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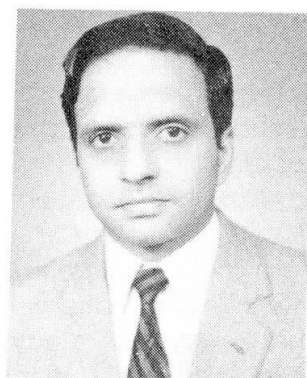
## Mathematical Modelling for Scour Around Bridge Piers

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### ABSTRACT

Many design relationships for scour are available as a result of studies carried out so far by several researchers. Lacey- Inglis method is mostly used for estimation of design scour depth in Indian Railways. However, the scour depth as estimated by this or many of the other methods is expected to occur after a very long time of scour activity. However, the flood discharge which is used for the estimation of design scour depth does not last that long. Thus, realistic estimation of scour depth does need the scour modelling for estimation of temporal variation of scour depth and the equilibrium scour depth. In the present paper a mathematical model is discussed for the estimation of temporal variation of scour depth and the equilibrium scour depth. Limitations of the Lacey-Inglis method for scour estimation in alluvial and boulder bed rivers are identified. Also the results available on effect of bed material cohesion on scour are reported.



## INTRODUCTION

Local scour around bridge piers and abutments is a problem of continuing interest. Huber (1991) has reported that since that year 1950, over 500 bridges have failed in the USA and a majority of them were the result of hydraulic conditions and primarily the scour of foundation material. Such data are not available for bridges of other countries including India. However, this has remained a matter of concern for all and the realistic estimation of scour depth around bridge piers is thus very important.

It is now well known that for given flow conditions scour at bridge abutments is smaller than that at bridge piers of same dimensions. Several design relationships for estimation of scour are available as a result of studies carried out so far by numerous investigators. These methods are useful in determination of the design scour depth at a bridge pier for steady flow conditions. However, the flow in a river during a flood is unsteady and discharge changes in it are quite rapid. Therefore, scour modelling in unsteady or flood flows is required for realistic estimation of the scour depth.

In the present paper, first a mathematical model for the estimation of temporal variation of scour depth in steady and unsteady flows is discussed. Use of this model for scour estimation under flood flow situations is demonstrated. Next, some available methods for estimation of design scour depth are reviewed. The processes of scour in cohesive and boulder bed rivers are also discussed.

## TEMPORAL VARIATION OF SCOUR DEPTH

The time required by the design discharge to scour to its full potential is generally much larger than the time for which it occurs. Therefore, computations on temporal variation of scour depth are important for design purposes. The horse-shoe vortex and associated downflow are considered to be the main agents causing scour at bridge piers. Based on the characteristics of horse-shoe vortex (see Fig. 1) a scheme is proposed by Kothyari et al. (1992, a & b) for computation of temporal variation of scour depth. Various steps involved in computations through this scheme for uniform size bed material are depicted in Fig. 2. The results produced through this scheme have been verified using experimental data of various investigators. However, river bed material invariably consists of sediments of varying size fractions i.e. it is non-uniform. The scheme depicted in Fig. 2 can also be used for computation of temporal variation in non-uniform sediments. For this the effective size  $d_e$  is defined as the size of uniform sediment that gets scoured at the same rate as the non-uniform material under the given flow and pier conditions. Kothyari et al. (1992, a) gave the following equation for  $d_e$ ,

$$\frac{d_e}{d_{50}} = 0.925 \sigma_g^{0.67} \quad (1)$$

Here  $d_{50}$  is the sediment size of non-uniform material such that 50 percent material is finer than this by weight and  $\sigma_g$  is the geometric standard deviation of the material. Use of  $d_e$  as given by Eq. (1) in place of  $d$  in Fig. 2 will give the temporal variation of scour depth in non-uniform sediments.

For computation of temporal variation of scour due to unsteady flows, the hydrograph causing unsteadiness can be discretised into steady segments as shown in Fig. 3. The scour depth is computed as per Fig. 2 in each segment of flow. Scour depth at the end of preceding segment becomes equal to the scour depth in the beginning of the next segment. Comparison of results obtained with the experimentally observed data is also given in Fig. 3.

