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**Autor:** Nakazaki, Shunzo / Yamaguchi, Hiroki  
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## Very Long Suspension Bridges Using the Temporary Mass Method

**Shunzo NAKAZAKI**  
Mgr  
Nippon Eng. Consultants Co. Ltd  
Tokyo, Japan

**Hiroki YAMAGUCHI**  
Prof. Dr  
Saitama Univ.  
Urawa, Japan

### Summary

This paper deals with the method to enhance the aerodynamic stability by adding a mass temporarily into the box girder of a suspension bridge. Studies were carried out for both single and twin deck suspension bridges. For a single deck suspension bridge, the most effective longitudinal position of such a mass to improve the aerodynamic stability was first determined, then suspension bridges were roughly designed using this method. As a result, this method could reduce much structural material in comparison with the conventional design method. It was found that this method was also effective for a twin deck suspension bridge.

### 1. Introduction

Very long suspension bridges with more than 2000m span length are being planned in the world. If the conventional design method is used in order to achieve the required aerodynamic stability, the construction cost of such suspension bridges will be prohibitively expensive. Therefore, some new design methods (ex. mono-duo cable system and cross hanger system [1], active control system [2] and others) are being developed.

This paper deals with the method to enhance the aerodynamic stability by adding a mass such as sea water temporarily into the girder of a suspension bridge during a storm. This method is expected to give the suspension bridge the mass effect, the increase of cable stiffness and the mode control, and consequently enhances the aerodynamic stability.

Studies were carried out for single and twin deck suspension bridges. For a single deck suspension bridge, the most effective longitudinal position for such a mass to improve the aerodynamic stability was first determined, then suspension bridges were roughly designed in order to compare with the conventional design method. Since this study needed many natural vibration modes, the extended Bleich theory [3] and a numerical integration method [4] were employed. In the case of twin deck suspension bridge, since constant added mass was fully applied along the bridge, the suspension bridge was analyzed as a 2-D (vertical and torsional) problem.



## 2. Single Deck Suspension Bridges

### 2.1 Suspension Bridges Studied

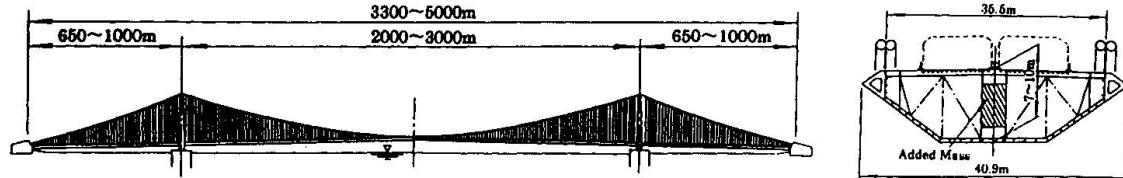


Fig.1. Single deck suspension bridge

Three suspension bridges with center span of 2000m, 2500m and 3000m were investigated in order to study the relation between the span and the effect of mass (Fig.1). As shown in Fig.1, the mass such as sea water was added at the center of box girder section, so that the added mass hardly changed the polar moment of inertia and the torsional frequency. The following conditions were assumed:

- The Theodorsen's unsteady aerodynamic force is acting on the girder.
- The required critical wind speed of the coupled flutter is 85m/s.

### 2.2 Selection of the Longitudinal Position for Added Mass

Seventeen loading cases shown in Fig.2 were analyzed to determine the most effective position of the added mass from the aerodynamic stability point of view. Case-0 has no added mass. Cases-1~9 have 0~100% mass length in the center span and full mass length in both side spans. Cases-10~17 are the same conditions as Cases-2~9 without the mass in side spans.

