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## Stochastic Modelling of Traffic Loads for Long-Span Bridges

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### **Summary**

Load traffic models usually given in the Codes cannot be used for the design of long-span bridges, because they result too much severe when the span length is bigger than 200 m. For this reason the evaluation of traffic loads on long span bridges requires special studies, based on probabilistic concepts, leading to suitable numerical procedures. In the paper a theoretical stochastic traffic model, based on special vehicles (super-vehicles), representing convoys crossing simultaneously the bridge, is illustrated. Numerical examples, illustrating the application of the model to a cable-stayed long-span bridge, show the potentiality of the model to describe effects induced by traffic running on one or in several lanes.

### **1. Introduction**

The design of long-span bridges requires special preliminary studies devoted to define suitable load traffic models. In fact, the load traffic models usually given in the modern Codes, intended to be used properly for the most common bridge types, whose span length is lesser than 200 or 300 m, result too much severe for long span bridges, which are highly sensitive to the magnitude of traffic loads.

The evaluation of traffic loads on long span bridges is generally based on probabilistic concepts, leading to numerical procedures, in which traffic samples, representing as well as possible the expected traffic on the bridge, are suitably handled according to reasonable deterministic traffic scenarios, in order to obtain the characteristic values of the relevant effects. Obviously, this procedure becomes more and more unsatisfactorily when, rather than the effects induced by the traffic running on one single lane, multi-lane traffics or combination of traffic loads with other action of different nature, for example, wind or temperature, must be taken into account, mainly because it is very difficult to foresee reasonable traffic scenarios, including jammed and flowing vehicle convoys of considerable length. Beside that, when geometric non-linearity effects become significant, further difficulties occur, as the effects cannot be superimposed and the whole set of vehicles running simultaneously on the bridge must be considered.

To solve the problem, a new theoretical stochastic traffic model is proposed, which allows to represent in a very general way traffic scenarios on one or several lanes, using the general results



of the stochastic process theory.

The model illustrated in the following is a further development of the one, based on an equilibrium renewal process of vehicle arrivals, previously proposed by the Authors [1] for pre-normative research studies. The theoretical method has been already validated [1] through an analytical-numerical procedure, starting from the traffic data recorded in Auxerre, used for the calibration of load models of EC1-part 3 [2], comparing the extreme values obtained with the proposed model with the target ones, for a wide set of different bridge schemes, spanning up to 200 m.

The traffic model needs some improvement to be extended to long span bridges, in particular to surmount the difficulties concerning the vehicle distribution and the treatment of second order effects.

At present, the problem has been completely solved as illustrated in the following, for all cases in which second order effects can be disregarded, introducing some kind of special vehicles (super-vehicles), representing convoys crossing simultaneously the bridge, while solution for bridges sensitive to geometric non-linearity requires further studies.

The solution has been found by means of a two step method, very similar to those described in [1] for span length up to 200 m. Provided that in long span bridges the traffic effects depend on the equivalent uniformly distributed load, the first step of the method consists in the set up, according to the aforesaid procedure, of some kind of super-vehicles, each one representing one long convoy (up to 200 m long), associated with the suitable PDF of its length and of its uniformly distributed weight. The super-vehicles so determined are then used in the second step to assemble super-convoys, which substitute convoys for long span bridge analysis. In this way the sensitivity of the results on the assumption concerning the intervehicle distance is strongly reduced: in fact, because intervehicle distance concerns only very long vehicles (the super-vehicles), its effect on the EUDL is practically insignificant, even if it varies in a wide range.

Numerical examples, illustrating the application of the model to a cable-stayed long span bridge, show the potentiality of the method.

## 2. The Theoretical Traffic Model

To represent reliably the traffic, satisfying at the same time the physical condition that the distance between two consecutive vehicles cannot be smaller than a limit value, depending on the vehicles themselves, the theoretical traffic model [1] has been derived assuming a stationary ergodic arrival process of vehicles, each one characterised by its own probabilistic distribution of length and total weight, described by a suitably modified equilibrium renewal process, so that the time interval between two consecutive arrivals is given by

$$f(x) = \int_{l_{\min}}^{l_{\max}} t(l) \cdot \sum_i (p_i \cdot b_i(l)) dl \quad (1)$$

in which  $t(l)$  is the truncated distribution of the intervehicle distance,  $b_i(l)$  is the truncated distribution of the length of the  $i$ -th standard vehicle, whose frequency is  $p_i$ , and  $l_{\min}$  and  $l_{\max}$  are the extreme values of the vehicle lengths.

