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Autor: Mola, Franco

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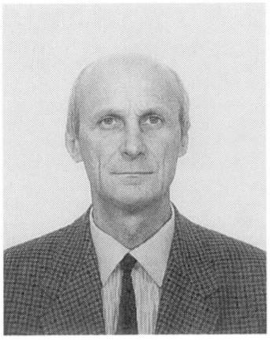
Model Code Format for long term analysis of R.C. and P.C. structures

Franco MOLA
Structural Civil Engineer
Politecnico di Milano
Milano, Italy



Franco Mola, born 1946, received his civil engineering degree from Politecnico di Milano in 1971. He is currently professor of Theory and Design of R.C. and P.C. Structures, Department of structural Engineering, Politecnico di Milano.

Summary

A Code Format approach for the long term analysis of concrete structures is presented. The formulation is based on the algebraic form of creep constitutive law recommended by CEB. In order to make feasible for practical applications the CEB algebraic law, the statements of a basic theorem proven by the author are applied. In this way a simple procedure exhibiting high accuracy and requiring only elastic calculations is derived. The application of the proposed procedure to the long term analysis of P.C. sections subjected to prestressing, external loads and shrinkage allows to derive feasible design formulas easily implementable in a Code Format.

1. Introduction

In CEB – FIP Model Code 90 /1/ the new CEB creep model has been introduced. This model, based on a product form of the creep function, shows a good agreement with experimental data and the related mathematical formulation is rather similar to the one pertaining to ACI model, /2/. The direct application of CEB MC90 model to structural analysis is quite complex as it requires the solution of systems of Volterra integral equations, making the use of general purpose computer programs mandatory. In Europe, after the publication of CEB MC90 some Design Aids have been prepared, /3/, in order to give to practitioners basic information regarding the correct way of approaching long term structural analysis, albeit for very simple structural arrangements. In order to investigate more complex systems without recurring to heavy calculations the algebraic form related to the so-called Age Adjusted Effective Modulus Method (AAEMM) has been proposed. The application of AAEMM drives to an elastic formulation including an imposed deformation linearly depending on the initial one, so this method cannot be directly applied by using current computer programs of structural analysis. In fact in their most popular configuration these programs can accept only inelastic imposed deformations. In searching for the possibility of directly apply AAEMM when using general computer programs, the author has proven that the solution can be obtained by combining three particular elastic solutions using a convenient time variable combination coefficient.
In this way AAEMM can be immediately implemented on general purpose computer programs as no particular imposed deformations are needed. In the present work a detailed discussion about this new way of proceeding is presented and the application to the long term analysis of P.C. sections will be developed, deriving simple Code Format design formulas.

2. General formulation of concrete viscoelastic law

According to McHenry Principle of Superposition /4/, the uniaxial creep stress-strain law for concrete can be written in the following form

$$\varepsilon(t) = \int_0^t d\sigma(t')\, J(t,t') + \bar{\varepsilon}(t) \qquad \sigma(t) = \int_0^t d(\varepsilon(t') - \bar{\varepsilon}(t'))\, R(t,t') \qquad (1)$$

with J(t,t') creep function, $\bar{\varepsilon}(t)$, imposed deformation, R(t,t') relaxation function. Introducing the creep coefficient $\varphi(t,t')$ representing the ratio between the delayed deformation due to creep and the initial elastic one, we can write

$$J(t,t') = 1/E(t')\,(1 + \varphi(t,t')) \qquad \int_0^t \frac{\partial\varphi(\tau,t')}{\partial\tau} R(t,\tau)\, d\tau = E(t') \qquad (2)$$

The application of eqs. (1) to structural analysis using the Force Method leads to systems of Volterra integral equations requiring appropriate algorithms for their solution. These algorithms are based on iterative procedures to be implemented in computer programs so they cannot be directly incorporated in a Model Code Format. A simplified form of the first of eqs. (1), feasible for the engineer approach to structural problems and exhibiting good reliability, is the following algebraic relationship stated by Trost /5/

$$\varepsilon(t) = \sigma(t)\,[1 + \chi(t,t_0)\,\varphi(t,t_0)]/E(t_0) + \sigma(t_0)\,\varphi(t,t_0)\,[1 - \chi(t,t_0)]/E(t_0) + \bar{\varepsilon}(t) \qquad (3)$$

