Aerodynamic Instability of Long-Span Cable-Stayed Bridges

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Summary

In this paper an analysis of the aerodynamic behaviour of fan-shaped long span cable-stayed bridges under nonstationary aerodynamic loads is developed. A numerical analysis is carried out, based upon time integration of the motion equations of the discretized structure. The main structural nonlinearity arising from the elastic response of stay is accounted for together with the nonlinear effects related to the assumed nonstationary model of the aerodynamic loads. Moreover, a continuous model of the bridge based on the hypothesis of a small spacing between stays is developed, whose analytical results are usefully compared to numerical ones.

1. Introduction

As is well known the cable-stayed bridge scheme evoked great interest as a valid solution for long spans, particularly regarding the so-called fan-shaped scheme of the self-anchored type.
The structural behaviour of this scheme is marked by a dominant state of axial tensions in the stays and of axial compression in the girder, while less important is the bending stress as a result of the prevailing truss behaviour of the scheme.
Moreover, in long-span bridges the analysis of the dynamic behaviour is the most important one. The influence of moving loads, the precence of seismic forces and the influence of aerodynamic effects must be carefully examined; in fact, the more dangerous stresses and deformations are related to these kinds of external action. Therefore, the fan shaped cable-stayed bridge scheme, suitable for long spans, requires an accurate analysis of the aerodynamic instabilities.
In this paper an anlysis of the dynamic instability of long-span cable-stayed bridges under non stationary aeodynamic loads is developed by using both a discrete model of the bridge and a continuous one based on the assumption of small spacing between stays. The intrinsic nonlinearity arising from Dischinger constitutive equation is taken into account together with the nonlinear effects arising from the deformation dependent nonstationary aerodynamic forces.The analysis is developed at first by analyzing the flexural and torsional oscillations of the bridge. Then, the critical wind speed, both in the case of flutter and stall flutter is investigated.

2. The structural model of the bridge

The fan-shaped scheme of cable-stayed bridge of Fig.1 is considered, in which the girder is simply supported at its ends and is hung to the tops of H-shaped towers by means of two stays curtains.
It is assumed that the stays spacing Δ, the girder width 2c and the stay curtain interval 2b are small quantities compared to the central span length L_C. The aspect ratios $r_1=L_c/H$; $r_2=L_s/H$ of span legths to the tower height are usually obtained on the basis of economy and of the anchor cable stability condition.
The longitudinal vertical plane *yz* is assumed to be a symmetrical one; in addition, the bridge is also symmetrical with respect to the midspan cross plane.
According to the usual erection procedures, girder and towers are assumed to be free from bending under dead load g. Then, the cross sectional areas A_S and A_O of the couple of diffused stays and of the anchor stays, respectively, are given by

$$A_s = \frac{g\Delta}{\sigma_g \sin\alpha}; \qquad A_0 = \frac{gL_s}{2\sigma_{g_0}}[1+(\frac{L_s}{H})^2]^{1/2}[(\frac{L_c}{2L_s})^2-1] \qquad (1)$$

where

$$\sigma_g = \sigma_a \frac{g}{p+g}; \qquad \sigma_{g_0} = \sigma_a \left\{1+\frac{p}{g}[1-(\frac{2L_s}{L_c})^2]^{-1}\right\}^{-1} \qquad (2)$$

and where σ_a is the allowable stress, α is the angle between a stay and its horizontal projection, and p denotes the live load.
We assume that towers and girder's axial elongations are negligible, and we apply the Euler-Bernoulli bending theory and the Saint-Venant torsion theory for the girder.
As far as the stays behaviour is concerned, the Dischinger modulus $E_s^*=E/(1+\gamma^2 l_0^2\, E/12\sigma_0^3)$ is used, where E is the Young modulus, γ is the specific weight, l_0 is the horizontal projection length of the stay and σ_0 is the initial tension. The tower is characterized by the flexural stiffness k_T.