Clearly, the geometry and the main statistical properties of each standard vehicle depend on the expected traffic and they should be chosen to fit at the best the real traffic.

The probability  $P_n(l_c, L)$  that a lonely  $n$ -vehicles convoy of length  $l_c$  is travelling on the bridge length  $L$  (see fig. 1) is then given by

$$P_n(l_c, L) = \int_0^{L-l_c} \frac{\mathfrak{I}(y)}{\mu} \cdot \int_0^{l_c} f_n(x) \cdot \mathfrak{I}(L-x) \cdot dx \cdot dy + \int_{L-l_c}^L \frac{\mathfrak{I}(y)}{\mu} \cdot \int_0^{L-y} f_n(x) \cdot \mathfrak{I}(L-x) \cdot dx \cdot dy \quad (2)$$

being  $f_n(x)$  the  $n$ -th convolution of (1),  $\mathfrak{S}(x)$  the survival function and  $\mu$  the mean arrival rate.

On the other hand, from the mechanical point of view, because the bridge behaves like a filter, converting the traffic process in the extreme effects process, each part of the time history of the

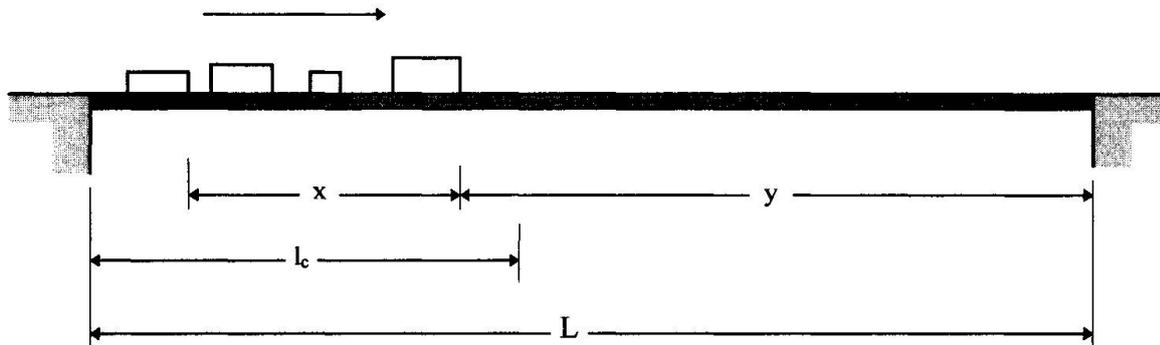


Fig.1 - Transit of a vehicle convoy on the bridge

actions on the structure can be considered as the consequence of the transit of an appropriate convoy, composed by one or several standard type vehicles, characterised by its length and its composition, in such a way that each convoy can be associated to the maximum value of the part of the history related to it. As the length  $L_c$  of the relevant loaded area of the bridge increases, the effects are governed mainly by the uniformly distributed weight of the convoy, being quite independent on the individual axle load values; so that the transfer function  $T$ , which relies the maximum effect induced by the convoy to its the uniformly distributed weights, depends only on the total length  $l_c$  of the convoy and on the total length  $L$  of the bridge. In particular, when second order effects can be disregarded,  $T(L, l_c)$  can be derived from the knowledge of the ordinates  $\eta(x)$  of the relevant influence line,

$$T(L, l_c) = \max_{0 \leq t < L - l_c} \left( \int_t^{t+l_c} \eta(x) dx \right), \quad l_c \leq L. \quad (3)$$

On the other hand, the distribution of the total weight of an  $n$ -vehicles convoy can be determined through the convolution of the PDFs of the total weight of each individual vehicle of the convoy itself. Being  $m$  the total number of vehicles of the standard set and  $\rho_i(W)$  the PDF of the total weight of the  $i$ -th standard vehicle, the PDF  $\rho^{(1)}(W)$  of the total weight  $W$  of one vehicle results

$$\rho^{(1)}(W) = \sum_{i=1}^m p_i \cdot \rho_i(W), \quad (4)$$

while the PDF  $\rho^{(n)}(W)$  of the total weight of an  $n$ -vehicles convoy is expressed by

$$\rho^{(n)}(W) = \sum_{\omega_i \in \Omega^n} \left[ \frac{\prod_{s=1}^n p_{i_s}}{\sum_{\omega_j \in \Omega^n} \prod_{k=1}^n p_{j_k}} \cdot \rho_{\omega_i}(W) \right], \quad \omega_i = (i_1, i_2, \dots, i_n), \quad (5)$$

where the sums are extended to all the possible choices of  $n$  vehicles in a set of  $m$ , the products are extended to all the  $n$  vehicles of the convoy and  $\rho_{\omega_i}(W)$  is the convolution of the PDFs  $\rho_{i_s}(W)$ .