$$\chi(t,t_0) = 1/(1 - R(t,t_0)/E(t_0)) - 1/\varphi(t,t_0) \qquad (4)$$

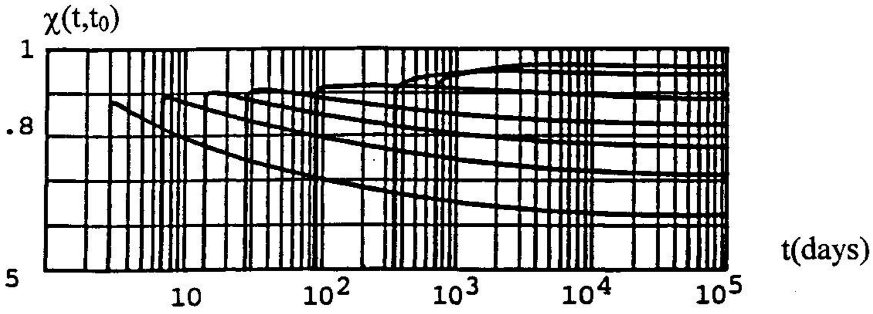


Fig.1 Ageing coefficent $\chi(t,t_0)$ according to CEB model

In fig.1, the ageing coefficent χ, calculated according to the CEB creep model /1/, is reported for particular values of the main parameters, namely the concrete strength f_{ck}, the notional thickness h_0 and the relative humidity RH. The χ coefficient lies at the interior of the interval $0{,}5 \leqslant \chi \leqslant 1$ and when considering time intervals t – t' sufficiently large, we can assume $\chi = 0{,}8$ as suggested by CEB MC90 for preliminary solutions.
Eq. (3) is recommended in /6/ for the long term analysis of R.C. and P.C. structures, nevertheless no indications are given about the most convenient way for applying it to typical problems of engineering practice. Furtherly no consideration is made about the reliability levels reached when using eq. (3). These two basic aspects will be discussed in the next section.

3. Algebraic approach to long term structural analysis

R.C. and P.C. structures can be regarded as homogeneous viscoelastic structures elastically restrained, so, indicating by $\underline{X}(t)$ the vector of the redundant elastic restraints, the application of eq. (3) drives to the following compatibility system of algebraic equations

$$[\underline{F}_c(t_0)\,(1+\chi\varphi) + \underline{F}_s]\,\underline{X} + \underline{F}_c(t_0)\,\varphi(1-\chi)\,\underline{X}_0 = -\,\underline{\delta}_g(1+\chi\varphi) - \underline{\delta}_{0g}\varphi(1-\chi) - \bar{\underline{\delta}} \qquad (5)$$

where $\underline{F}_c(t_0)$, $\underline{F}_s$ are the elastic deformability matrices of the viscoelastic part and of the elastic restraints and $\underline{\delta}_g$, $\bar{\underline{\delta}}$ are the vectors of the relative displacements produced by external loads and imposed deformations. At initial time $\varphi = 0$, so eq. (5) becomes

$$[\underline{F}_c(t_0) + \underline{F}_s]\ \underline{X}_0 = -\ \underline{\delta}_{0g} - \overline{\underline{\delta}}_0 \tag{6}$$

Combining eqs. (5), (6) we finally obtain

$$[\underline{F}_c(t_0)(1+\chi\varphi) + \underline{F}_s]\underline{X} = -\ \underline{\delta}_g(1+\chi\varphi) - \underline{\delta}_{0g}\varphi(1-\chi) - \overline{\underline{\delta}} + \underline{F}_c(t_0)[\underline{F}_c(t_0) + \underline{F}_s]^{-1}\cdot[\underline{\delta}_{0g} + \overline{\underline{\delta}}_0]\cdot\varphi(1-\chi) \tag{7}$$

Applying the principle of superposition the solution of the algebraic system (7) can be expressed in the following form

$$\underline{X} = \underline{X}_1 + \underline{X}_2 \tag{8}$$

with $\underline{X}_1$, $\underline{X}_2$ partial solutions satisfying the subsequent systems

$$[\underline{F}_c(t_0)\ (1+\chi\varphi) + \underline{F}_s]\ \underline{X}_1 = -\ \underline{\delta}_g(1+\chi\varphi) - \overline{\underline{\delta}} \tag{9}$$

$$[\underline{F}_c(t_0)\ (1+\chi\varphi) + \underline{F}_s]\ \underline{X}_2 = \varphi(1-\chi)\ \{-\ \underline{\delta}_{0g} + \underline{F}_c(t_0)\ [\underline{F}_c(t_0) + \underline{F}_s]^{-1}\cdot[\underline{\delta}_{0g} + \overline{\underline{\delta}}_0]\} \tag{10}$$