## EQUATIONS FOR PREDICTION OF EQUILIBRIUM SCOUR-DEPTH

The equilibrium scour depth can be obtained from the temporal variation as the scour at a large time. It is, however, desirable from practical considerations to have a simple relationship for equilibrium



scour depth. Available equations for equilibrium scour depth are expected to provide conservative estimates of the scour depth in unsteady flows. A brief description of important equations is given below :

### **Lacey -Inglis Equation**

Lacey -Inglis approach for scour estimation is used by Indian Railways and other government organizations. During the early part of the present century, Lacey (1929) analyzed the data of stable irrigation canals flowing through loose noncohesive sandy materials in Indo-Gangetic plains and obtained the following equation for flow depth (or hydraulic radius)  $D_{LQ}$

$$D_{LQ} = 0.47 (Q/f)^{0.33} \quad (2)$$

Here  $Q$  is the discharge in  $m^3/s$ ,  $D_{LQ}$  is the depth in m and  $f$  is Lacey's silt factor which is related to median size of the bed material  $d$  as below.

$$f = 1.76 \sqrt{d} \quad (3)$$

Here  $d$  is in mm. On the basis of analysis of scour data on 17 bridges in alluvial rivers in North India, Inglis (1949) found that the maximum scour below the water level  $D_{se}$  is related to computed value of  $D_{LQ}$  as

$$D_{se} = K D_{LQ} \quad (4)$$

where  $K$  varies from 1.76 to 2.59 with an average value of about 2.0 . When bridge pier foundations are to be designed, this equation will be used for a flood discharge of return period 50 to 100 years, even though Eq. (4) is at best valid for bankful discharge. Also value of coefficient  $K$  in Eq. (4) should depend upon pier shape and size, sediment gradation, obliquity of flow etc. Since these factors are not explicitly taken into account, Lacey-Inglis method should not be used outside the range of data on which it is based.

### **Laursen-Toch Equation**

The equation proposed by Laursen and Toch (1956) for prediction of equilibrium scour depth below river bed level  $d_{se}$  is give as below.

$$d_{se}/D = 1.35 (b/D)^{0.70} \quad (5)$$

Here  $D$  is flow depth and  $b$  is pier diameter.

### **Melville and Sutherland Equation**

Melville and sutherland (1988) assumed that the largest possible scour depth around bridge piers is given as below :

$$d_{se} = 2.4 b \quad (6)$$

This scour depth is reduced by multiplying factors which depend upon whether clear-water or live-bed conditions exist, flow depth is shallow and sediment is graded. The multiplying factors are determined from the analysis of experimental data covering a wide range of pertinent variables.



### **Kothyari -Garde - Ranga Raju's Method**

Based on the analysis of extensive laboratory data collected using uniform, non-uniform and stratified sediments and steady and unsteady flows, Kothyari et al. (1992, a & b ) have proposed the following equations for scour estimation

Clear- water condition :

$$\frac{d_{sc}}{b} = 0.66 \left(\frac{b}{d}\right)^{-0.25} \left(\frac{D}{d}\right)^{0.16} \left(\frac{U^2 - U_c^2}{\frac{\Delta\gamma_s}{\rho} d}\right)^{0.4} \alpha^{-0.30} \quad (7)$$

where the average critical velocity  $U_c$  is given by

$$\frac{U_c^2}{\frac{\Delta\gamma_s}{\rho} d} = 12 \left(\frac{b}{d}\right)^{-0.11} \left(\frac{D}{d}\right)^{0.16} \quad (8)$$

and opening ratio  $\alpha$  is given as  $\alpha = (B-b)/B$

Here  $U$  is flow velocity,  $\rho$  is mass density of water,  $\Delta\gamma_s = \gamma_s - \gamma_b$ ,  $\gamma_s$  and  $\gamma_b$  are specific weights of sediment and water respectively,  $\alpha$  is opening ratio and  $B$  is center to center spacing of piers.

Live-bed Condition :

$$\frac{d_{sc}}{b} = 0.88 \left(\frac{b}{d}\right)^{-0.33} \left(\frac{D}{d}\right)^{0.4} \alpha^{-0.3} \quad (9)$$

It may be seen that in sediment transporting flows, the scour depth is not dependent on velocity. When the sediment is non-uniform, effective sediment size  $d_e$  can be used in Eq. (7) and (9) instead of  $d$ , the former being given by Eq. (1).