Obviously, this procedure is independent on the length  $L$ , but it becomes very cumbersome and time expensive, as the number of vehicles or the length of the convoy, i.e. the number of convolutions, increases. Nevertheless, the procedure can be considerably simplified introducing some kind of super-vehicle, representing convoys whose length is lesser than an assigned value  $l^*$ , for example 100 m or 200 m, which is a convenient fraction of the relevant length  $L$ . The



significant parameters concerning the super-vehicle can be then easily derived, using the formulae given above, pointing out that the distribution  $w_{sv}(W)$  of the total weight  $W$  of the super-vehicle itself is given by

$$w_{sv}(W) = \frac{\sum_{i=1}^r \int_0^{l^*} P_i(l_c, l^*) \cdot dl_c \cdot \rho^{(i)}(W)}{\sum_{i=1}^r \int_0^{l^*} P_i(l_c, l^*) \cdot dl_c}, \quad (6)$$

where  $\int_0^{l^*} P_i(l_c, l^*) \cdot dl_c$  represents the CDF, evaluated according (2), of the transit of a lonely  $i$ -vehicles convoy on the given length  $l^*$  and  $\rho^{(i)}(W)$  is the PDF (5) of its weight, while  $r$  is the maximum number of vehicles in  $l^*$ .

Obviously, in this way all the convoys whose length is lesser than  $l^*$  are considered as a single fictitious vehicle, the *super-vehicle*, characterised by the length  $l^*$  and the weight distribution  $w_{sv}(W)$ . The supervehicle can be then introduced in the standard vehicle set, provided that the annual flows of the standard vehicles are suitably rearranged in order to maintain unchanged the annual traffic flow. The general procedure can be then applied, considering the significant length of the influence surface  $L$ , to the so modified standard vehicle set, in such a way that, using properly formulae (1), (2), (4) and (5), the characteristic parameters of new convoys, the *super-convoys*, are obtained.

The PDF  $f_m(\xi)$  of the maximum value  $\xi$  of the considered effect, induced by the given traffic running on one lane, is so

$$f_m(\xi) = \sum_{i=1}^s \int_0^L \left[ \bar{P}_i^*(L, l_c) \cdot \bar{\rho}^{(i)}(W) \cdot \frac{l_c}{T(L, l_c)} \right] \cdot dl_c, \quad W = \frac{\xi \cdot l_c}{T(L, l_c)}, \quad (7)$$

where  $\bar{P}_i^*(L, l_c)$  is the probability that an  $l_c$ -long lonely  $i$ -vehicles super-convoy is on  $L$ ,  $\bar{\rho}^{(i)}(W)$  is the PDF of the total weight of the  $i$ -vehicle super-convoy and  $T(L, l_c)$  is the transfer function, expressed by (3).

At this point, it must be emphasised that appropriate input traffic data should take into account various scenarios, including both flowing and jammed traffics. Since flowing traffics only can be directly derived from traffic measurements, jam situations, characterised by reduced intervehicle distances and even by the rise of the percentage of the heavy vehicles, are artificially modelled, introducing reasonable a priori hypotheses and manipulating the data records accordingly [2], [3], so that a general representation of the actual traffic can be obtained, mixing suitable percentages of flowing and jammed traffic.

Said  $q_1$  and  $q_2=(1-q_1)$ , respectively, the probabilities of the flowing and jammed reference traffics and  $f_{m1}(\xi)$  and  $f_{m2}(\xi)$  the PDFs of their extreme effects, the general PDF  $f_{gm}(\xi)$  of the maxima induced by traffic running on one lane results

$$f_{gm}(\xi) = q_1 \cdot f_{m1}(\xi) + q_2 \cdot f_{m2}(\xi), \quad (8)$$

and the CDF of the maxima  $F_\tau(\xi)$  in an assigned return period  $\tau$  is then obtained, starting from the CDF of  $f_{gm}(\xi)$ ,  $F_{gm}(\xi)$ , using the usual formula of the statistics of extreme

$$F_\tau(\xi) = [F_{gm}(\xi)]^{N(\tau)}, \quad (9)$$

in which  $N(\tau)$  represents the total number of lonely super-convoys crossing the bridge on the given lane, in the time interval  $\tau$ .

