

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 83 (1999)  
  
**Artikel:** Analysis of the ultimate response of externally prestressed beams  
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**DOI:** <https://doi.org/10.5169/seals-62926>

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## Analysis of the Ultimate Response of Externally Prestressed Beams

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### ABSTRACT

The deterioration of existing bridges due to progressive structural aging, severe weathering conditions, and corrosion of reinforcement has become a major problem around the globe. Also, many of these bridges are structurally deficient due to fatigue damage caused by increased traffic volume and truck loads beyond those estimated during the time of their design. To remedy such problems, several technologies and new materials have been developed for strengthening existing structures and have grown recently to occupy a significant share of the construction market. These include the use of high strength steel, continuous fiber reinforced plastics (FRP), carbon fiber reinforced polymer (CFRP) as reinforcement and prestressing material, and FRP laminated products glued to the weak tensile zones of the structural members. Among these technologies, external prestressing has proven to be efficient and cost-effective. Also, because of the economy and ease of maintenance associated with the use of external tendon system, external prestressing has been recommended recently in the design and construction of new segmental bridge structures.

In externally prestressed members, the prestressing tendons are unbonded to the concrete because they are located outside the concrete member. Consequently, the behavior of externally prestressed members is expected to be influenced by the same parameters that are known to influence the behavior of beams with internal unbonded tendons. However, there is one major difference between the behavior of internal and external unbonded tendon systems. That is, unlike internal unbonded members where the tendon eccentricity remains practically constant during the load-deflection response, the tendons in externally prestressed members are free to displace vertically relative to the axis of the beam between the deviator points or between the deviator points and the anchorages. This leads to progressive change in their eccentricity with increasing member deformation to failure and therefore may influence the response as compared to beams with internal unbonded tendons.

It is possible to develop accurate numerical models to predict the entire load-deflection response of externally prestressed members by using constitutive material stress-strain relationships for the concrete and steel and a multilevel iterative procedure to satisfy the deformation and force equilibrium requirements at any stage during the response. These models are however cumbersome, and relatively complex, particularly if only the ultimate limit state characteristics are of interest.

In this paper, a simple and computationally efficient analytical model, based on strain compatibility approach, is proposed to evaluate the nominal flexural strength characteristics of concrete members prestressed using external tendons. The analysis is based on idealized curvature distribution along the member length and makes use of the requirements of force and moment equilibrium as well as compatibility between the strain in the external tendons and their elongation between the anchorage ends at nominal strength. The analysis takes into account the reduction in the depth of the external

tendons with increasing member deformation to failure, and considers most of the important parameters that influence the response. These include: areas of external prestressing steel and internal bonded reinforcement, geometry of applied load, and span-depth ratio of the member.

The validity and accuracy of the proposed strain compatibility analysis were verified by comparing with experimental results of externally prestressed members reported in the literature. The stresses in the external prestressing steel and nominal moment capacities predicted by the analysis were in very good agreement with the experimental results. Also, with its simplicity in application and considerable efficiency in computation, the discrepancy between the results predicted by the analysis and the experimental results is quite identical to the discrepancy that could be obtained using more elaborate, and presumably more accurate, nonlinear analysis methods developed in the technical literature.

A limited parametric study was undertaken using the strain compatibility analysis to evaluate the effect of several design variables on the stress response and nominal moment capacity of externally prestressed members. These include content of tension reinforcement or reinforcing index (between 0.1 and 0.35); geometry of applied load (two-third point loads, uniform load, single concentrated load); deviator configuration (undeformed tendons, deformed tendons); and span-depth ratio of the member (between 10 and 50).

Based on the results of the parametric evaluation, it was found that irrespective of the geometry of applied load, the stress increase  $\delta f_{ps}$  in the external prestressing steel (above effective prestress) decreases with increasing content of tension reinforcement or reinforcing index as expected. However, the magnitude of  $\delta f_{ps}$  is very much dependent on the length of plastic region of the member expected to develop at nominal flexural strength. Accordingly, two-third point loads and uniform loads produce significantly larger  $\delta f_{ps}$  in comparison with single concentrated loads, particularly at low level of reinforcing index. Also, within the practical range of reinforcing index, the magnitude of the nominal steel stress  $f_{ps}$  remains below yield, which is similar to the behavior of beams with internal unbonded tendons.

Because the depth of external undeformed tendons decreases with increasing beam deflection to failure, beams with undeformed tendons mobilize significantly lower  $\delta f_{ps}$  and also nominal moment capacity  $M_n$  as compared to beams with deformed tendons. Since the nominal deflection produced by two-third point loads is considerably larger than that produced by single concentrated load (larger equivalent plastic region length), the reduction in steel stress and nominal moment capacity associated with the use of undeformed tendons in comparison with deformed ones are much more substantial for two-third point loads in comparison with single concentrated loads.

Also, as the member span-depth ratio increases,  $\delta f_{ps}$  decreases, which is similar to the behavior of beams with internal unbonded tendons. However this trend is negligible for beams loaded with two-third point loads, and it tends to be most significant for beams loaded with single concentrated load; reinforced with low reinforcing index; and having span-depth ratios less than about 20.