Grade and Kothyari (1997) have tested the above methods for scour estimation by using field data from 17 bridges in India, 55 bridges in USA, 6 bridges in New Zealand and 5 bridges in Canada. Results obtained through these are summarized in Table - 1.

**TABLE 1: Comparison of Accuracy of Prediction of Scour Depth by Different Methods**

Methods	% of Data points falling within given error band		
	$\pm 30$	$\pm 50$	$\pm 90$
Lacey-Inglis	59	85	100
Laursen- Toch	38	65	98
Melville-Sutherland	79	95	100
Kothyari - Garde- Ranga Raju	86	96	100



It can thus be seen that among the methods tested, the methods by Melville-Sutherland and Kothyari et al. give results of better accuracy. These methods indeed take into account the effects of flow depth, velocity, pier shape and size and the size distribution of river bed material.

## SCOUR IN COHESIVE SOILS

When the river bed consists of clayey material, forces also act between soil particles imparting cohesion into it which resists the dislodgment of particles by the flow. Therefore, scour in cohesive materials is more complex and less understood than the scour in noncohesive sandy material. The rate and amount at which clayey material gets eroded due to flow depends upon type and percentage of clay, antecedent moisture conditions in clayey beds, quality of water etc. Some investigators have tried to relate the scour in cohesive soils to plasticity index, vane shear strength and other such properties, but these attempts are not very successful. Some basic work on scour in cohesive soils has been undertaken by the writers. Preliminary results reveal that for the given pier, scour in cohesive soils can even be more than that in cohesionless soils depending upon moisture state of the soil prior to the start of scouring, See Fig. 4 (Sarfaraz, 1998).

## SCOUR IN GRAVEL-BED RIVERS

Gravel-bed rivers are characterised by relatively large median size and large standard deviation. When a bridge pier is constructed in such strata, the coarser particles would accumulate in the scour hole thus forming an armor layer and partly inhibiting further development of scour. Hence scour depth obtained would be smaller than that in uniform material having the same  $d_{50}$  size.

The IRC-78 (1979) code recommends that scour depth in gravel-bed rivers be taken as a multiple of flow depth estimated by using Lacey-Inglis approach involving discharge intensity  $q$  (i.e. river discharge per unit width), as below:

$$D_{Lq} = 1.33 \left( \frac{q^2}{f} \right)^{1/3} \quad (12)$$

and a silt factor of 24. In this connection, it may be stated that basically, Inglis-Lacey relation for flow depth was derived for sandy bed rivers, and gravel bed rivers are not expected to follow the same. Published data of gravel bed rivers indicate the depth of flow relationship as below (Hey and Heritage, (1993)

$$D = a_0 Q^{a_1} d_{50}^{a_2} \quad (11)$$

Here, the coefficient  $a_1$  is found to vary between 0.33 and 0.49 and  $a_2$  between -0.03 and -0.12. This is thus different from Lacey's relation i.e. Eq. (11). Also bed material size of gravel bed rivers varies over a wide range and an armor layer may be formed during scour. Hence use of a constant silt factor of 24 in Eq. (11) is thus questionable. Also Lacey's method does not take into account the effect of pier shape and size on scour.

The methods of Kothyari -Garde- Ranga Raju and Melville-Sutherland, however take into account the effect of pier size, shape, sediment non-uniformity and hence armoring effects. Nevertheless, there is a need to collect scour data from gravel-bed rivers for studying the relative accuracy of available methods.

## CONCLUSIONS

Available methods for computation of design scour depth around bridge piers are reviewed. It is seen that the computation of temporal variation of scour depth is required for realistic estimation of design scour depth. Enough information is not found to exist on scour around bridge piers in clayey and boulder bed rivers. It is concluded that Lacey-Inglis method should be used for sand bed rivers only precisely within



the range for it was developed. This should be used with caution, if at all, in rivers with clayey or gravel beds.