The general PDF  $f_{gm}(\xi)$  represents the theoretical solution of the problem, even if it can be

evaluated only numerically. In fact, the PDFs of the maxima induced by traffic on several lanes as well as those concerning the combinations of the traffic actions with actions of different nature can be evaluated, as soon as the relevant PDF  $f_{gm}^{(i)}(\xi)$  for each lane is known. Of course, the evaluation of the PDFs pertaining to traffic combinations on two or more lanes can be evaluated through the convolutions of the relevant  $f_{gm}^{(i)}(\xi)$ , provided that the number of super-convoys travelling simultaneously on the considered lanes can be determined, for example using the formulae derived in previous studies [4].

### 3. Numerical Application of the Model

The theoretical procedure described above has been used to determine the characteristic values of traffic effects in several relevant cross sections of the long span cable-stayed concrete bridge, illustrated in fig. 2. The four lane bridge is characterised by a total length of about 575 m, a central span of about 440 m and a total width of the cross section of about 26 m, while the 130 m height Y-shaped pylon is made in concrete, except the upper part, which is made in steel. The static behaviour of the bridge subject to the traffic loads is sensibly linear.

The analysis has been carried out using the Auxerre traffic, previously assumed as reference traffic for the calibration of load models of EC1.3 [2], [5] by reason of its very aggressive composition: in fact, on the basis of the present trends, Auxerre traffic represents the natural evolution of the long-distance commercial traffic in continental Europe.

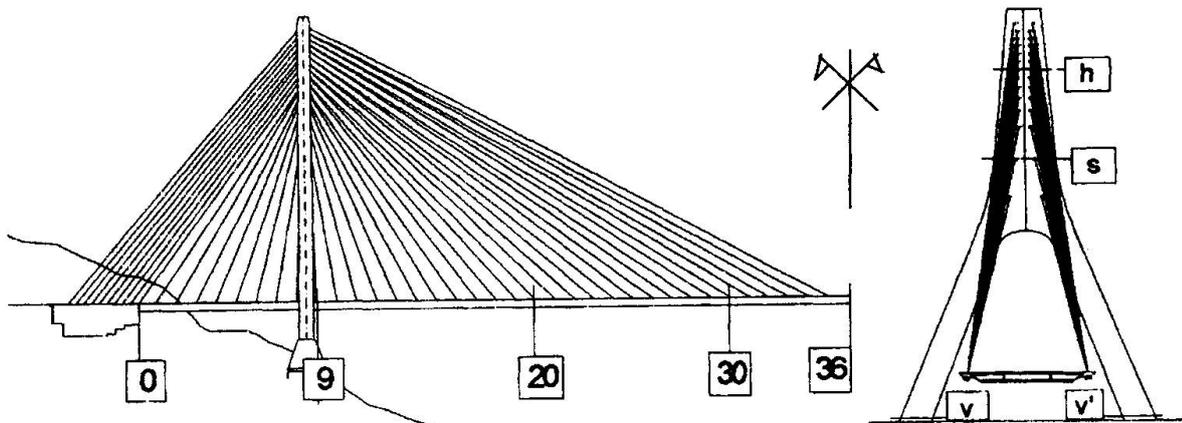


Fig. 2 – Cable stayed bridge

The effects which has been considered in the simulation refer to axial forces  $S$  in stays n. 5, 9, 15, 20, 25, 30 and 35 and in the anchor stays n. 1 and 10, as well as axial forces  $N$ , bending moments  $M$ , shear forces  $V$  and torque  $T$  in the relevant section 0, 9, 20, 30, 36 (the number correspond to the stay ordering) of the girder and in the relevant sections  $h$ ,  $s$ ,  $v$  and  $v'$  of the pylon, as indicated in fig. 2, considering one or two loaded lanes, with a jammed traffic percentage of 20%. Of course, effects characterised by base length of the influence surface lesser than 200 m has been disregarded, because they are just covered by the model proposed in [1].