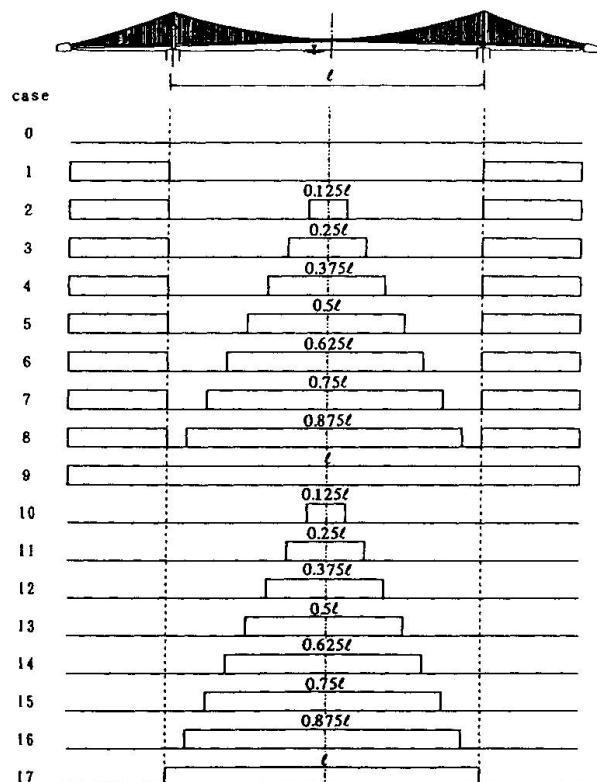


Fig.2 Added mass loading cases

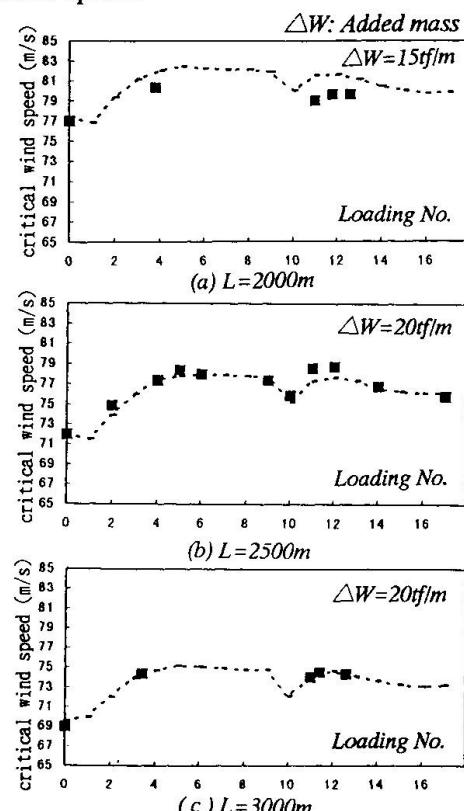


Fig.3 Critical wind speed for various loading conditions

The critical wind speeds of the coupled flutter were analyzed for the 17 cases shown in Fig.2. In the analysis of the flutter of a bridge, the extended Bleich theory and a numerical integration method were employed, as stated in introduction. In Fig.3, all cases were analyzed by the former and some cases were analyzed by the latter method. From Fig.3, the following trends were found:

- Results of both analytical methods show good agreement.
- The maximum increase of the critical wind speed due to the added mass is 6 to 7m/s.
- Cases-4 and 5 and Cases-10 through 13 are configurations with better locations for the added mass, because their critical wind speed are higher and their added mass are less than others.

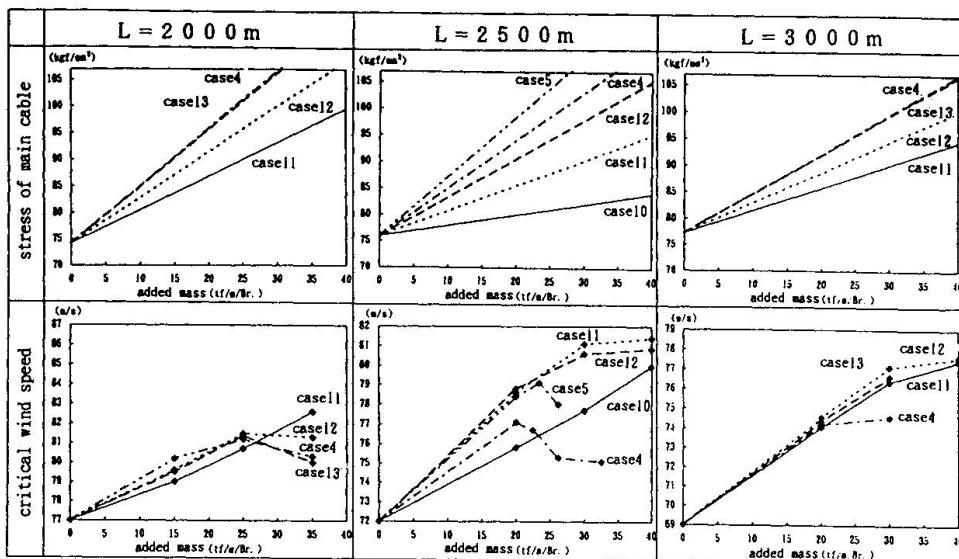


Fig.4 The effect of added mass intensity on the main cable stress and the critical wind speed