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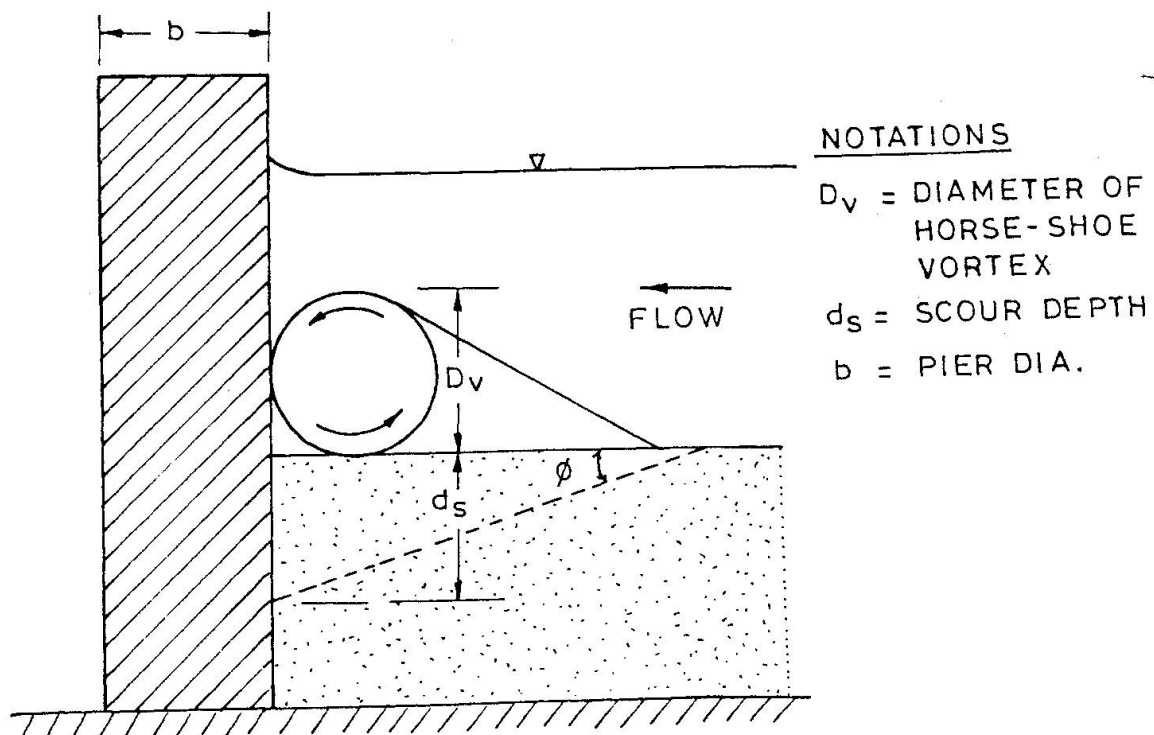