In the figures 3÷8 are reported some significant influence surfaces, concerning, in particular, the axial forces  $N$  in the stay n. 35 (fig. 3) and in the anchor stay n. 1 (fig. 4), the bending moments  $M$  in the box girder at the midspan (fig. 5) and at the support (fig. 6), as well as the torque  $T$  at the support of the box girder, (fig. 7), and the axial force  $N$  in the cross section  $s$  of the pylon (fig. 8).

In order to present in a comprehensible way the whole set of the theoretical results, it has been decided to determine the equivalent uniformly distributed load (EUDL) to be considered on the



notional lane, 3 m width, together with the corresponding axle loads of the EC1.3, to reproduce the characteristic effects, characterised by a return period of 1000 years, determined with the proposed method.

The EUDL so determined are shortly summarised, depending on the base length of the influence surface, in figures 9 and 10, concerning the first and the second lane, respectively. It must be noted however, that the second lane effects are evaluated as difference between the total effects induced by two-lane traffics and the effects induced by the traffic on the main lane.

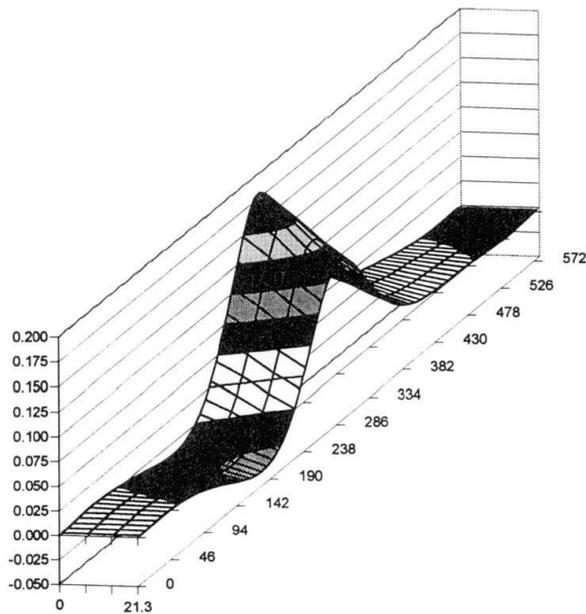


Fig. 3 – Influence surface of axial force  $N$  in the stay n. 35

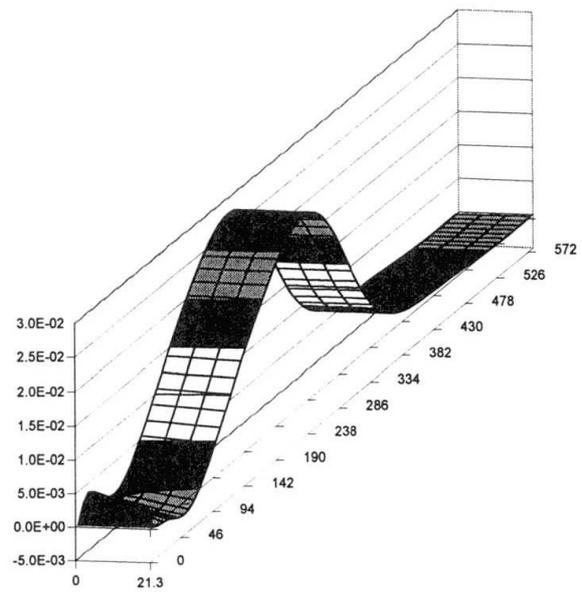


Fig. 4 – Influence surface of axial force  $N$  in the anchor stay n. 1

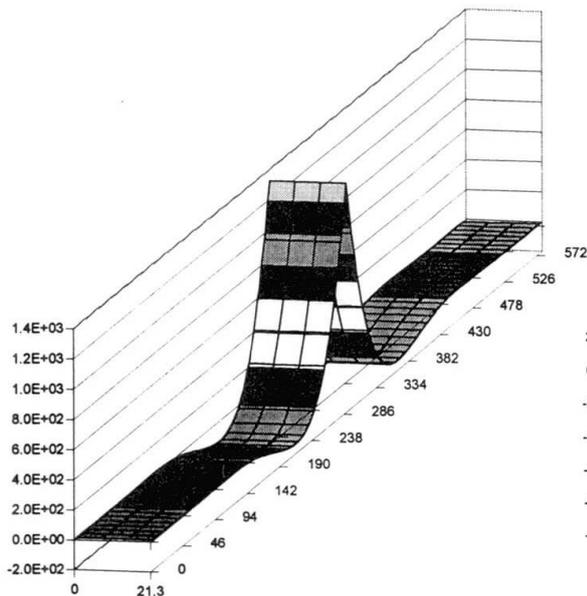


Fig. 5 – Influence surface of bending moment  $M$  at midspan (section n. 36)