The mass intensity of Cases-4 and 5 and Cases-10 through 13 can be increased, because the main cable stresses in these cases are much lower than the allowable stress of Case-9. Then, the effect of increasing added mass intensity on the critical wind speed was investigated, and at the same time, the stress in the main cable was studied(Fig.4). For the analysis, a numerical integration method was used. Reviewing Fig.4, the following results can be noted:

- The increase of added mass intensity doesn't always increase the critical wind speed. For example, the critical wind speed of Cases-4 and 5 for L=2500m tends to decrease from 20tf/m.
- It seems that Case-11 (25% of center span) is the best configuration, because its aerodynamic stability is high and the main cable stress is low.

Based on Fig.4, loading position Case-11 and added mass intensity shown in Table 1 were selected.

### 2.3 Preliminary Design

After determining the position and intensity of the added mass, the conventional type and the added mass type suspension bridge were designed to find the economic consequences of the added mass method. Table 1 summarizes the results of design investigated. The increase in the critical wind speed was 6~9m/s. And, the decrease in the steel weight of a suspended structure was 9~12%.

L	Added Mass	$\Delta V^{*1}$	$\Delta W^{*2}$
2000m	38tf/m/Br.	6.0m/s	-9%
2500m	30tf/m/Br.	9.0m/s	-12%
3000m	40tf/m/Br.	8.6m/s	-11%

\*1: Increase of the critical wind speed  
 \*2: Decrease of the steel weight of a suspended structure

Table 1 Main results of design



### 3. Twin Deck Suspension Bridges

#### 3.1 Suspension Bridges Studied

A suspension bridge with 2500m center span and two 1250m side spans was investigated in order to study the effect of added mass (Fig.5a). Four cables system (Fig.5b) and two cables system (Fig.5c) were assumed as cable support systems for twin deck. The twin deck serves four lanes.

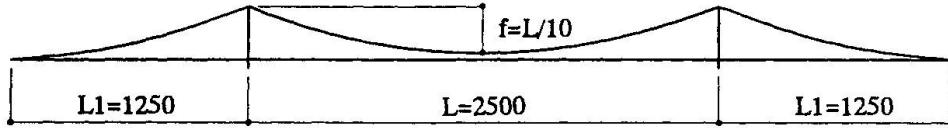


Fig.5a Twin deck suspension bridge (Unit:m)

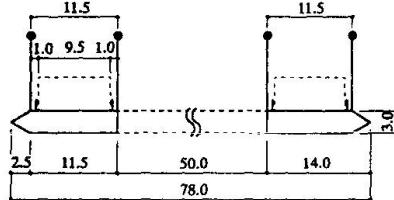


Fig.5b Four cables system

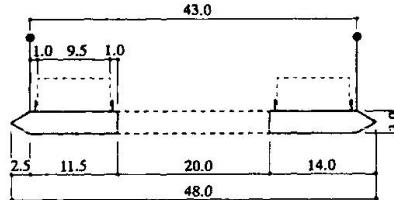


Fig.5c Two cables system

#### 3.2 Analytical Methods

For the calculation of the coupled flutter, the following analytical methods were employed.

**METHOD-1 (M-1):** This method is quoted from reference [5] which is based on the quasi-steady theory. This theory gives a smaller critical wind speed than the unsteady theory. Air forces are assumed to act on a flat plate.

**METHOD-2 (M-2):** This method is based on the equation (1) considering Theodorsen's air forces on a flat plate. An underlined term in the equation (1) is the effect of the air force dumping due to pitching.

If the aerodynamic interference between the two boxes is completely neglected, the coefficient  $c$  equals 1. But, in this case, even though  $a$  equals  $b$ , namely,  $D$  equals zero, an underlined term remains. Accordingly, we give the coefficient  $c$  which becomes zero when  $D=0$ .