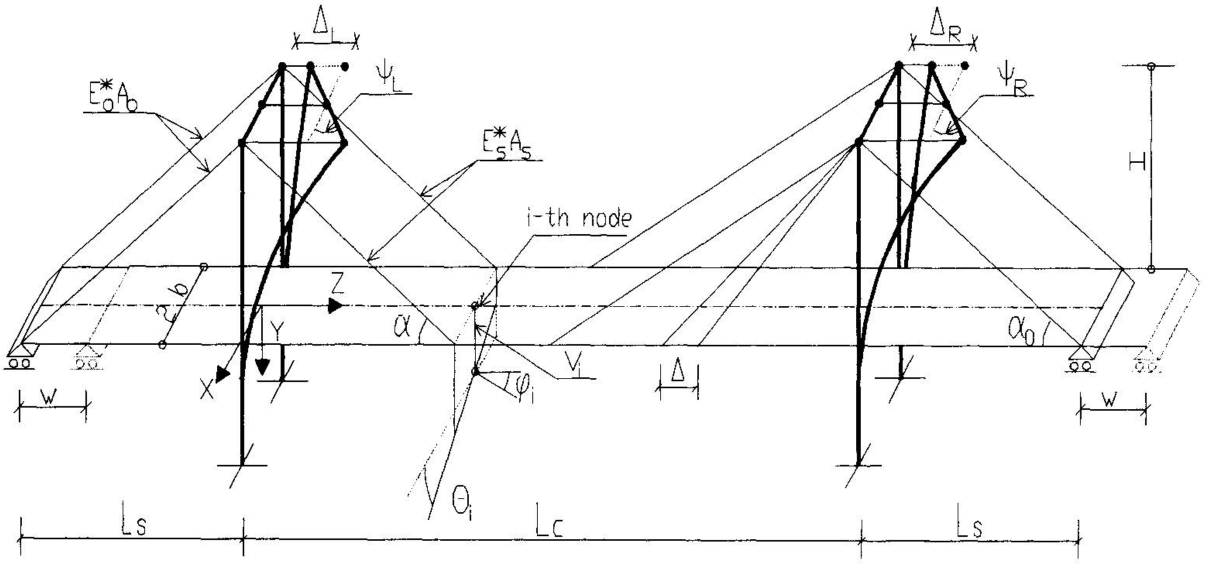

Fig.1. Cable stayed bridge scheme

At first we develop a discrete model of the bridge based on a finite element discretization of the girder by using hermitian cubic interpolation functions for transverse deflection and linear interpolation functions for torsional deformation.
Therefore, for the H-shaped towers scheme (Fig.1), the deformation of the bridge can be described by the following displacement parameters:
- the axial displacement w of the girder;
- the axial displacements Δ_L, Δ_R and the torsional rotations Ψ_L, Ψ_R around the vertical axis of the towers tops;- at each internal node i of the girder, where a couple of stays act on the girder:
  -the vertical deflection v_i;

-the torsional rotation θ_i;
-theflexural rotation φ_i around the x-axis.
Now the air forces acting on the bridge are considered. To give an accurate analysis of the aeroelastic behaviour of the bridge, a sound evaluation of the aerodynamic loads must be used.
It is widely accepted that the nonstationary formulation is the most adequate one for predicting the aeroelastic behaviour of the girder with good accuracy. Moreover, to obtain simple formulas we refer to the simple thin airfoil theory.
According to these assumptions the aerodynamic lift l and torque m per unit length acting on the cross section of the bridge in a laminar approaching flow with zero mean angle of attack can be expressed by:

$$l=\frac{1}{2}\rho V_0^2(2c)[kH_1^*\frac{\dot{v}}{V_0}+kH_2^*c\frac{\dot{\theta}}{V_0}+k^2H_3^*\theta] \tag{3}$$

$$m=\frac{1}{2}\rho V_0^2(2c^2)[kA_1^*\frac{\dot{v}}{V_0}+kA_2^*c\frac{\dot{\theta}}{V_0}+k^2A_3^*\theta] \tag{4}$$

