Simplified Model for Wind Speed/Height Relationship and Design of High Rise Structures

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Summary

A new simplified model for wind speed/height relationship is hereby reported in this technical paper. The applications of the model to the development of wind engineering codes and standards for the specifications of design wind speeds and dynamic wind pressures, for the structural design of high-rise structures, in different parts of the world, with different climatic conditions, are discussed briefly.

Introduction

The need to provide simplified but accurate models for extreme wind speed/height relationship is an important consideration in the development of appropriate design methodology in structural engineering design application. A new simplified model which can easily be linearized, in order to simplify computational work, has been developed and hereby presented in this technical paper. Having identified the wind profile for any topographical zone: namely, urban, semi-urban, or rural zone, and using appropriate field data, good statistical correlations have been established between the actual predictions of the model and recommended field data which can be used in structural engineering design.

THE NEW MODEL

Results of recent research are hereby presented for the characterization of wind speed/height relationship which can be used in structural design of high-rise structures. The new model gives results, which agree with Davenport's power laws model (1); which can be represented by:

$$y(h) = \frac{V(h)}{V(H_G)} = \left| \frac{h}{H_G} \right|^c \tag{1}$$

In equation (1), V(h) is the wind speed at a height h above the ground level, H_G is known as the gradient height. C and H_G are constants which depend on environmental surface roughness of any of the three topographical zones; namely (i) urban, (ii) semi-urban and (iii) rural. The variable, y(h), is a function of the height h, as shown in equation (1). The values of these constants are as follows (1):

	Zone	The Exponent C	Gradient Height, H_c (feet)	The Drag Coefficient, K
(i)	Urban	0.40	1700	0.050
(ii)	Semi-Urban	0.28	1300	0.015
(iii)	Rural	0.16	900	0.005

In general, wind speed is generally a function of height h and wind gust duration t; however at a given value of wind gust duration, the mathematical form of the new simplified model is as follows; for any particular zone:

$$y(h) = \frac{h}{a_1 h + b_1} = \frac{1}{a_1 + \frac{b_1}{h}} \tag{2}$$

Where a_1 and b_1 are constants for any particular zone. Equation (2) can be linearized to give the simple expression:

$$\left(\frac{h}{y(h)} \right) = a_1 h + b_1 \tag{3}$$

When multivariate regression analysis was carried out on the data of h/y(h) against h excellent correlations were recorded, thereby demonstrating the validity of the model of equation (2). Results of the multivariate regression analysis are as follows:

	Zone	a_1	b_1	Correlation Coefficient
(i)	Urban	0.9184	336.44	0.988
(ii)	Semi-Urban	0.9433	165.92	0.985
(iii)	Rural	0.9647	62.81	0.992

The data of y(h) were generated using equation (1) and the recommended values of C and H_G given above.

In the above analysis h is in feet. Equations (1) and (2) are indeed simple mathematical models, which can be linearized, and easily applied to the prediction of wind speed/height relationship, for a given wind gust duration t and for any of the three zones indicated. Using this model, the variations of wind speed/height relationships, for each of these three zones are as shown in Figure 1. These wind speed/height profiles are applicable to both normal and extreme wind speeds.

Some important characteristics of the model given by equation (2) are as follows: When h is large the value of $y(h)$ tends to a_1^{-1}. This means practically that when h approaches the gradient height H_{G1} where the gradient wind speed V_1 occurs, $y(h)$ becomes 1.0, when $a^{-1} = V_G$, the gradient wind speed V_G relevant to the zone. This aspect of the model agrees with the field data (3). Furthermore by differentiation, it can also be shown that the value of the slope d/dh $[y(h)]$ is b_1^{-1} when h=0.