Unit Dead Load	Cable	tf/m	13.6
	Girder	tf/m	14.0
	Pavement, etc.	tf/m	6.0
	Total	tf/m	33.6
Section Properties	Cable Area $A_c$	$m^2$	1.5
	Girder $I_x$	$m^4$	1.4
	Girder J	$m^4$	4.0
Polar Moment of Inertia		tf $\cdot$ m $\cdot$ s $^2$ /m	1177

Table 2 Structural condition(Bridge)

$$L = -2sv^2 \left[ f_1 \left( \alpha + \frac{h}{v} \right) + f_2 \cdot \frac{b}{2v} \cdot \dot{\alpha} \right] \quad (1)$$

$$M = sbv^2 \left[ f_1 \left( \alpha + \frac{h}{v} \right) - \left( f_3 \cdot \frac{b}{2v} + \underline{f_1 \cdot \frac{2a^2}{bv} \cdot c} \right) \dot{\alpha} \right]$$

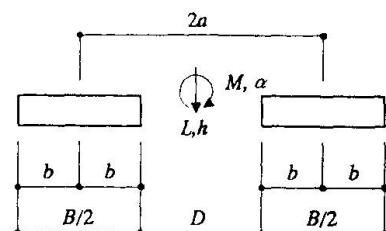


Fig.6 Idealized twin deck

where,  $L, M$ : unsteady aerodynamic lift and pitching moment,  $f_1 = C(k)$ ,  $f_2 = 1 + C(k)$ ,  $f_3 = -C(k)$ ,  $C(k)$  : Theodorsen function,  $s = 2\pi\rho b$ ,  $\rho$ : air density,  $b$ : semi-chord of a deck,  $a$ : semi-distance between deck centers,  $v$ : wind speed,  $h$  and  $\alpha$ : vertical and torsional displacements,  $c$ : coefficient for the aerodynamic interference between two boxes ( $1 - (b/a)^2$  or  $1 - b/a$ ).

**METHOD-3 (M-3):** This method uses the equation (2) quoted from reference[6]. This equation was obtained from the wind tunnel test of twin deck with wind screen.

$$V_F = \left[ 1 + 1.2438 \cdot \left( \frac{D}{B} \right)^{1.4712} \right] \cdot V_{sel} \quad (2)$$

where,  $V_F$  : the critical wind speed of flutter,  $V_{sel}$  : the flat plate critical wind speed obtained from the Selberg formula,  $D$  : slot width,  $B$  : solid deck width.

In order to find the differences among above methods, calculations were carried out for the two cables system (Fig.5c), using datum shown in Table 2. Fig.7 shows the results of analyses M-2 and M-3 about the relationship between  $D/B$  and  $V_F/V_{sel}$ . Moreover, M-2.1 and M-2.2 use  $[1 - (b/a)^2]$  and  $[1 - (b/a)]$  for the coefficient  $c$ , respectively. Fig.7 shows that M-2.2 and M-3 agree well with each other. On the other hand, M-2.1 shows the excessive value of  $V_F/V_{sel}$ , compared with other methods.

For the calculation of the divergence speed, the equation (3) quoted from reference [5] was used.

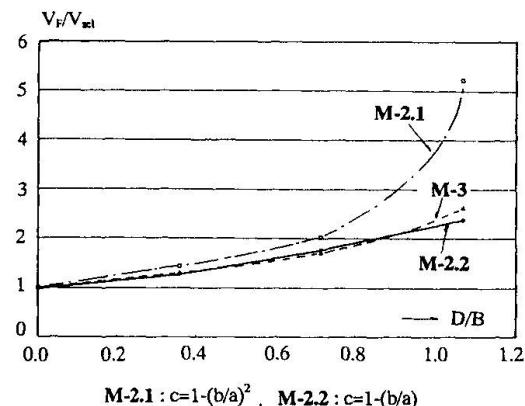


Fig.7 Comparison of flutter analytical methods for two cables system

$$V_D = \omega_t r \sqrt{\mu} \quad (3)$$

where,  $V_D$  : the divergence speed,  $\omega_t$  : torsion frequency,  $r$  : radius of gyration of bridge,

$$\mu = \frac{m}{4\pi\rho b^2}, \quad m : \text{mass / unit span.}$$

### 3.3 Four Cables System

In this system, the bending frequency  $\omega_b$  nearly equals to the torsion frequency  $\omega_t$ . It is known that the flutter is eliminated in this case. Fig.8 shows  $V - \delta$  (wind speed - logarithmic decrement) curves analyzed by M-1 and M-2.1. Both methods never show the flutter and the logarithmic decrement of M-1 shows smaller values than that of M-2.1.