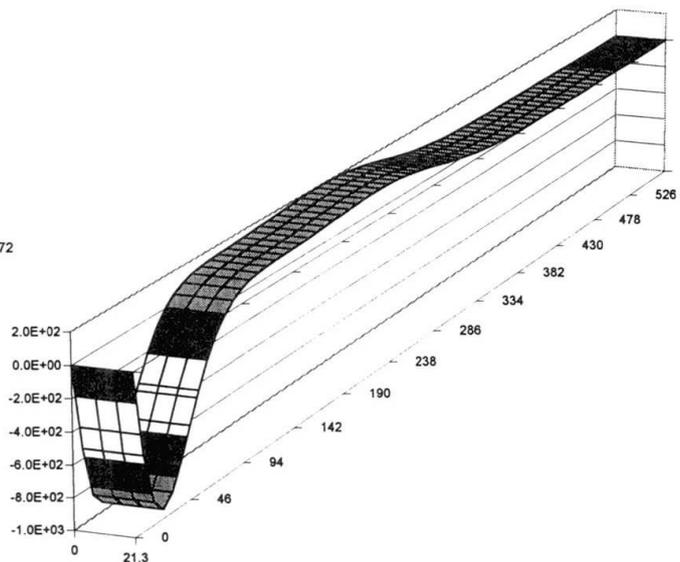


Fig. 6 – Influence surface of bending moment  $M$  at the support (section n. 0)

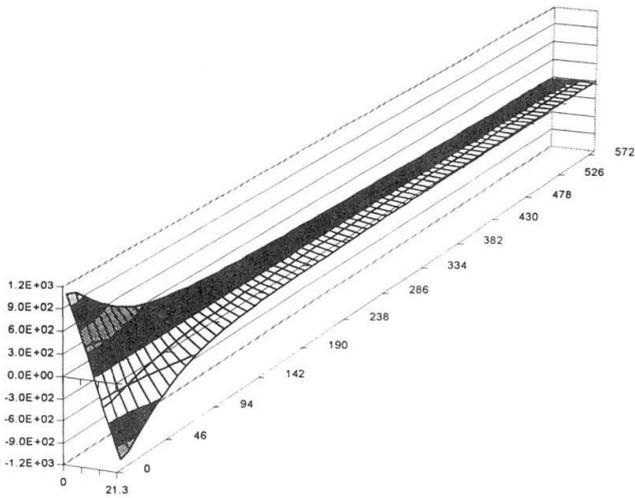


Fig. 7 – Influence surface of torque  $T$  at the support (section n. 0)

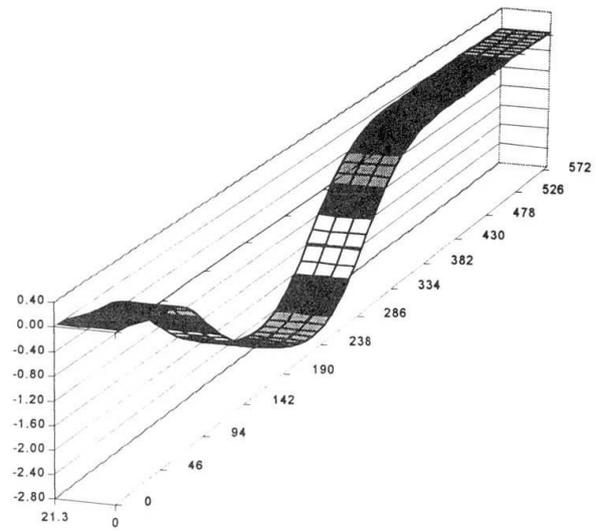


Fig. 8 – Influence surface of the axial force in the pylon (section  $s$ )

