Earthquake Response of Elevated Storage Tanks

Giuseppe DE MARTINO
Prof.
Naples Univ., Federico II
Naples, Italy

Giuseppe De Martino, born 1935, received his hydraulic eng. degree from Naples Univ. in 1962. He is currently Prof. of Hydraulic Constr. at Naples Univ. Federico II.

Maurizio GIUGNI
Assoc. Prof.
Naples Univ., Federico II
Naples, Italy

Maurizio Giugni, born 1953, received his hydraulic eng. degree from Naples Univ. in 1979. He is currently Assoc. Prof of Hydraulic Special Plants at Naples Univ. Federico II.

Summary

With a view to assessing the hydrodynamic effects caused by seismic motions on an elevated storage tank (a vessel fixed to the ground by a single vertical anchorage), we devised a mathematical model representing the tank as an elastic cantilever with a concentrated mass at the head. In our paper, we aim to give a brief outline of this model, corroborating it with additional results obtained from elevated, cylindrical cross-section tanks in concrete.

1. Introduction

Waterworks must always be provided with an earthquake-proof design as they frequently constitute a vital link in a community's lifeline and so must be kept in working order even during emergency situations. A designer engineer must therefore accurately assess:

- design response spectrum and hydrodynamic effects [1] [2] (needless to say, these should be added to inertial loads induced by structural masses);
- structural behaviour and a full analysis of construction details (for example, dampers) to help keep damage to a minimum.

In previous articles, we dealt with the technical problems arising from an assessment of the hydrodynamic effects caused by seismic motion on hydraulic works (e.g. [3]). With particular emphasis on liquid-containing tanks, we initially took a look at ground-supported, flat-bottomed tanks, assumed to follow a rigid pattern [4]. At a later stage, we tried to account for tank wall flexibility (liquid-shell coupling), which is responsible for considerably more dramatic consequences than those in rigid tanks [5] [6].

In this paper, we wish to dwell on the seismic analysis of an elevated storage tank: a vessel fixed to the ground by a single vertical anchorage. To this end, we have come up with a mathematical model representing the tank as an elastic cantilever with distributed mass and a head with a concentrated mass (the value varies according to the extent to which the tank is filled). We based ourselves on the hypothesis that the vessel (assumed rigid) and the liquid in it contained, both absorb energy generated by ground acceleration through the stiffness of the anchorage. As a consequence, the vessel's hydrodynamic response can be assumed comparable to that of a ground-surface tank, provided, however, that the ground acceleration value is replaced by the shell-liquid system acceleration. Our mathematical model therefore covers:

- preliminary structural design (vertical anchorage and vessel itself);
- dynamic component, to assess in the first instance the natural pulsation of the system and in the second the acceleration transmitted to the vessel;
- hydraulic component, to calculate the hydrodynamic effects on the vessel (pressures and

resultant thrust, slosh height).
Once the model has been adequately described, we will then proceed to list the results obtained for cylindrical cross-section, elevated tanks in concrete.

2. The liquid motion in the vessel

A variety of mathematical models (for example, Jacobsen, Housner, Bratu, Veletsos [4]) can be used to assess the hydrodynamic effects caused by an earthquake on a rigid container placed on the ground. Faced with the task of devising a mathematical model to assess the hydrodynamic effect on elevated water tanks, we chose the most broad-based, i.e. the Bratu model [7], which was originally developed for a rectangular water tank stressed by horizontal seismic acceleration acting in any direction relative to the wall. Bratu worked with standard and simplified hypotheses (listed below) which proved to be technically reliable on the whole, as experiments and research bear witness to [8]:

- rigid container and two-dimensional motion;
- homogenous, isotropic and non-viscous liquid;
- non-compressible liquid and negligible surface tension;
- irrotational motion of the elementary particles;
- low wave amplitude of the harmonic sort in the fundamental mode[1].

Using the Laplace equation solution combined with the above hypotheses, Bratu came up with the boundary conditions listed below:

- separation between the storage tank bottom and the liquid;
- water particles sticking to the tank walls as they move;
- lack of horizontal components in the gravitational force.