The divergence speed  $V_D$  is calculated from the equation (3).  $V_D$  of a bridge shown in Fig.5b increases from 63m/s to 77m/s by adding a mass of 16tf/m/Bridge. If  $V_D$  of 77m/s is to be accomplished by increasing the length of cross beam, it's length must be increased from 50m to 62m. In this system, A-shaped rigid tower can also increase the bending frequency, and consequently enhance the divergence speed.

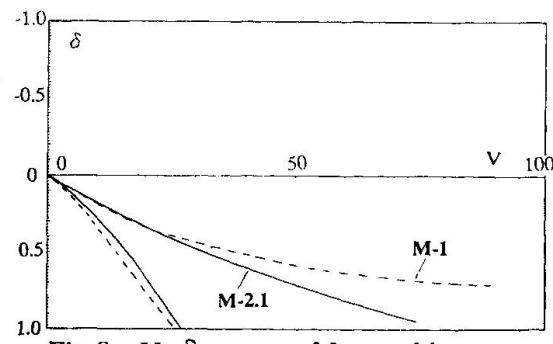


Fig.8  $V - \delta$  curves of four cables system



### 3.4 Two Cables System

In this system ( Fig.5 c ), the torsion frequency is larger than the bending frequency, because the radius of gyration of bridge  $r$  is much smaller than the semi-distance between two main cables. In order to find out the most effective position of added mass in deck for the wind resistant design,  $V_D$  and  $V_F$  are calculated in 3 cases of added mass position. The changes of  $V_D$  and  $V_F$  by adding temporary mass (16tf/m) are shown in Table 3.  $V_F$  is calculated using M-3 method. From Table 3, the following results can be noted.

- Added mass affects not only  $V_D$  but also  $V_F$  unlike four cables system.
- Added mass increases  $V_D$ , but the added mass position hardly affects  $V_D$ .
- The position of added mass changes  $V_F$ , and the value of  $V_F$  is the maximum for the Case 1.

Items	Case-0	Case-1	Case-2	Case-3	Case-0
$f_b$ (Hz)	0.0541	0.0532	0.0532	0.0532	
$f_t$ (Hz)	0.0891	0.0912	0.0872	0.0759	
$f_t / f_b$	1.647	1.714	1.639	1.427	
$V_D$ (m/s)	70.5	77.3	77.1	77.0	
$V_F$ (m/s)	63.5	75.6	72.1	60.7	

Table 3 Added mass effects for two cables system

### 4. Conclusion

For single deck suspension bridges, it could be concluded, that

- the middle 25% part of the center span was the most effective position for the temporary mass from the aerodynamic stability point of view.
- this method increased the coupled flutter speed by 6~9m/s for a given condition.
- this method reduced the steel weight of a suspended structure by approximately 9~12%, compared with the conventional method.

For twin deck suspension bridges, it could be concluded, that

- the coupled flutter speeds predicted by the methods M-2.2 and M-3 showed good agreement.
- in the case of four cables system, added mass was effective to increase the divergence speed.
- in the case of two cables system, added mass was effective to increase not only the divergence speed but also the coupled flutter speed.

### References

- [1] Astiz,M.Z. and Andersen, E.Y., On wind stability of very long spans in connection with a bridge across the Strait of Gibraltar, Strait crossing, Ed. J.Kroekborg, Balkema, 1990, pp.257-264.
- [2] Ostenfeld, K.H., Innovative structural systems for the Gibraltar strait crossing project, IASS-CSCE International Congress, Toronto, July, 1992, pp.231-242.
- [3] Bleich,et al., The mathematical theory of vibration suspension bridges, Dept. of U.S.Gov.Print Office, 1950.
- [4] Bell,A.J. and Brotton,D.M., A numerical integration method for the determination of flutter speeds, Int.J.Mech.Sci.15, 1973, pp.473-483.
- [5] Richardson, J.R., The development of the concept of the twin suspension bridge, National Maritime Institute, NMI R125, 1981.
- [6] Larsen,A.,Ostenfeld, K.H. and Antiz,M.A., Aeroelastic stability study for the Gibraltar Bridge feasibility phase, 4<sup>th</sup> Int'l Colloquium on the Gibraltar Strait Fixed Link, May 1995, pp.99-108.