It is immediate to observe that the solution of system (9) is very simple as it coincides with the elastic one assuming for concrete the varied modulus $E' = E_c(t_0)/(1+\chi\varphi)$. On the contrary the solution of eq. (10) is quite involved as the known term at right member has to be calculated in advance and cannot be directly introduced in a standard computer program for structural analysis. For this reason the algebraic form (3) of the creep constitutive law cannot be directly incorporated in a Model Code Format. By means of a basic theorem, recently proven /7/ /8/, eq. (10) can be reduced to a very simple form leading to a general solution obtained by solving three elastic problems using standard computer programs. As will be shown in detail in the following, this allows to consistently incorporate the algebraic form (3) in a Model Code Format.

4. The basic theorem governing long term algebraic structural analysis

Let us consider eqs. (9) (10). As previously said eq. (9) can be solved by means of an elastic analysis performed at time t, assuming for concrete the varied modulus E'. Regarding eq. (10), according to eq. (5), we write it in the following form

$$[\underline{F}_c(t_0)\ (1+\chi\varphi) + \underline{F}_s]\ \underline{X}_2 = -\ \underline{\delta}_{0g}\ \varphi(1-\chi) - \underline{F}_c(t_0)\ \varphi(1-\chi)\ \underline{X}_0 \tag{11}$$

For the solution of eq. (11) we assume the subsequent expression

$$\underline{X}_2 = \underline{X}' + \mu\underline{X}_0 \tag{12}$$

with $\underline{X}'$ unknown vector and μ unknown coefficient depending on t, t_0. Substituting eq. (12) in eq. (11) we obtain

$$[\underline{F}_c(t_0)\ (1+\chi\varphi) + \underline{F}_s]\ (\underline{X}' + \mu\underline{X}_0) = -\ \underline{\delta}_{0g}\ \varphi(1-\chi) - \underline{F}_c(t_0)\ \varphi(1-\chi)\ \underline{X}_0 \tag{13}$$

adding and subtracting at second member the quantity $\mu[\underline{\delta}_{0g}(1+\chi\varphi)+\overline{\underline{\delta}}_0]$, we see that eq. (13) can be splitted in the two following systems

$$[\underline{F}_c(t_0)\ (1+\chi\varphi) + \underline{F}_s]\ \underline{X}' = \mu[\underline{\delta}_{0g}\ (1+\chi\varphi) + \overline{\underline{\delta}}_0] \tag{14}$$

$$[\underline{F}_c(t_0)\underline{X}_0 + \underline{\delta}_{0g}]\ [\mu\chi\varphi + \varphi(1-\chi)] = \underline{0} \tag{15}$$

Indicating by X_{10} the elastic solution obtained assuming the initial values of the applied actions and concrete modulus E', the solution of eq. (14) can be immediately written in the following form

$$\underline{X}' = -\ \mu\underline{X}_{10} \tag{16}$$

Regarding eq. (15), we derive that it is satisfied when assuming for μ the following expression

$$\mu = -(1-\chi)/\chi \tag{17}$$

Consequently, the general solution of eq. (5) can be put in the following compact form

$$\underline{X} = \underline{X}_I + \mu(\underline{X}_0 - \underline{X}_{I0}) \tag{18}$$

In particular, for actions constant in time we have $X_I = X_{I0}$, so eq. (18) assumes the simpler form

$$\underline{X} = \underline{X}_I(1-\mu) + \mu\underline{X}_0 \tag{19}$$

The two relationships (18) or (19) together with eq. (17) allow to reduce the long term analysis of concrete structures to the superposition of three elastic analyses so that they can be easily incorporated in a Code Format.

5. Applications and worked examples

By means of the first numerical application we want to investigate about the accuracy of eqs. (18) (19). At this scope let us consider the basic problem regarding the calculation of the evolution in time of the state of stress in the prestressed tie of fig. 2.