where :
-V_0 is the approaching wind speed;
-ρ is the air density;
- $\kappa = c\omega/V_0$ is the reduced frequency, where ω is the frequency of the oscillating bridge deck.
-H_i^*, A_i^* are the nondimensional Theodorsen aerodynamic coefficients, given in real notation, according to Scanlan, by:

$$\begin{cases} kH_1^*=-2\pi F \\ kH_2^*=-\pi(1+F+\frac{2G}{k}) \\ k^2H_3^*=-2\pi(F-\frac{kG}{2}) \end{cases} \qquad \begin{cases} kA_1^*=\pi F \\ kA_2^*=-\frac{\pi}{2}(1-F-\frac{2G}{k}) \\ k^2A_3^*=\pi(F-\frac{kG}{2}) \end{cases} \tag{5}$$

Then, the dynamical equilibrium equations of the discrete structure can be put in the following matrix form:

$$\mathbf{M}\ddot{\mathbf{s}}+\mathbf{K}(\mathbf{s})\mathbf{s}=\mathbf{F}(\dot{\mathbf{s}},\mathbf{s},t) \tag{6}$$

where **M** is the mass matrix, **s** is the displacement vector and **F** is the external load vector.
The above nonlinear problem was solved numerically by using the Newmark integration scheme. Moreover an algorithm based on the predictor-corrector method was used.
It must be observed that forces **F** depend on the reduced frequency k, that is on the deck oscillation ω. Due to the low sensitivity of the aerodynamic forces with respect to the variation of ω, k is updated only after one cycle of the midspan deflection (for the evaluation of l) or of the midspan torsional rotation (for the evaluation of m). The updated value of ω is then obtained from the wavelength of the corresponding oscillation. This procedure, when t tends to infinity, gives the flexural ω_v and torsional ω_θ frequencies converging to the unique critical value ω_c when V_o tends to its critical value V_c.
To determine the critical wind speed, integration stats with zero speed. Moreover, the initial conditions at time t=0 are choosen as the first flexural and the first torsional eigenmodes.
After some oscillation cycles, an increment is given to the wind speed, and integration stats up again assuming as initial conditions and as ω value, the final displacements, velocities, and ω values of the previous wind-speed step. The computation goes on step by step by means of wind-speed increments, and at each step w is updated as previously discussed and when ω convergence is reached, the motion character (damped or undamped) is estimated in order to determine the critical condition.
Now a continuous model of the bridge is employed to obtain simple formulas able to capture the main features of the bridge behaviour [1,2].This model is founded on the assumption that the spacing Δ is very small compared to the main span length L_c; this allows the development of a continuous structural model assuming a continuous distribution of stays along the deck. In this case, for

symmetrical motions with respect to midspan, the deformation of the girder is described by the flexural v(z) and torsional $\theta(z)$ displacement functions, respectively, together with the scalar displacement parameters $\Delta_L=-\Delta_R$, $\Psi=\Psi_L=-\Psi_R$ with w=0.
To give analytical developments, the following quantities are introduced:

$$\xi = \frac{z}{H};V(\xi,t) = \frac{v(z,t)}{H};U(t) = \frac{u(t)}{H};\ \varphi(\xi) = \frac{1}{(1+a\xi^2)(1+\xi^2)} \tag{7}$$

$$a = \frac{\gamma^2 H^2 E}{12\sigma_g^3};\frac{\varepsilon^4}{4} = \frac{I\sigma_g}{H^3 g};\tau^2 = \frac{C_t\sigma_g}{Eb^2H_g};M = \frac{\mu H\sigma_g}{Eg};J_0 = \frac{I_0 H\sigma_g}{b^2 Eg} \tag{8}$$

$$\chi = \frac{k^T\sigma_g}{Eg};\chi_0 = \frac{E_0^{*}A_0}{E}\frac{\sigma_g}{gH}\sin\alpha_0\cos^2\alpha_0;\rho = \int_L \frac{\cos^2\alpha}{1+a\xi^2}d\xi + \chi_0 \tag{9}$$

where:

- μ is the mass per unit length of the girder;
- I, I_o, C_t are the flexural inertia, the polar moment of inertia of mass and the torsional rigidity factor of the girder cross section