Bratu discovers the expression for velocity potential function ϕ and introduces into it seismic acceleration. At this stage, Bratu proceeds to write expressions for the contour of the liquid free surface, the hydrodynamic pressure distribution acting on the walls, which turns out to be parabolic in shape, and the resulting thrust. One point Bratu neglects to take into account, however, is the gravitational term gz when applying the linearized motion equation:

$$-\frac{\partial \phi}{\partial t}+\frac{p}{\rho}+g \cdot z=0 \tag{1}$$

The result of this, as can be gathered from a series of appropriate calculations carried out specially, is an irregularity in the pressure distribution. In fact, the distribution pattern takes on a value not equal to zero at the free surface, whereas, given the boundary conditions a zero value would have been expected. So much is this so that when the seismic period is shorter than that of the liquid sloshing (a frequent occurrence in real life) it takes on a negative value. It continues to be negative to a depth which varies according to the geometric features of the tank and the seismic period.
By applying formula (1) in its complete form, we were able to eliminate this distribution irregularity. We then proceeded to apply the Bratu method to cylindrical section containers (see fig 1): in this case, the radial symmetry of the tank rendered seismic acceleration direction irrelevant, thus simplifying the velocity potential expression [9]. With reference to the liquid fundamental sloshing mode, the following relationships have been obtained to assess the impulsive pressure distribution, p_w, the resulting thrust, S_w, and the peak rise, η, at the walls (see fig 1):

[1] The model actually allows us to account for a number of vibration modes without significantly influencing the results relating to the overall motion [4].

$$p_w(z) = \frac{4}{\pi} \cdot \gamma \cdot C \cdot R \cdot \frac{T_w^2}{T_s^2 - T_w^2} \left[\frac{\cosh \cdot k(z+H)}{\cosh \cdot kH} - 1 \right] \tag{2}$$

$$S_w = \frac{4}{\pi} \cdot \gamma \cdot C \cdot H \cdot R \cdot \frac{T_w^2}{T_s^2 - T_w^2} \cdot \left[\frac{\tanh(kH)}{kH} - 1 \right] \tag{3}$$

$$\eta = -\frac{4}{\pi} \cdot C \cdot R \cdot \left[1 + \frac{T_w^2}{T_s^2 - T_w^2} \right] \tag{4}$$

In the above relationships, the following symbols have been used:

- γ : specific liquid weight;
- C : seismic intensity coefficient (ratio between seismic and gravity acceleration);
- R: tank radius;
- H : water height;
- z : depth (from the liquid surface at rest);
- $k = \frac{\pi}{2R}$
- $T_w = \frac{2\pi}{\sqrt{gk \tanh(kH)}}$: fundamental period of liquid mass sloshing;
- $\frac{T_w^2}{T_S^2 - T_w^2}$:cyclic amplification factor, with T_s being the fundamental period of the earthquake for a tank placed on the ground or that of the structural system for elevated water tanks.

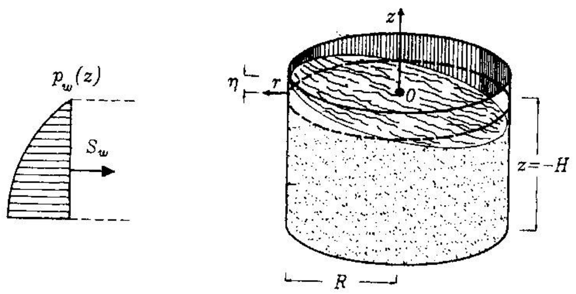

Fig. 1- Cylindrical tank: scheme of reference

The method above, unlike simpler ones, allows us to take into account the interaction between the T_s period and that of the liquid mass using the amplification factor in the relationships (2), (3) and (4). This way, the different hydrodynamic responses according to the different geometrical tank features and the point to which it is filled can be accurately monitored.

3. Elevated water tanks

First of all, it should be pointed out that the column-tank water system can be represented in two ways:

- elastic pattern, accounting for both the elasticity of the column and that of the tank;
- composite pattern, assuming that the column is elastic and the tank is rigid [2].

One must obviously take into account the structural features of the actual column-tank system being looked at: shape, building materials, and size of the various components. At this particular stage of the research project, we chose to look at an elevated water tank in ordinary reinforced concrete with a circular tank and a squared, cone-shaped connection fixed to the column so as to ensure a rigid anchorage (see fig 2). Bearing in mind, the structural features in question, we felt

[2] Only for squat water towers could we talk about a rigid pattern (non-elastic column-tank structure) with a dynamic behaviour similar to that of a monolithic block.

that we could safely overlook the tank elasticity (composite pattern) without significantly changing the outcome.

