Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 79 (1998)

Artikel: Strength optimisation and crack resistance of RC structures

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DOI: https://doi.org/10.5169/seals-59993

Strength Optimisation and Crack Resistance of RC Structures

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Summary.

The calculation of strength for reinforced concrete members is at present executed pursuant to requirements [1], where for efforts each of combinations in sections, in view of position of the section relatively to longitudinal axis of member (normal, inclined, spatial) the appropriate formulas for calculation of strength are adduced. The aid [2], made in development of [1], contains the auxiliary tables oriented on «hand-operated» calculations. However such approach is badly combined with opportunities of computer simulation.

1. Optimum Formulation

At the same time, all problems of calculation of strength, submitted in [1] and [2] have sufficient generality, as follows: in each of them, at given efforts, overall dimensions of section, classes of concrete and reinforcement, it is necessary to find the minimum area of reinforcement at which the conditions of strength, as well as parametrical and structural restrictions, intended in [1], are provided. Such formulation of the problem of calculation of strength quite correspondents to the problem of nonlinear mathematical programming (NMP), as it represented, for example, in [3]:

$$\min\left\{F(\bar{x})\middle| g_i(\bar{x}) \geq 0, i = 1,..q; h_j(x) = 0, j = 1,..,P\right\}. \qquad (1)$$

Here: $\bar{x} = x_i(1,..,n)$ - n-dimensional the vector of unknown variables; $\Phi(\bar{x})$ - scalar, in general case - nonlinear functions of all several variables x_i; $g_i(\bar{x}), h_j(\bar{x})$ - scalar, in general case - nonlinear functions of all or some variables forming system of restrictions, correspondingly, in form of unequations and equations.

It is important to note that formalization of wide class of strength problems in form of optimization model [1] permits during the construction of appropriate computing algorithms to use enough general dependencies which describes stress-strained state of reinforcement and concrete in section, without additional simplifications, stipulated by the limited opportunities of «hand-operated» calculation.

This article enters the improved diagram of dependence of stress in longitudinal reinforcement σ_s from the relative depth of compressed zone of concrete ξ for bending and eccentric compressed members from concrete having class B30 and below, with reinforcement of class A-I, A-II, A-III (see fig.1, curve 1). The stress σ are described by not direct but broken line in segment $\xi \in [\xi_R, 1]$ with point of inflection in {} in contrast to known simplified diagram, adduced in [4] and used in [1]. By this it is provided the better coincidence with exact formula of stress [4].

Here: $\bar{\sigma} = \frac{\sigma_s}{R}$ -reduced stress in reinforcement placed at extended or less compressed edge of member.

Beside it is used a single expression for determination of stresses in longitudinal reinforcement

$$\sigma_s = R_s \cdot F(\xi), \xi \in [0,1.1] \qquad (2)$$

where $F(\xi)$ - function, approximated the considered diagram. The interpolation polinom of 6-th degree is applied as such function.

$$F(\xi) = a_0 + \sum_{1}^{6} a_i \xi^i \qquad (3)$$

Values of coefficients $a_i (i = 0,..,6)$ for classes of concrete B12,5 and B15 by $\gamma_{B2} = 0{,}9$ are calculated on computer and listed in table.

Table.

a_i	Class of Concrete	
	B12.5	B15
a_0	0.987019	0.987970
a_1	3.036522	2.868721
a_2	-36.030163	-34.394078
a_3	140.012541	135.665211
a_4	-224.673126	-222.200063
a_5	149.077017	151.287490
a_6	-33.430986	-35.232187

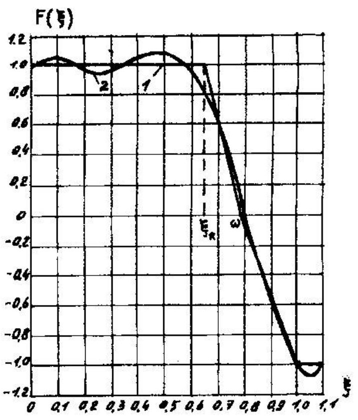

fig 1.