In practical cases the nondimensional flexural ε and torsional τ stiffness parameters are very small ($\varepsilon\leq0.3$, $\tau\leq0.1$). This corresponds to a prevailing truss behaviour of the bridge in which girder's bending and torsion are of local nature, while axial forces and overall displacements are well defined on the truss bridge scheme($\varepsilon=\tau=0$). This enables terms in ε and τ to be disregarded with respect to others in the equilibrium equations.
With these assumptions the dynamic equilibrium equations for the continuous model are:

$$\begin{cases} \varphi V - \xi\varphi U = -M\ddot{V} + Q_1\theta + Q_2\dot{\theta} + Q_3\dot{V} \\ -(\rho+\chi)U + \int_L \xi\varphi V d\xi = 0 \\ -\varphi\theta + \xi\varphi\psi = J_0\ddot{\theta} - \mu_1\theta - \mu_2\dot{\theta} - \mu_3\dot{V} \\ \psi = \dfrac{1}{\rho+\chi}\int_L \xi\varphi\theta d\xi \end{cases} \tag{10}$$

with

$$\begin{cases} Q_1 = \frac{1}{2}\rho V_0^2(2c)k^2H_3^{*}\frac{\sigma_g}{Eg} \\ Q_2 = \frac{1}{2}\rho V_0^2(2c)kH_2^{*}\frac{c}{V_0}\frac{\sigma_g}{Eg} \\ Q_3 = \frac{1}{2}\rho V_0^2(2c)kH_1^{*}\frac{1}{V_0}\frac{\sigma_g H}{Eg} \end{cases},\qquad \begin{cases} \mu_1 = \frac{1}{2}\rho V_0^2(2c^2)k^2A_3^{*}\frac{H\sigma_g}{Egb^2} \\ \mu_2 = \frac{1}{2}\rho V_0^2(2c^2)kA_2^{*}\frac{c}{V_0^2}\frac{H\sigma_g}{Egb^2} \\ \mu_3 = \frac{1}{2}\rho V_0^2(2c^2)kA_1^{*}\frac{1}{V_0}\frac{H^2\sigma_g}{Egb^2} \end{cases} \tag{11}$$

The aerodynamic instability and the corresponding critical wind speed can be obtained by putting the solution of eqn (10) in the form:

$$V(\xi,t) = \bar{V}(\xi)e^{st};U(t) = \bar{U}e^{st};\theta(\xi,t) = \bar{\theta}(\xi)e^{st};\psi(t) = \bar{\psi}e^{st} \tag{12}$$

where a purely immaginary value of $s=\alpha+i\omega$ corresponds to flutter.
Substituting eqn (12) in (10) a linear homogeneous system in the time independent displacement variables introduced in (12) is obtained; putting its determinant equal to zero and disregarding less relevant terms, the following frequency equation is obtained:

$$\sigma^4 + \sigma^3\beta\Omega\frac{\pi}{k}(2F+\gamma G_1) + \sigma^2(1+\varphi^2 - \beta\gamma\Omega^2 G_2 + \beta^2\gamma\Omega^2 G_3) +$$

$$\sigma\beta\Omega\frac{\pi}{k}(2\varphi^2 F+\gamma G_1)+(\varphi^2-\beta\gamma\Omega^2 G_2)=0 \tag{13}$$

with

$$\beta=\frac{\rho c^2}{\mu};\qquad \gamma=\frac{\mu c^2}{I_0};\qquad \varphi=\frac{\omega_{0\theta}}{\omega_{0v}};\qquad \Omega=\frac{\omega}{\omega_{0v}};\qquad \sigma=\frac{s}{\omega_{0v}} \tag{14}$$

$$G_1(k)=\frac{1}{2}(1-F-2\frac{G}{k})\qquad G_2(k)=\frac{\pi}{k^2}(F-\frac{kG}{2});\qquad G_3(k)=\frac{2\pi^2}{k^2}F; \tag{15}$$