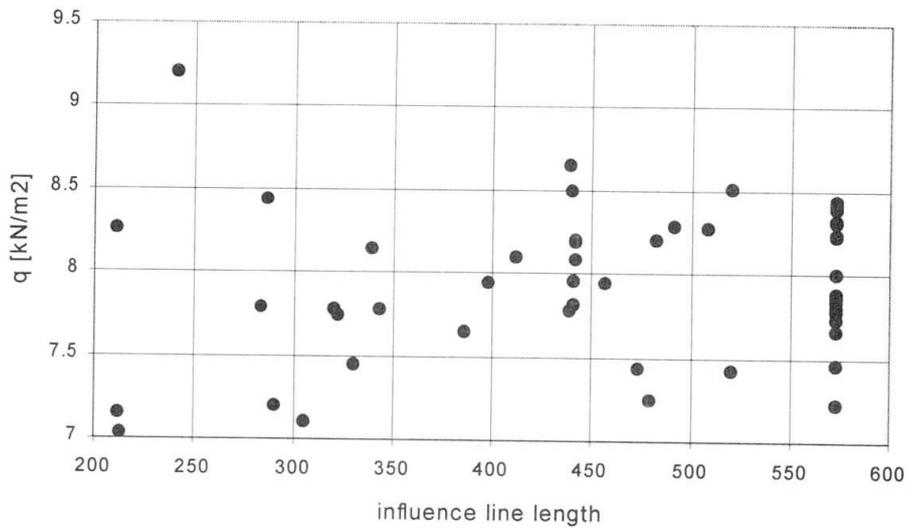


Fig. 9 – EUDL on the first lane vs base length of the influence lines

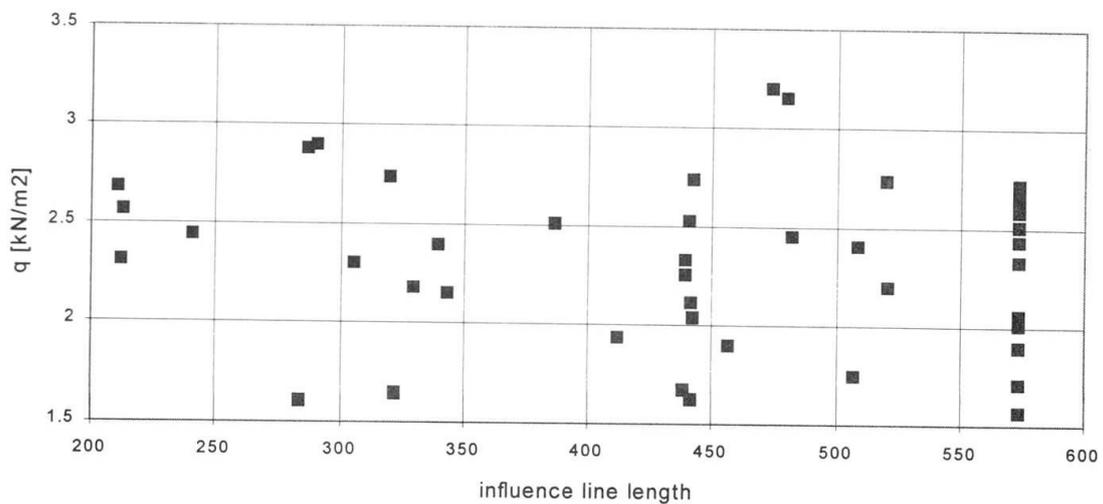


Fig. 10 – EUDL on the second lane vs base length of the influence lines



Obviously, as expected, the EUDL values are scattered, since a big number of effects, N, M, T and V, characterised by different shapes of the influence surfaces are simultaneously considered in the diagrams, so that the definition of some kind of target values requires further analyses, which are out of the scope of the present paper, to optimise the EUDL versus influence line length functions. Nevertheless, it clearly appears that the EUDL values tend to reduce increasing the base length.

## 5. Conclusions

A refined stochastic model to evaluate theoretically the probability distribution function of the extreme values of the traffic induced multilane effects in long span bridges has been presented. The procedure, which is based on the stochastic process theory, is a considerable improvement of the previous one concerning short and medium span bridges, proposed by the Authors in [1].

The main feature of the method consists in the introduction of special vehicles (super-vehicles) and special convoys (super-convoys), allowing to reproduce very easily probabilistic traffic scenarios including free flowing as well as jammed traffic, which can happen during the bridge life, under arbitrarily assigned traffics.

The fully developed numerical example, concerning a long span cable stayed concrete bridge, emphasise the possibilities of the method.

At present, the method cannot be applied to bridges which are sensitive to second order effects, further developments are still in progress to consider these cases too.

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