Fig.1 Definition diagram of horse-shoe vortex



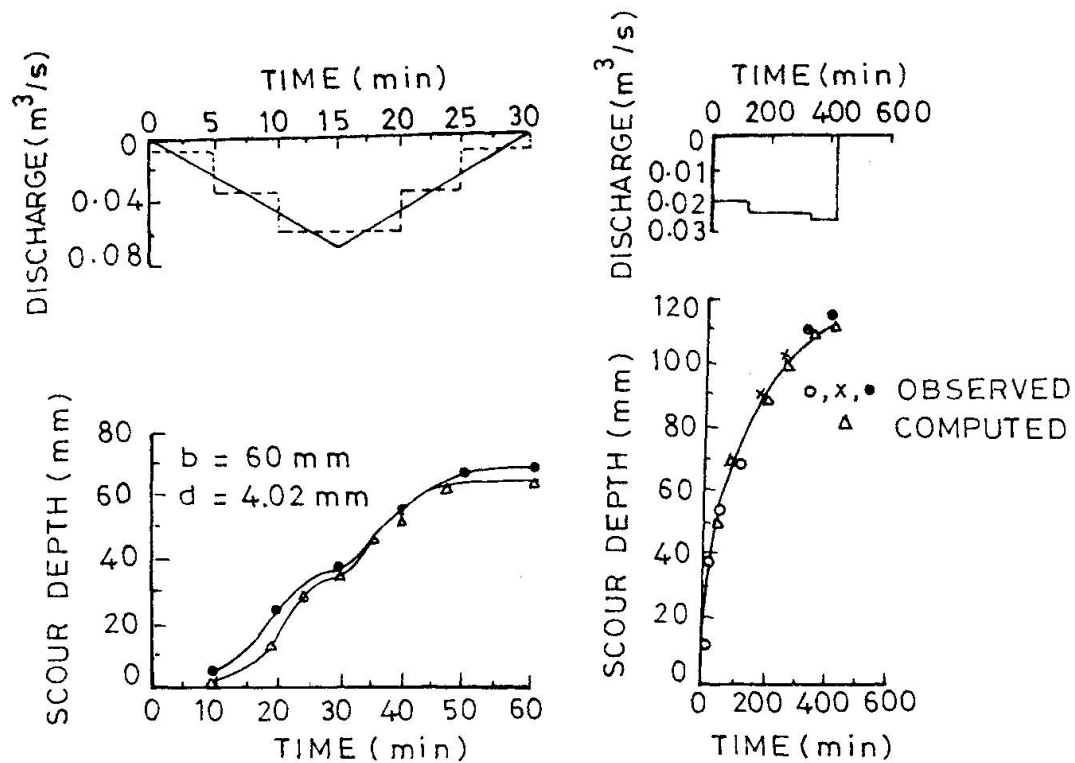


Fig. 3 Temporal variation of scour depth in unsteady flows

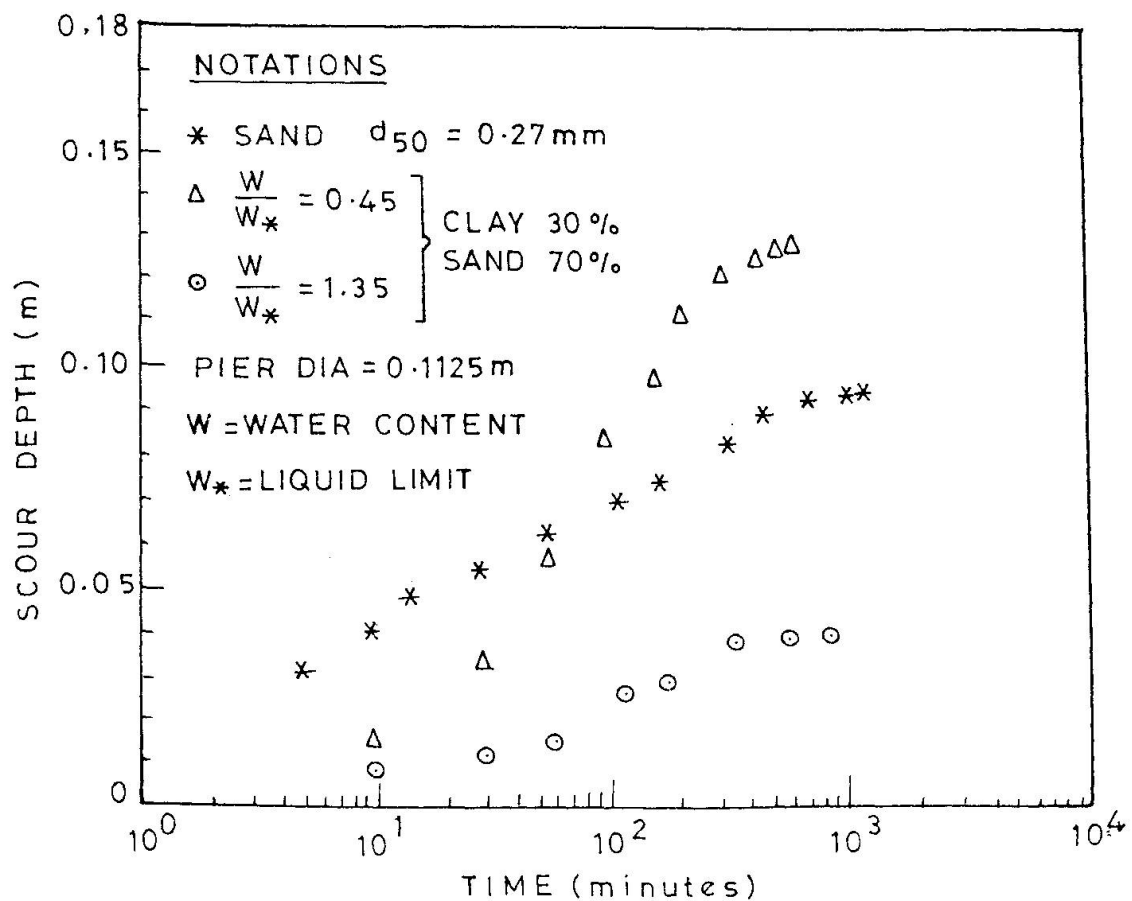


