Three-Dimensional Response Spectrum Analysis for Multicomponent Seismic Excitation

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Summary

Within the framework of response spectrum analysis, a general solution for the three-component orthotropic seismic excitation problem is presented. The contributions from three different orthogonal earthquake components are combined in a rational manner to the maximum and minimum values of any structure response quantity. The critical orientation of the seismic input associated with these values is also determined. The method incorporates in its formulation the Penzien-Watabe model of ground motion. Therefore, in contrast to the SRSS rule, it can explicitly account for the correlation of the three seismic components, which makes it particularly usefull in the dynamic analysis of curved bridges. All given relations are easy to implement in current standard dynamic analysis software.

1. Introduction

According to the model of Penzien and Watabe [1], the three translational seismic motion components on a specific point of the ground are statistically uncorrelated along a well-defined orthogonal system of axes whose orientation remains reasonably stable over time during the strong motion phase of an earthquake. This system of principal axes of the ground motion is oriented such that the major principal axis "p" is horizontal and directed towards the epicenter, the intermediate principal axis "w" is in the transverse (orthogonal) direction, and the minor principal axis "v" is vertical (The choosen notation shall remind of $\underline{P}$enzien-$\underline{W}$atabe model). This orthotropic ground motion is described by three generally independent response spectra S^a, S^b and S^c, with $S^a > S^b > S^c$.

In the special case of equal horizontal components $S^a = S^b$, the extreme values of the structure's response quantities do not depend on the direction "a" of the epicentral seismic component [2,3]. However, in the general case of $S^a > S^b$, the extreme value of a response quantity strongly depends on this direction. Therefore, the determination of the most unfavorable (critical) epicentral direction for each response quantity is of great practical interest. Smeby and Der Kiureghian [4] determined the critical direction in case of analogous response spectra $S^a = \gamma S^b$, where $0 <= \gamma <= 1$. Also, Anastassiadis [3] and Lopez and Torres [5] determined the critical direction for the more general case of arbitrary response spectra.

In this paper, on the basis of the Penzien-Watabe idealization, the tensorial properties of the extreme values of response are presented. The critical epicentral direction as well as the correspondent maximum and minimum values of an arbitrary response quantity can be straightforwardly deduced from these properties.

2. Notation

A fixed global orthogonal reference system Oxyz is used for the structure. The spectra S^a or S^b are applied individually in the direction of the x- or y-axis according to Figure 1. The corresponding peak probable values of a typical response quantity R (force or displacement) are symbolized as $R_{,xa}$, $R_{,xb}$, $R_{,ya}$ and $R_{,yb}$ (Figure 1a,b,c,d), where the first subscript refers to motion in direction x or y and the second (a or b) to the input earthquake spectrum (S^a or S^b). $R_{,x}$ and $R_{,y}$ symbolize the extreme values of a typical response quantity R produced from a bidirectional excitation with epicentral direction along the axis x or y respectively (Figure 1e,f). An analogous notation is used for the variable system of principal axes Opwv.
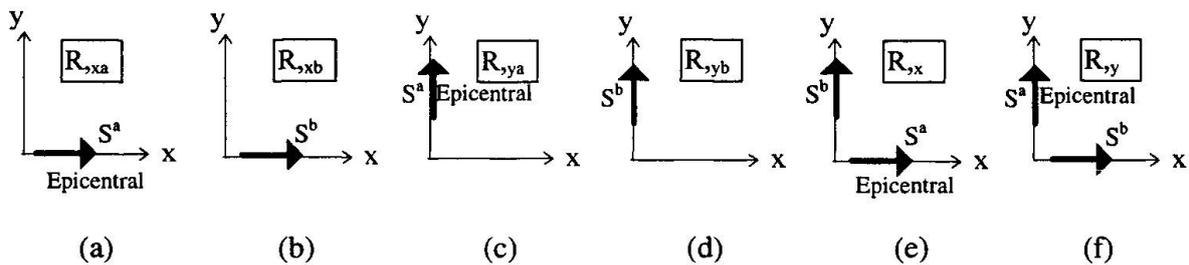

Figure 1. Response parameter notation

3. Tensorial properties - Critical direction

We assume that the epicentral principal axis p of the ground motion is defined in terms of an angle θ relative to the x axis of the fixed reference system of the structure (Figure 2). If S^a is the design spectrum in the direction of p axis and S^b the design spectrum in the direction of w axis, the probable extreme value of a response quantity R is [6] :
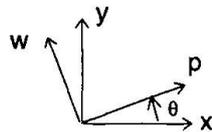
Figure 2. Definition of angle θ

$$R_{,p}^2 = R_{,pa}^2 + R_{,wb}^2 = \sum_i \sum_j \varepsilon_{i,j} \left(R_{i,pa} R_{j,pa} + R_{i,wb} R_{j,wb} \right) \tag{1a}$$

In the above expression, $\varepsilon_{i,j}$ denotes the correlation coefficient between the responses in modes i and j, and $R_{i,pa}$ and $R_{i,wb}$ denote the modal values of quantity R for the excitations defined by the indices after comma. If S^b is the design spectrum in the direction of p axis and S^a the design spectrum in the direction of w axis, we obtain the probable extreme value

$$R_{,w}^2 = R_{,pb}^2 + R_{,wa}^2 = \sum_i \sum_j \varepsilon_{i,j} \left(R_{i,pb} R_{j,pb} + R_{i,wa} R_{j,wa} \right) \tag{1b}$$

and the correlation term

$$R_{pw} = R_{pw,a} - R_{pw,b} = \sum_i \sum_j \epsilon_{i,j} \ (\ R_{i,pa} R_{j,wa} - R_{i,pb} R_{j,wb} \ ) \tag{1c}$$