STATISTICAL MODEL FOR EXTREME WIND SPEEDS

Using Type I Gumbel asymptotic distribution of extreme values in mathematical statistics, the maximum annual wind speed $V(h,t)$ at a given height h and gust duration t can be shown to have a cumulative distribution function, which can be expressed as (2):

$$G[V(h, t)] = exp \cdot -\alpha \, exp[-\beta V(h, t)] \cdot$$

(4)

Where α and β are constants which can be obtained from field data; by linearizing equation (4) to give:

$$ln(-lnG \, [v(h, t)]) = ln\alpha - \beta V(h, t)$$

(5)

By carrying out multivariate regression analysis on ln {-ln G [v(h,t)]} against $V(h,t)$ we can obtain the constants α and β. Having obtained α and β for any given location, the return period in years, T_V, for any annual maximum wind speed $V(h,t)$ can be expressed as (2) for the same location.

$$T_V = \frac{1}{\alpha} \, exp[\beta V(h, t)]$$

(6)

The values of the constants α and β for any given location can also be computed from the mean μ and the standard deviation σ of the recorded data of the annual maximum extreme wind speeds which are (2).

$$\mu = \frac{1}{\beta}(ln\alpha + 0.577) \qquad\qquad \sigma = \frac{1.282}{\beta} \qquad (7) \qquad (8)$$

From equations (7) and (8), the values of the constants α and β can be evaluated. In general, the annual maximum wind speed data for $V(h, t)$ are normally collected at height $h = h_1 = 33$ feet (10 m) at several meteorological stations. Equations (1) and (2) can be made use of in order to evaluate the corresponding maximum wind speed at any other height, for the same location, and for the same wind gust period.

In order to allow for the possible statistical variabilities in the maximum wind speeds V(h,t), it is necessary to evaluate the relationship between the intensity of wind turbulence, I_V, at a height h, and the normalized spectrum of wind turbulence evaluated at a height h_1 = 33 feet (10 m), as follows (1):

$$I_V = \frac{\sigma_V(h, t)}{V(h, t)} = 2.45 K^{\frac{1}{2}} \left(\frac{h}{h_1} \right)^{-\gamma}$$

(9)

where K is the drag coefficient, of the particular zone, as indicated earlier in this technical paper.

APPLICATIONS IN STRUCTURAL DESIGN

In practical design of high rise structures against wind loads the Codes of Structural Engineering Practices have always recognized the major variables which can account for design wind speed. A typical specification of design wind speed, V(h,t), at a given height h in meters and wind gust duration, t, is as follows (3):

$$V_D(h, t) = V_B(h_1, t) F_1 \times F_2(h, t) \times F_3$$

(10)

Where $V_B(h_1,t)$ is referred to as basic wind speed, which is the maximum wind speed, on the average, which occurs once in 50 years, (i.e. Tv = 50 years), see equation (6). F_1 is the design factor or constant due to topographic influences of the environment; $F_2(h,t)$ is a design factor or variable due to surface roughness of the environment, wind gust duration t appropriate to the size of the structure, and the height h of the structure and components above the ground level, where the wind loading is to be considered; F_3 is a design factor or constant due to the probabilistic considerations of the design life of the structure. Relevant tables (3) have also been provided in these Codes of Practices which had specified appropriate values of F_1, F_2(h,t) and F which should be used in the design of high-rise structures. Design wind pressure, P, is also defined as follows:

$$P = \frac{\rho}{2g} [V_D(h, t)]^2$$

(11)

Where ρ is the density of the air, and g is the acceleration due to gravity appropriate to the location. From equation (10), it is quite clear that the appropriate specification of the random variable $F_2(h,t)$ is a critical issue in the characterization of the design wind speed, for structural design application. Based on results of recent research in this field, which can be verified using appropriate statistical data this paper therefore proposes the use of the same mathematical form of the model in equation (2), which will now be built into the process of defining the appropriate random variable $F_2(h,t)$ as follows:

$$F_2(h, t) = h[m(t)h + n(t)]^{-1}$$

(12)

Where m(t) and n(t) are the model constants at a specific value of wind gust duration, t, relevant to the height h, where the design wind loading is acting on the structure. Equation (12) can be linearized as follows:

$$\frac{h}{F_2(h, t)} = m(t)h + n(t)$$

(13)

Multivariate statistical regression analysis of the variable $h/F_2(h,t)$ has been carried out against the variable, h; using appropriate data from the British Code of Practice, see Reference (3) for the different zones of (I) urban, (ii) semi-urban and (iii) rural zones. This statistical analysis yields excellent correlation coefficients, see Table I, which therefore confirms that the new simplified linearized model of equation (13) from equation (12), gives results which agree very well with recommended practical field data.