Fig. 4 Temporal variation of scour depth in cohesive soils



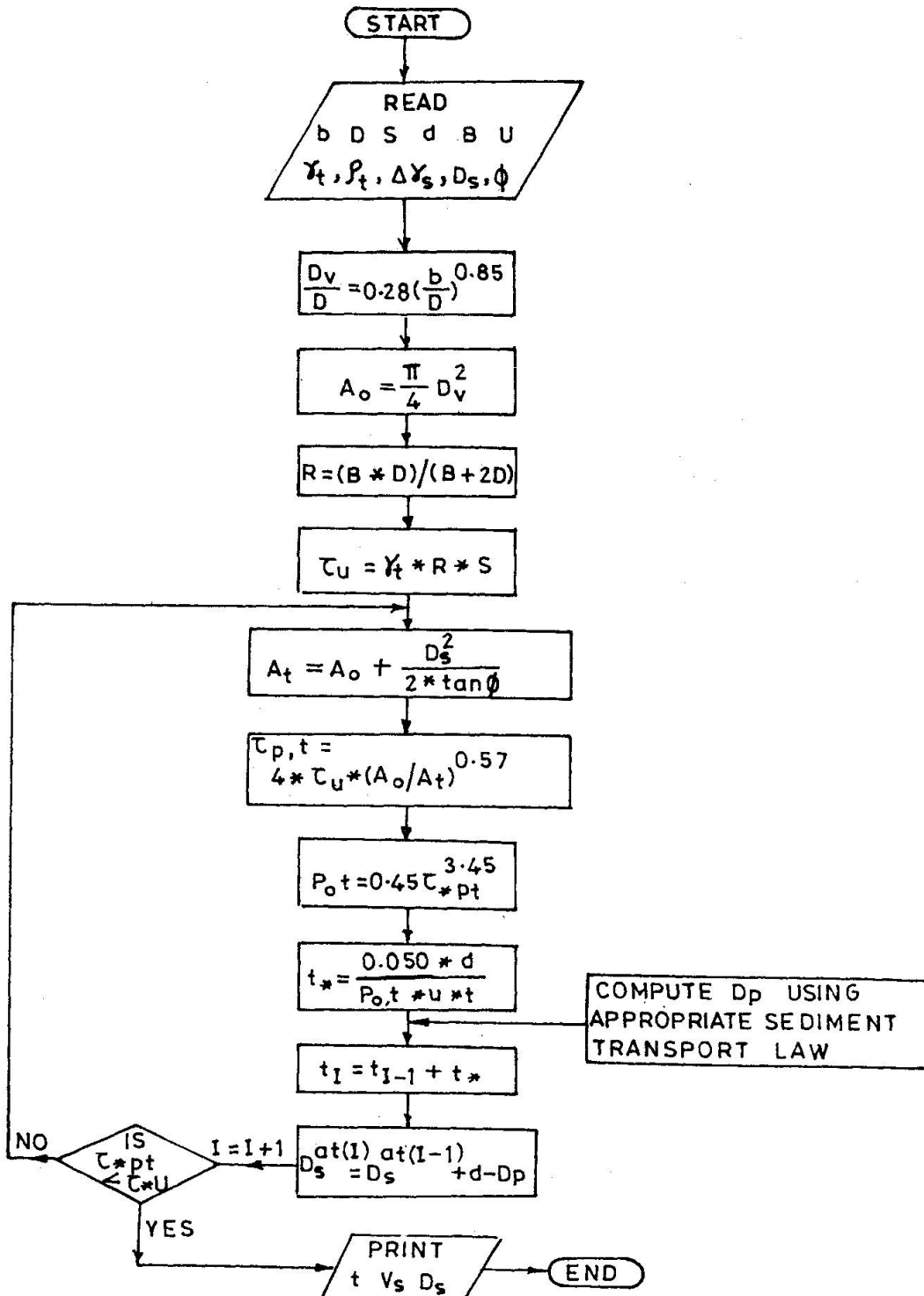


Fig.2 Algorithm for calculation of temporal variation of scour depth