The modal values $R_{i,p}$ and $R_{i,w}$ are connected to the modal values $R_{i,x}$ and $R_{i,y}$ through the following relations, which are independent of the used earthquake spectrum S^a or S^b :

$$R_{i,p} = + R_{i,x} \cos\theta + R_{i,y} \sin\theta \tag{2a}$$

$$R_{i,w} = - R_{i,x} \sin\theta + R_{i,y} \cos\theta \tag{2b}$$

Inserting these relations in the right-hand terms of (1) we obtain

$$R_{,p}^2 = R_{,x}^2 \cos^2\theta + R_{,y}^2 \sin^2\theta + R_{xy} \sin2\theta \tag{3a}$$

$$R_{,w}^2 = R_{,x}^2 \sin^2\theta + R_{,y}^2 \cos^2\theta - R_{xy} \sin2\theta \tag{3b}$$

$$R_{pw} = - (\tfrac{1}{2}) (R_{,x}^2 - R_{,y}^2) \sin2\theta + R_{xy} \cos2\theta \quad , \tag{3c}$$

where

$$R_{,x}^2 = R_{,xa}^2 + R_{,yb}^2 = \sum_i \sum_j \epsilon_{i,j} \ (\ R_{i,xa} R_{j,xa} + R_{i,yb} R_{j,yb} \ ) \tag{4a}$$

$$R_{,y}^2 = R_{,xb}^2 + R_{,ya}^2 = \sum_i \sum_j \epsilon_{i,j} \ (\ R_{i,xb} R_{j,xb} + R_{i,ya} R_{j,ya} \ ) \tag{4b}$$

$$R_{xy} = R_{xy,a} - R_{xy,b} = \sum_i \sum_j \epsilon_{i,j} \ (\ R_{i,xa} R_{j,ya} - R_{i,xb} R_{j,yb} \ ) \tag{4c}$$

It is important to note that relations (3) are similar to the transformation rules for the components of a symmetric second order tensor. Consequently, the four quantities $R_{,x}^2$, $R_{,y}^2$ and $R_{xy} = R_{yx}$ can be considered as components of a symmetric second order tensor, expressed analytically by matrices

$$\begin{bmatrix} R_{,x}^2 & R_{xy} \\ R_{yx} & R_{,y}^2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} R_{,p}^2 & R_{pw} \\ R_{wp} & R_{,w}^2 \end{bmatrix}$$

in the Oxy and Opw reference system respectively. Due to its tensorial character, the arbitrary response quantity R is characterized by the following properties which are common to all symmetric second order tensors :

(a) The trace and the determinant of the above matrices are not dependent on the orientation of the earthquake excitation :

$$R_{,x}^2 + R_{,y}^2 = R_{,p}^2 + R_{,w}^2 \quad \text{and} \quad R_{,x}^2 R_{,y}^2 - R_{xy}^2 = R_{,p}^2 R_{,w}^2 - R_{pw}^2$$

(b) There is a specific earthquake orientation defined by the axes (I, II) for which the correlation term R_{pw} vanishes. This specific orientation is determined by the critical angle (see eq. 3c) :

$$\theta_{cr} = (\tfrac{1}{2}) \tan^{-1}(2R_{xy}/(R_{,x}^2 - R_{,y}^2)) \tag{5}$$

The corresponding response quantity R takes the following maximum and minimum values :

$$\max R^2 = R_{,I}^2 = (R_{,x}^2 + R_{,y}^2)/2 + \sqrt{[(R_{,x}^2 - R_{,y}^2)/2]^2 + R_{xy}^2} \tag{6a}$$

$$\min R^2 = R_{,II}^2 = (R_{,x}^2 + R_{,y}^2)/2 - \sqrt{[(R_{,x}^2 - R_{,y}^2)/2]^2 + R_{xy}^2} \tag{6b}$$

(c) The correlation term R_{pw} takes its maximum value

$$\max R_{pw} = R_{12} = (\tfrac{1}{2})(R_{,I}^2 - R_{,II}^2)$$

for a seismic excitation along axes (1,2) defined by the angle bisecting the axes (I, II). For these seismic directions, a response quantity R takes the value

$$R_{,1}^2 = R_{,2}^2 = (\tfrac{1}{2})(R_{,I}^2 + R_{,II}^2)$$

i.e., the interchange of the input design spectra S^a and S^b along the axes 1 and 2 does not affect the peak value of R.

It is clear from the preceding considerations that the calculation of the maximum and minimum values of an arbitrary response quantity requires four independent dynamic analyses of the structure, applying input spectra S^a and S^b as shown in Figures 1a to 1d. All necessary terms, e.g., the modal values in the right-hand sides of equations (4), are routinely calculated by current standard linear dynamic analysis programs. Then, using (4), $R_{,x}^2$, $R_{,y}^2$ and R_{xy} can be computed, and from (6a,b) the maximum and minimum values of any response quantity R can be immediately obtained, with no need to previously calculating the critical angle θ. Finally, the contribution of the vertical seismic component is to be added to the above values, according the SRSS combination rule. It is obvious that all mentioned relations can be easily implemented in current standard software for multicomponent seismic analysis.

4. Conclusions

A general solution for the three-component orthotropic seismic excitation problem is presented. It offers, within the framework of response spectrum analysis, a rational procedure for determining the maximum and minimum values of any given response quantity R of a structure. It also provides a simple means of determining the critical orientation θ associated with the extreme values of R. In contrast to the SRSS rule prescribed by many design codes, the presented method can explicitly account for the correlation of the different seismic components by incorporating in its formulation the Penzien-Watabe model of ground motion. This fact makes it particularly usefull in the dynamic analysis of curved bridges. All necessary relations are of a computationally simple form and can be easily implemented in current standard dynamic analysis software.

References

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