The statistical analyses also give the relevant values of the new model constants $m(t)$ and $n(t)$ at different values of wind gust duration, t. The summary of the significant results of the statistical analyses are shown in Table I of this paper. Similar studies carried out on recommended field data of other regions of the world have yielded encouraging excellent correlations in the ongoing research in this field, at The Ohio State University, Columbus, Ohio, USA.

CONCLUSIONS AND RECOMMENDATIONS

The new simplified model developed in this paper is recommended for use in different parts of the world. The same statistical studies, which have been carried out in this paper should also be carried out on similar field data in different regions of the world, with different climatic conditions, in order to establish the relevant constants of this model which can be applied in practical structural engineering design of high-rise structures, in different parts of the world. The results of such studies should also be made available to the international technical groups and organizations in structural engineering.

ACKNOWLEDGMENT

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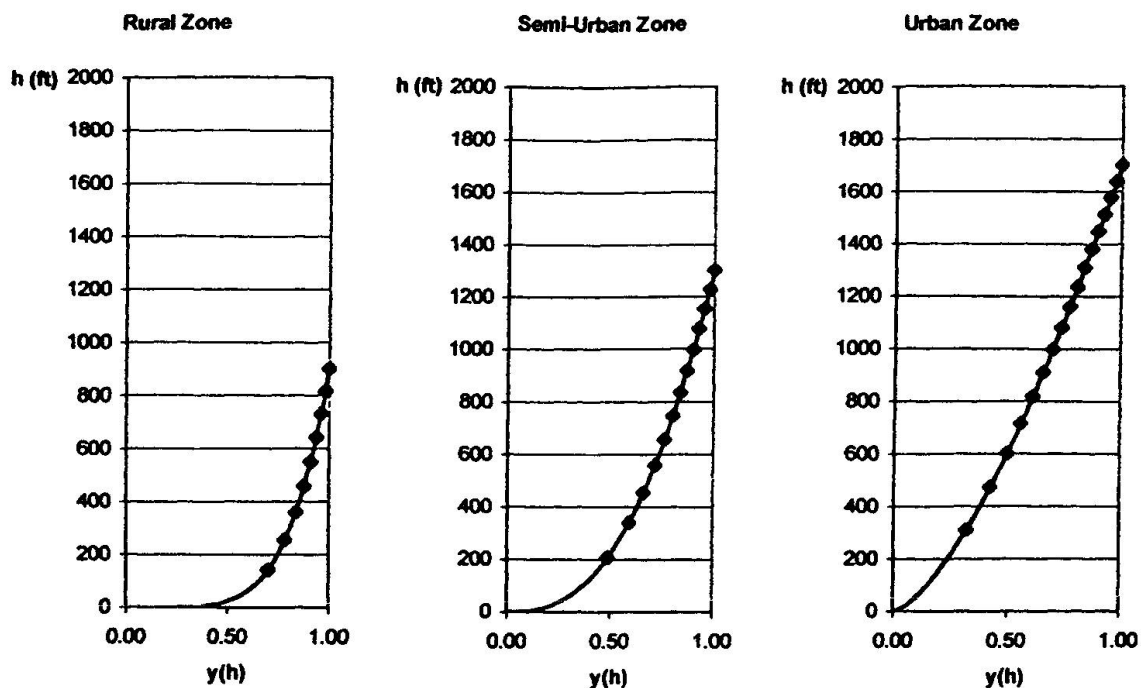
Rural Zone   Semi-Urban Zone   Urban Zone

FIGURE 1: MEAN VELOCITY PROFILE, y(h) USING THE NEW MODEL.

Table I: Statistical Analyses of Data of Reference (3)

$$\frac{h}{F_2(h,t)} = m(t)h + n(t); \qquad \text{(h in meters)}$$

Zone	t=3 secs gust		t=5 secs gust		t=15 secs gust		Correlation Coefficient
	m(t)	n(t)	m(t)	n(t)	m(t)	n(t)	
Urban	0.7943	7.8190	0.8088	9.1890	0.8222	10.4981	0.9974
Semi Urban	0.7917	5.3979	0.7948	6.0970	0.8173	7.0315	0.9986
Rural	0.7804	3.5050	0.7984	3.9210	0.8166	4.6122	0.9988