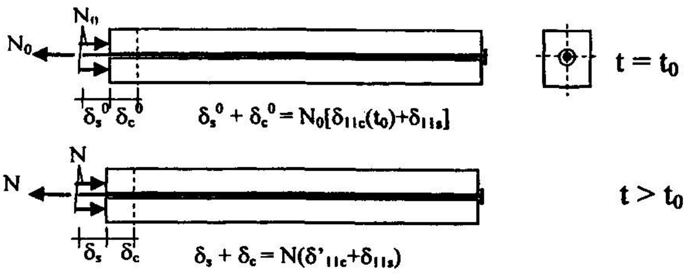


Fig. 2 Prestressed tie

Indicating by N_0 the initial prestressing force and by N the prestressing force at time t, the exact solution of the problem is obtained by solving the following integral equation

$$\int_0^t dN(t')\,(\delta_{11c}(t_0)\cdot E_c(t_0)\,J(t,t') + \delta_{11s}) = N_0\,(\delta_{11c}(t_0) + \delta_{11s}) \tag{20}$$

with $\delta_{11c}(t_0) = 1/(E_c(t_0)A_c)$, $\delta_{11s} = 1/(E_sA_s)$

Introducing the coupling coefficient

$$\omega = \delta_{11c}(t_0)/(\delta_{11c}(t_0) + \delta_{11s}) = (1+1/(n\rho_s))^{-1} \quad n = E_s/E_c(t_0),\ \rho_s = A_s/A_c \tag{21}$$

and putting $\xi_N(t) = N(t)/N_0$, from eq. (20) we derive

$$\int_0^t d\xi_N(t')\,[\omega E_c(t_0)\,J(t,t') + 1 - \omega] = 1 \tag{22}$$

Applying the approximate form (19) we on the contrary obtain

$$N_I(\delta_{11c}(t_0)\,(1+\chi\varphi) + \delta_{11s}) = N_0(\delta_{11c}(t_0) + \delta_{11s}) \tag{23}$$

and remembering eq. (21), for the ratio $\xi_{IN} = N_I/N_0$ we write

$$\xi_{IN} = 1/(1+\chi\omega\varphi) \tag{24}$$

At initial time $N(t_0) = N_0$, $\xi_{0N} = 1$, so eq. (19) gives

$$\xi_N = \xi_{1N}(1-\mu) + \mu\xi_{0N} = (1+\mu\chi\omega\varphi)/(1+\chi\omega\varphi) \qquad (25)$$

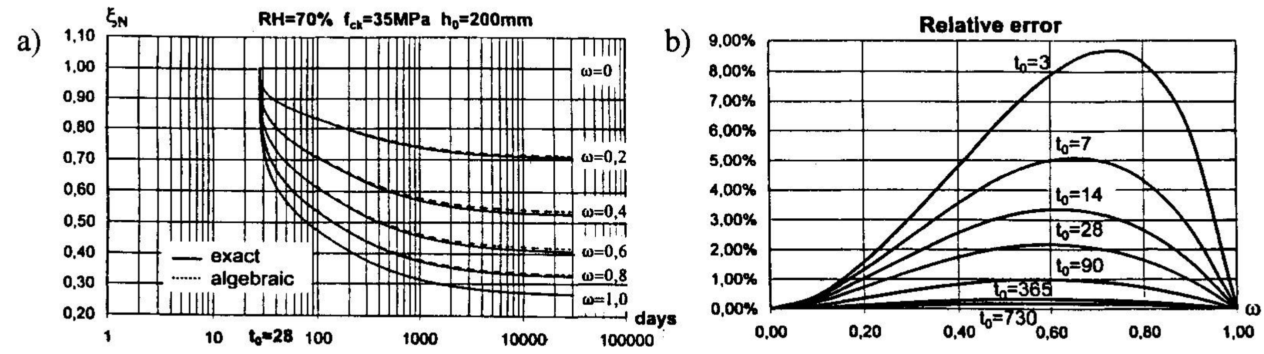


Fig.3 a) Function $\xi_N(t,t_0)$, b) Relative error be tween exact and approximate solution of eq.(22)