where ω_{0v} and $\omega_{0\theta}$ denote the flexural and torsional free oscillation frequencies in still air [2].
The flutter condition is formulated by putting $s=i\Omega_c$ in eqn (13). We obtain the flutter condition:

$$\Omega_c^4(-G_1+2F\beta G_2-2F\beta^2 G_3)+\Omega_c^2(2G_1-2F\beta G_2)-G_1=0 \tag{16}$$

and

$$\varphi^2=\frac{\Omega_c^2(2F+\gamma G_1)-\gamma G_1}{2F}, \tag{17}$$

and the critical wind speed is given by the relation : $(V_c/c\omega_{0v})^2=(\Omega_c/k_c)^2$
Now, the single degree of freedom torsional mode of flutter of stalled airfoils is considered. In this case, according to the Ragget theory, the aerodynamic moment can be expressed by:

$$m=\frac{1}{2}\rho V_0^2(2c^2)[k\bar{A}_2^* c\frac{\dot\theta}{V_0}+k^2\bar{A}_3^*\theta] \tag{18}$$

with

$$k\bar{A}_2^*=-\frac{S_m}{2}(\frac{\pi}{S_m}-F_m-2\frac{G_m}{k})\qquad k\bar{A}_3^*=S_m(\frac{F_m}{k}-\frac{G_m}{2}) \tag{19}$$

$$F_m(k)=1+\frac{\frac{\pi}{S_m}-1}{1+\left(\frac{0.3}{k}\right)^2}-\frac{\frac{\pi}{S_m}}{1+\left(\frac{3}{k}\right)^2}\qquad G_m(k)=\frac{\frac{0.3}{k}(\frac{\pi}{S_m}-1)}{1+\left(\frac{0.3}{k}\right)^2}-\frac{\frac{3}{k}\frac{\pi}{S_m}}{1+\left(\frac{3}{k}\right)^2} \tag{20}$$

where S_m is the slope of the steady state moment versus angle of attack, approching π as the angle of attack vanishes. In this case dynamical equilibrium equations (10) can be rewritten accounting only for the torsional displacement and force parameters. After some algebra we get the following frequency equation for stall flutter:

$$\sigma^2+\Gamma_1\Omega\sigma+(1-\Gamma_2\Omega^2)=0 \tag{21}$$

where

$$\beta=\rho c^2,\ \gamma=\frac{c^2}{I_0},\ \Omega=\frac{\omega}{\omega_{0\theta}},\ \sigma=\frac{s}{\omega_{0\theta}},\quad \Gamma_1=\frac{\beta\gamma}{k}\frac{S_m}{2}(\frac{\pi}{S_m}-F_m-2\frac{G_m}{k});\ \Gamma_2=\frac{\beta\gamma}{k^2}S_m(F_m-k\frac{G_m}{2})$$

The flutter condition : $s=i\Omega_c$ in this case gives

$$\frac{\pi}{S_m}-F_m-2\frac{G_m}{k_c}=0;\qquad \Omega_c^2=\frac{1}{1+\frac{\beta\gamma}{k^2}S_m(F_m-\frac{k_c G_m}{2})} \tag{22}$$

and after some algebra

$$S_m=\pi\left\{1-\frac{3}{17}\frac{[1+(0.3/k_c)^2][(1+2/3(3/k_c)^2]}{[1+(3/k_c)^2][0.3/k_c]^2}\right\};\Omega_c^2=\left\{1+\frac{15}{17}\pi\frac{[(3/k_c)^2+3/2][(3/k_c)^2-2]}{[(3/k_c)^2+1][3/k_c]^2}\frac{\beta\gamma}{k_c^2}\right\}^{-1}$$

and the critical wind speed is given by the relation : $(V_c/c\omega_{0\theta})^2=(\Omega_c/k_c)^2$.

3. Numerical results and concluding remarks

Here we analyze a bridge scheme characterized by the following parameters:
r_1=5; r_2=5/3; L_C=750m; L_S=250m; H=150m; Δ=25m; b=c=17m; I=11.69m^4; I_0=918,378t_mm k_T=2350t/m; μ=4.8t_m/m; g=47t/m; p=28t/m; E=21x10^{6}t/m^2; σ_a=72x10^{3}t/m^2; ε=0.3; τ=0.2.

