_1}{\underline{\binom{110}{100}}} = \underset{\gamma'_2}{\underline{\binom{101}{011}}} + \underset{\gamma'_1}{\underline{\binom{110}{011}}},$$

$$(\sqrt{x_1y_1}) = \binom{011}{011} = \underset{x_1}{\underline{\binom{100}{100}}} + \underset{\xi_1}{\underline{\binom{111}{111}}} = \underset{\xi_2}{\underline{\binom{100}{111}}} + \underset{x_2}{\underline{\binom{111}{100}}} = \underset{\xi_6}{\underline{\binom{010}{010}}} + \underset{x_6}{\underline{\binom{001}{001}}} =$$

$$= \underset{\xi_3}{\underline{\binom{010}{110}}} + \underset{x_3}{\underline{\binom{001}{101}}} = \underset{x_5}{\underline{\binom{101}{001}}} + \underset{\xi_5}{\underline{\binom{110}{010}}} = \underset{x_4}{\underline{\binom{101}{110}}} + \underset{\xi_4}{\underline{\binom{110}{101}}}$$

font reconnaître qu'on peut poser

$$\sqrt{x_1} = \sqrt{s-1},\quad \sqrt{x_2} = \sqrt{z+1},\quad \sqrt{y_1} = \sqrt{\xi_1} = \sqrt{s+1}.$$

$$\sqrt{y_2} = \sqrt{\xi_2} = \sqrt{z-1},\quad \sqrt{z_1} = \sqrt{\gamma^0_1} = \sqrt{s-z+\varepsilon},$$

$$\sqrt{x} = \sqrt{x_3} = \sqrt{z-i},\quad \sqrt{y} = \sqrt{\xi_3} = \sqrt{z+i}$$

et qu'alors les deux caractéristiques

$$(k_1) = (\sqrt{xyz_1}) = \binom{001}{101} + \binom{010}{110} + \binom{001}{111} = \binom{010}{100},$$

$$(k'_1) = (k') + (\sqrt{yz_1}) = \binom{010}{101} + \binom{010}{110} + \binom{001}{111} = \binom{001}{100}$$

sont paires. En introduisant ces fonctions et caractéristiques dans les formules générales, il vient

$$\mp(1+i)\sqrt{\frac{\chi_{\binom{001}{101}}}{\chi_{\binom{010}{101}}}} = \frac{\vartheta_{\binom{001}{100}}}{\vartheta_{\binom{010}{100}}} \cdot \frac{\vartheta_{\binom{001}{101}}(v_1, v_2, v_3)}{\vartheta_{\binom{010}{101}}(v_1, v_2, v_3)},$$

où

$$\sqrt{\chi_{\binom{001}{101}}} = \Sigma \pm x\sqrt{x},\ z_1{}^{(1)}\sqrt{x^{(1)}},\ \sqrt{y^{(2)}x_1{}^{(2)}y_1{}^{(2)}},\ \sqrt{y^{(3)}x_2{}^{(3)}y_2{}^{(3)}},$$

$$\sqrt{\chi_{\binom{010}{101}}} = \Sigma \pm \sqrt{xx_1x_2}, \sqrt{x^{(1)}y_1{}^{(1)}y_2{}^{(1)}}, \sqrt{y^{(2)}x_1{}^{(2)}y_2{}^{(2)}}, \sqrt{y^{(3)}x_2{}^{(3)}y_1{}^{(3)}}.$$

Il serait inutile d'insister encore sur le problème de Jacobi, attendu qu'à l'aide de ce qui vient d'être dit, le lecteur suivra facilement jusqu'au bout l'ouvrage si souvent cité. Notre travail peut donc s'arrêter ici, d'autant plus que les deux problèmes de Jacobi et de Riemann seront repris, dans un second mémoire, à un point de vue tout différent et spécialement approprié au cas particulier qui a fait l'objet de cette étude.