In fig. 3a) the exact solution of eq. (22) and the approximate one given by eq. (25) are reported versus time assuming ω as parameter. The high accuracy of eq. (25) is quite evident and this is emphasized by the graphs of fig. 3b) representing the relative error between the two solutions for $t\to\infty$ assuming ω as independent variable. The relative error does not exceed 9% and for prestressed elements, connected to small values of ω ($\omega \leq 0{,}08 \div 0{,}10$), the error is lesser than 2%. The algebraic approach stands very reliable for the analysis of prestressed elements so we can try to derive a feasible Code – Format design formula of general purpose. At this scope let us consider the prestressed section of fig. 4, subjected to an initial prestressing force N_0, a permanent bending moment M_g and shrinkage deformation ε_{cs}. Applying eq. (19) and the superposition principle we see that the prestressing force varies according to eq. (25) when introducing for the parameter ω the following more general expression

$$\omega = (1 + e^2/r^2)/(1 + e^2/r^2 + 1/n\rho_s) \qquad (26)$$

where e is the eccentricity of the cable and r is the gyration radius of the section. Regarding M_g and ε_{cs}, they do not produce stresses in the cable at initial time so we have only to calculate the related effects at time t by means of the two following relationships

$$N_{1g}(\delta'_{11c} + \delta_{11s}) = M_g e\varphi/(E_c(t_0)A_c r^2) \qquad N_{1cs}(\delta'_{11c} + \delta_{11s}) = \varepsilon_{cs} \qquad (27)$$

Remembering that $\delta_{11c}(t_0) = (1 + e^2/r^2)/(E_c(t_0)A_c)$ and introducing the following quantities

$$c = e^2/r^2, \qquad \alpha_g = M_g/N_0\, e, \qquad \alpha_{cs} = \varepsilon_{cs}\, E_c(t_0)A_c/N_0 \qquad (28)$$

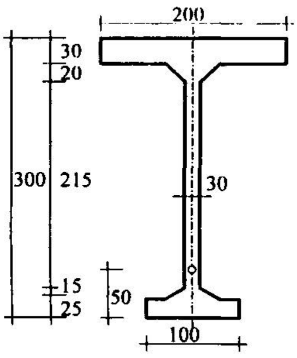
the superposition of ξ_N given by eq. (25) and of the solutions of eqs. (27), according to eq. (19) gives the following general design formula

$$N = N_0\,[1+ \mu\chi\omega\varphi + \alpha_g\omega c\varphi/(1+c) + \alpha_{cs}\omega/(1+c)]/(1+ \chi\omega\varphi) \qquad (29)$$

Eq. (29) allows to obtain by means of simple calculations the final state of stress in the prestressing cable taking into account the mutually interacting effects connected to prestressing, external loads and shrinkage.
Introducing the numerical values of fig. 4 we immediately obtain

$$N/N_0 = (1-0{,}43\cdot0{,}7\cdot0{,}063\cdot2{,}6+0{,}707\cdot0{,}063\cdot0{,}595\cdot2{,}6-1{,}99\cdot0{,}063/2{,}467)/(1+0{,}7\cdot0{,}063\cdot2{,}6) =$$
$$= 0{,}853 + 0{,}062 - 0{,}046 = 0{,}87$$

We see that the total reduction of the initial stress is 13%. This value derives from the superposition of three contributions, namely a reduction of 14,7% connected to prestressing, an increase of 6,2% due to external load and a decrease of 4,6 due to shrinkage. Eq. (29) exhibits simplicity in use, detailed quantification of the various effects and high accuracy. Remembering that it derives from the superposition of simple elastic calculations we can conclude that it can be recommended as reliable Model Code Format design formula of general use for long term analysis of prestressed sections.



A_c=16500cm^2; e=129.47cm; J_c=188.32·10^{6}cm^4;

r=106.83cm; c=1.467; c/(1+c)=0.595;

A_s=83.4cm^2; ρ_s=0.00505; $E_c(t_0)$=36.16GPa;

ε_{cs}=-40·10^{-5}; n=5.4; ω=0.063; N_0=12000kN;

M_g=10980kNm; α_g=0.707; α_{cs}=-1.99; φ=2.6;

χ=0.7; μ=-0.43

Fig. 4 Prestressed post-tensioned section

6. Conclusion

The algebraic approach to long term structural analysis of concrete structures drives to feasible Code Format solutions when applied in the form allowing to consider in an indirect way the effect of the initial deformation. The proposed procedure, requiring to superimpose three elastic solutions obtained referring to the actual modulus of concrete or to a reduced value of it, does not introduce the initial deformation as an imposed one, so it can be easily operated recurring to usual computer programs dealing with elastic analysis. Further works devoted to the analysis of more complex systems have to be programmed in order to extended the basic principles now discussed, allowing the practitioner to approach in a rational and reliable way the long term structural analysis of concrete structures. In particular the analysis of structures incorporating two rheologically nonhomogeneous concrete parts and elastic parts like precast beams collaborating with cast in situ slabs is of significant interest.

7. References

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