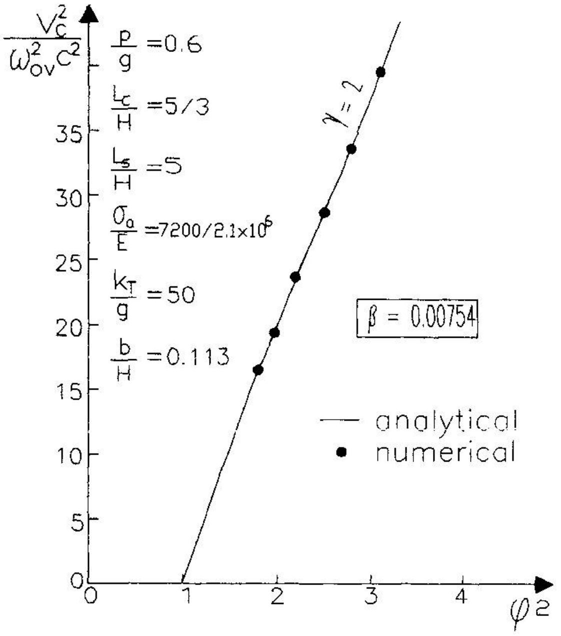

Fig.2: Critical wind speed versus frequency ratio φ.

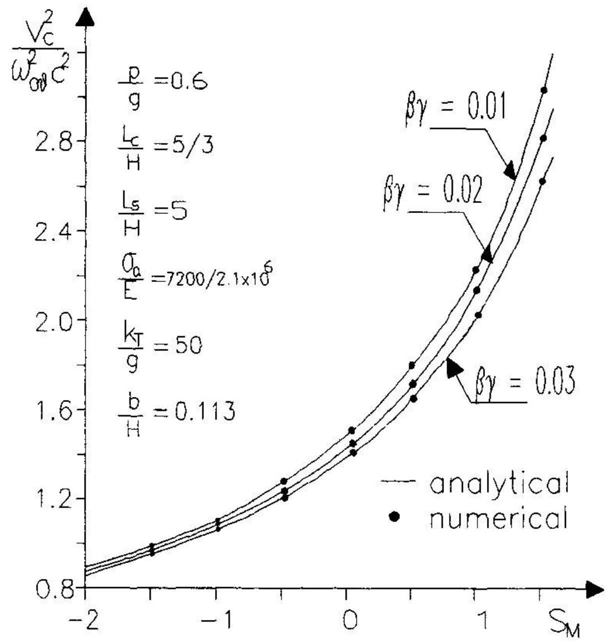

Fig.3: Critical wind speed versus slope S_M

In Fig.2 the flutter critical wind speed V_c is plotted versus the frequency ratio parameter φ; it can be observed that for $\varphi=\omega_{0V}/\omega_{0\theta}=1$, any wind speed is critical. In addition, a very high sensitivity of the bridge aerodynamic behaviour emerges with respect to variations of φ. In Fig.3 the torsional mode of flutter of a stalled airfoil is examined; in particular, the critical wind speed V_c is plotted versus the slope S_M of the steady state moment-angle of attack. It can be observed that the effect of the mass coefficients β and γ is very small and become practically negligible for negative values of S_M. Moreover, the numerical results obtained by the discrete model of the bridge well agree with the analytical ones obtained by the continuous model of the bridge.
In conclusion, the two models here established, that is the discrete model of the bridge and the continuous one, seem to work in a good agreement. Moreover, it can be observed that the discrete model allows us to analyze more complex situations, where the continuous theory is hard to apply, that is, for instance, non constant cross section of the girder, variable live loads, nonlinearities of stays. However, the continuous model seems to be capable of capturing the main features of the dynamical bridge behaviour,which is a useful tool to validate numerical results obtained by FEM computer codes.

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