The diagram $F(\xi)$, having been received for concrete B15, is represented on fig.1 (curve 2). Let's consider some problems of calculations of reinforced concrete members formulating as a problem of NMP using dependencies (2) and (3). All the designations, except of specially mentioned, are accepted from [2].

2. The strength calculation for normal sections of bended members

The relative depth of compressed zone of concrete, as well as tensioned and compressed areas of reinforcement are accepted as unknown variables of the NMP problem:
$x_1 \equiv \xi, x_2 \equiv As, x_3 \equiv A's$. The NMP problem accepts the following air:
Find the minimum of object-function:

$$\min \Phi(x) = x_2 + x_3 \tag{4}$$

by executing of:

a) conditions of strength

$$x_1(1-0.5x_1) + \frac{x_3 R_s (h_0 - a')}{R_B b h_0^2} - \frac{M}{R_B b h_0^2} \geq 0 \tag{5}$$

b) equation of balance

$$\frac{\left(x_2\left(a_0 + \sum_1^6 a_i x_1^i\right) - x_3\right)}{x_1} - \frac{R_B b h_0}{R_s} = 0 \tag{6}$$

c) parametrical and structure restrictions

$$\left.\begin{array}{l} x_1 > 0, \\ x_1 < 1.1, \\ x_2 - 0.0005bh0 \geq 0, \\ x_3 \geq 0. \end{array}\right\} \tag{7}$$

Solutions of problem (4)-(7) is illustrated by control examples taken from [2].
The example 1: b=30cm; h=80cm; a=5cm; a'=3cm; concrete B15 ($\gamma_{B2} = 0{,}9$); reinforcement AIII; M=20.0 $\text{toneforce} \cdot \text{m}$. The results (the correspondenced values from [2] are indicated in brackets):
$x_1 \equiv \xi = 0.323(0.322)$; $x_2 \equiv A_s = 11.36(11.34)\text{cm}^2$
The example 2: b=30cm; h=80cm; a=5cm; a'=3cm; concrete B15 ($\gamma_{B2} = 0{,}9$); reinforcement AIII; M=80.0 $\text{toneforce} \cdot \text{m}$. The results :
$x_1 \equiv \xi = 0.538(0.550)$; $x_2 \equiv A_s = 35.96(35.91)\text{cm}^2$; $x_3 \equiv A_s' = 10.62(10.00)\text{cm}^2$.

3. The strength calculation for eccentric stressed members

Unknown variables of the NMP problem are accepted as in previous example:
$x_1 \equiv \xi; x_2 \equiv A_s; x_3 \equiv A_s'$.
The object-function has a previous air and conditions of strength is presented as:

$$\frac{2(R_B b h_0^2 x_1(1-0.5x_1) + R_s(h_0 - a')\cdot x_3)(K_1 + K_2(x_2 + x_3) - N)}{2e_0(K_1 + K_2(x_2 + x_3)) + (h_0 - a')(K_1 + K_2(x_2 + x_3) - N)} \geq N \tag{8}$$

Here :

$$K_1 = \frac{1.6 E_B bh}{3\left(\frac{l_0}{h}\right)^2 \varphi_1}\left(\frac{0.11}{0.1 + \delta_e} + 0.1\right),$$

$$K_2 = \frac{1.6E_S}{\left(\frac{l_0}{h}\right)^2}\left(\frac{h_0 - a'}{h}\right)^2,$$

Then the equation of balance is:

$$\left(x_2\left(a_0 + \sum_1^6 a_i x_1^i\right) - x_3\right) + \frac{N}{R_S x_1} - \frac{R_B b h_0}{R_S} = 0 \tag{9}$$