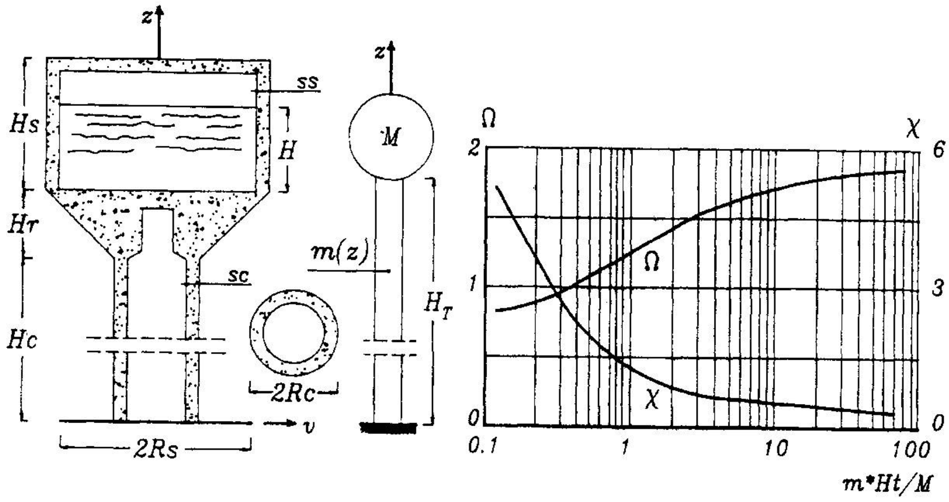

Fig. 2 - Elevated storage tank: system considered

In view of this, it should be noted that whilst a ground-surface tank is subjected to direct stress by seismic acceleration at the base, in an elevated tank the masses involved (support + container + liquid) absorb energy generated by the earthquake and transmitted from the foundation block by means of column elasticity. We can, therefore, assume that the liquid's hydrodynamic response is equivalent to that of a ground-surface tank provided we replace the ground acceleration with elevated storage tank values. In dynamic terms, we take our reference from a theoretical model of an elastic cantilever with distributed mass m(z) and a head with a concentrated mass M (see fig 2). The liquid has a fundamental period of T_w, as if it were resting on the ground. As explained earlier, the T_w expression wholly depends on the tank's geometrical features and on the quantity of liquid contained. The classical differential equation covering the free vibration problem for this sort of constant column system and the frequency equation are expressed in [10]:

$$EI \cdot \frac{d^4 v}{dz^4} - m \cdot \omega_s^2 \cdot v(z) = 0 \tag{5}$$

$$\Omega \cdot \frac{(\sin\Omega \cdot \cosh\Omega - \cos\Omega \cdot \sinh\Omega)}{1 + \cos\Omega \cdot \cosh\Omega} = \frac{m \cdot H_T}{M} \cong \frac{m \cdot (H_c + H_r + 0.5 \cdot H_s)}{M} \tag{6}$$

in which:

- $\Omega = H_T \cdot \sqrt[4]{\frac{\omega_s^2 \cdot m}{EI}}$
- ω_s : structural system frequency;
- M : liquid and tank (including the connecting element) mass;
- m : mass per unit length of the column;
- $H_T \cong H_c + H_r + 0.5 \cdot H_s$ (see fig 2);
- H_c: column height;
- H_r: height of the column-tank connection element;
- H_s: tank height;
- E : Young's elasticity modulus of the material;
- I : moment of inertia of the cross section of the column.

With reference to the fundamental mode, the values of Ω are shown as a function of mH_T/M in the diagram in figure 2; the same applies to the values of the relevant participation factor χ. The expressions relating to the buckling at the tank centre, $v(H_T)$, and to the corresponding inertial force, $F(H_T)$, have a value of:

$$v(H_T) = \sin\Omega - \sinh\Omega - \frac{\sin\Omega + \sinh\Omega}{\cos\Omega + \cosh\Omega}(\cos\Omega - \cosh\Omega) \tag{7}$$

$$F(H_T) = M \cdot \chi \cdot v(H_T) \cdot S_a \tag{8}$$

in which, S_a represents spectral acceleration, obtained from the design response spectrum and according to the natural period T_s and the damping factor in the structure under examination. As a result, the seismic intensity coefficient C for elevated water tanks to be introduced in equations (2), (3) and (4) is:

$$C_p = C \cdot v(H_T) \cdot \chi \tag{9}$$

which can be defined as the factor of structural amplification.

First of all, we concentrated our efforts on elevated water tanks in reinforced concrete with cylindrical vessels. This preliminary area of investigation put us in a position to analyse the different dynamic response of a range of column features (height, radius and width), different tank features (height and radius) and the extent to which it was filled.

Using the design response spectrum contained in current Italian standards and that of the 6/5/78 Friuli earthquake (the Tolmezzo Station one [11] with peak ground acceleration reaching 0.122 g) and damping factor of the structure equal to 2% or 5%, our calculations gave the following results:

- the vibration period of the structure T_s heavily depends on the column characteristics. More specifically, the less rigid the column, the higher the value of T_s; in other words, when the radius or the column thickness decreases or when the height is greater. T_s likewise increases, albeit to a lesser extent, along with the increase in water volume. An increase in the vibration period within the structure brings about an intensification of the cyclic amplification factor and consequently an increase in the hydrodynamic effects (thrust, water rise);
- for practical applications in this sort of elevated water tank T_s takes on a value ranging between $0.3 \div 0.5$ s and $1.2 \div 1.5$ s: the mathematical model does not cater for earthquake resonance, which is theoretically a possibility, nor does it take into account the subsequent intensification of hydrodynamic effects because of the dissipation effects of the ground-foundation-structure system;
- taking as a reference the Italian standard earthquake spectrum, the K ratio between the hydrodynamic and corresponding hydrostatic thrust takes on values ≤ 0.2 in practical applications for elevated water tanks of the sort being examined. If, on the other hand, we use the Friuli spectrum, K assumes much higher values, ranging between 0.20 and 0.45. For example, in figure 3 the K value, calculated according to the Friuli spectrum (with a 5% damping factor), is shown for a water tank with a radius of 5m and a height of 6.5m ; both the column height (between 10 to 30 m) and the extent to which it was filled H are varied. In order to provide figures for comparison, the diagram also shows the K value for ground-surface containers of the same size: obviously, as confirmed by other calculations, the hydrodynamic

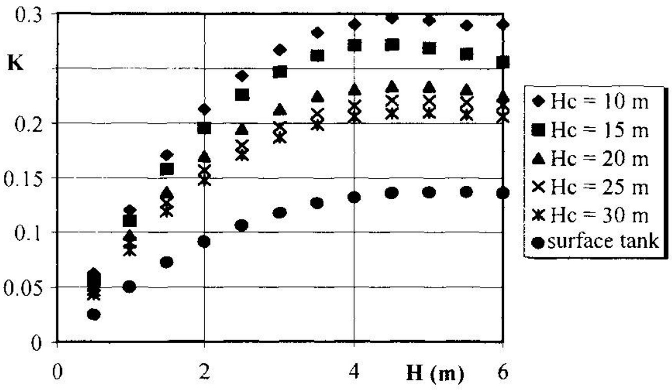

Fig. 3 - K ratio versus H for various Hc values

effects for elevated water tanks in earthquake zones are considerable greater than those for surface or underground tanks. This means that a meticulous assessment must be carried out of these effects;
- the water rise in elevated water tanks, albeit greater than those obtained for ground-surface tanks or underground tanks, is in any case quite small (it does not exceed 0.15÷0.20 m) and therefore, is most certainly contained within the freeboard - not under 0.50 m - generally allowed for in earthquake zones.

4. Conclusions

An analysis of the dynamic response of elevated water tanks using the mathematical model for systems in reinforced concrete and with a cylindrical vessel, will undoubtedly show us that hydrodynamic effects can reach very high levels and that in any case, they exceed those obtained for ground surface tanks of the same size by a long shot: therefore, any calculations must carefully account for them.
Research is currently underway. New materials (such as steel) and new design shapes are being looked at for both for the column and the tank. Moreover, the calculus model is being perfected to account for the dissipation effects of the ground-foundation-structure system. It would be interesting to research further into dissipation systems (e.g. dampers) with an eye to reducing even more the earthquake effects.

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