Parametrical and structure restrictions are recorded as

$$\left.\begin{aligned} & x_1 > 0, \\ & x_1 \le 1.1, \\ & x_2 - 0.0005bh_0 \ge 0, \\ & x_3 - 0.0005bh_0 \ge 0. \end{aligned}\right\} \tag{10}$$

It is accepted that in case of symmetric reinforcement $x_2 = x_3$ in problem (4),(8)-(10).
The example 3: b=40cm; h=60cm; a= a' =4cm; concrete B25
($\gamma_{B2} = 0{,}9$); $l_0 = 6.0\text{m}$; $N_1 = 70.0\,\text{tonforce}$; $E_B = 275000\text{kgforce}/\text{cm}^2$; reinforcement AIII;
$M_1 = 21.3\,\text{toneforce}\cdot\text{m}$.. The results (value from [2] are in brackets):
$x_1 \equiv \xi = 0.235(0.235)$; $x_2 = x_3 \equiv A_s = 6.51(7.6)\text{cm}^2$;

4. The strength calculation of inclined sections of bended members

The length of projection of inclined crack $C_0 \equiv x_1$, the length of projection of inclined section $C \equiv x_2$ and the area of cross reinforcement within the limits of inclined crack, referred to space of cross bars $\frac{A_{sw}}{S} \equiv x_3$ are accepted as unknown variables of the NMP problem.
The NMP problem has the following air:
The object-function:

$$\min \Phi = x_1 \cdot x_3 \tag{11}$$

condition of strength:

$$\frac{(1+\varphi_f)\varphi_{B2}R_{Bt}bh_0^2}{x^2} + R_{sw}\cdot x_1x_3 + q_1\cdot x_2 - Q_{max} \ge 0 \tag{12}$$

restriction on maximal value of $Q_B \le Q_B^{\max}$:

$$x_2 - \frac{\varphi_{B2}h_0}{2.5} \ge 0 \tag{13}$$

restriction on minimum value of $Q_B \ge Q_B^{\min}$:

$$x_2 - \frac{\varphi_{B2} h_0}{\varphi_{B3}} \leq 0 \quad (14)$$

restriction on minimum of web reinforcement:

$$x_3 - \frac{(1+\varphi_f)\varphi_{B3} R_{Bt} b}{2R_{SW}} \geq 0 \quad (15)$$

restrictions on value of C_0:

$$x_2 - x_1 > 0 \quad (16)$$

$$x_1 - \frac{\varphi_{B2} h_0}{2.5} \geq 0 \quad (17)$$

$$x_1 - 2h_0 \leq 0 \quad (18)$$

and, at least, condition of equality $Q_B = Q_S$:

$$\frac{x_1^2 x_3 - (1+\varphi_f)\varphi_{B2} R_{Bt} b h_0^2}{R_{SW}} = 0 \quad (19)$$

The example 4: b=20cm; h=40cm; a=3cm;concrete B15;
($\gamma_{B2} = 0,9$); cross reinforcement A-I;
$Q_{max} = 13.75$ tonforce;
$q_1 = 3.2$ tonforce / m.
The results (value from [2] are in brackets):
$x_1 \equiv C_0 = 45.2(44.8)\text{cm}^2$; $x_2 \equiv C = 107(108)\text{cm}$;
$x_3 \equiv A_{SW} / S = 0.07(0.07)\text{cm}^2 / \text{cm}$.

Conclusions

As is obviously, the results of solution of examples 1-4 having been received approach, have good coincidence with results from [2].
The all numerical experiments were conducted by computer IBM PC in environment of programming «EUREKA» [5] and have shown the high efficiency of offered approach. By this, the possibility and expediency of unification of calculation problems of strength for reinforced concrete members by reducing them to corresponding problems of NMP. Such approach can be fruitful also for the problems of calculation of strength in which the new physical models of reinforced concrete with plenty of unknown parameters are used and for which the «hand-operated» calculations are highly difficult. The new method of calculation of strength for reinforced concrete structures under the action of cross forces, offered in [6], can serve as example of such problem.

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