

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 78 (1998)  
  
**Artikel:** Design of reinforcement to control cracking due to imposed strains  
**Autor:** Nagy, Agnes  
**DOI:** <https://doi.org/10.5169/seals-59033>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 05.09.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## Design of Reinforcement to Control Cracking Due to Imposed Strains

**Agnes Nagy**  
PhD

Dept. of Struct. Engineering  
Lund University, Sweden



Agnes Nagy, born 1963, received her civil engineering degree 1986, the degree of licentiate of engineering 1994 and the PhD degree 1997.

During the period 1990-1997 she published 4 articles in Materials and Structures and in ASCE Journal of Material in Civil Engineering.

### Summary

The amount of reinforcement to control cracking of concrete structures is investigated. The effect of non-uniform cooling and shrinkage is studied with regard to the external and internal restraint acting on the structure. By means of a simple model several different cooling processes and restraint degrees are simulated for different wall sections. The results of the computation give the force actually arising in the structure during the cooling or shrinkage process and the range of softening and cracking in the concrete section. The aim is to investigate the parameters having major impact on the magnitude of this force and consequently influence the design of minimum reinforcement. The investigation concludes some simple guidelines regarding the design of minimum reinforced concrete structures in practical cases.

### 1. Introduction

Most existent concrete structures are provided with a significant amount of reinforcement to control cracking due to imposed strains. The amount of reinforcement necessary for crack control depends on three factors: the magnitude and distribution of imposed strains in the section, the degree of restraint of the structure and the deformation capacity of the concrete. Especially the first two factors are difficult to estimate and in practical design it is usually assumed on the safe side that the imposed strains are large and that the restraint is almost complete without further analysis. The consequence of such an overestimation in massive structures is a large amount of reinforcement, which makes the casting process difficult and counteracts durability performance.

Present design rules require that the capacity of the minimum reinforcement at yield should be higher than the force necessary to create a new crack [1]. In this way the reinforcement redistributes the stresses after the appearance of the first crack so that several narrower cracks arise instead of a few wide-opened [2].

In this paper investigations are made regarding walls with restraint exposed to different cooling processes. Rapid as well as slow temperature changes on the surface, the influence of wind and daily temperature cycles for summer and winter are considered. The spatial distribution of imposed strain developed from sudden cooling is roughly equivalent with the imposed strain from drying shrinkage [3], i.e. the results and conclusions regarding sudden cooling can also be applied for drying shrinkage.



## 2. Constitutive Model

In this paper a medium-thick wall, characterized by the geometry shown in Fig.1, is used as a reference structure. The external restraint acting on the wall is modeled with a set of springs characterized by the spring coefficient  $k$  (N/m<sup>2</sup>) related to the spring force  $P_z$  (N/m). Cooling is modeled as a linear thermal diffusion process, one dimensional and symmetric with respect to the mid plane of the wall. The wall is assumed to be in a state of generalized plane strain in the  $y$  and  $z$  directions.

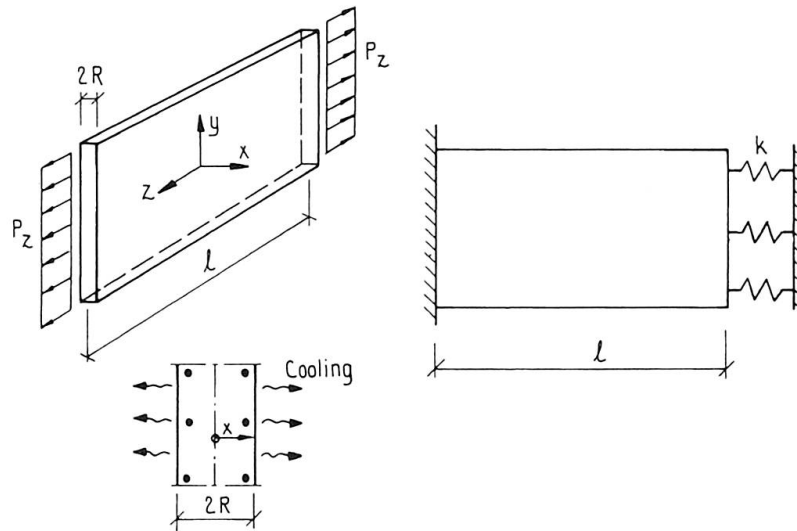


Fig.1 Geometry of the medium-thick wall.

The overall equilibrium in the  $z$  direction gives

$$\int_0^R \sigma_z dx + \sigma_{zs} \cdot A_s = \frac{P_z}{2} \quad (1)$$

where  $\sigma_z$  is the stress in the concrete,  $\sigma_{zs}$  is the stress in the steel,  $A_s$  is the reinforcement area/unit length at each surface of the wall and  $P_z$  is restraint force/unit length. The assumed stress-strain relation for the concrete in tension is shown in Fig.2. The steel is assumed to be linearly elastic during the whole cooling process.

The restraint force  $P_z$  during the cooling process and its maximum value  $P_{zmax}$  may be computed under these assumptions. This value may be compared with the maximum possible force in the section,  $P_z^{lim}$  - the nominal tensile capacity, corresponding to the case when the tensile strength of the concrete is reached simultaneously over the whole section. The following relation is valid for  $P_z^{lim}$ :

$$\frac{P_z^{lim}}{2} = Rf_{ct} + E_s A_s \epsilon_{sT_{tot}} \quad (2)$$

where  $f_{ct}$  is the tensile strength of the concrete,  $E_s$  is the Young's modulus of the steel and  $\epsilon_{sT_{tot}}$  is the total imposed thermal strain in the steel. In reality the actual force  $P_{zmax}$  is usually less than  $P_z^{lim}$  due to successive softening and cracking over the section. For further details regarding the principles of this analysis, see [4].

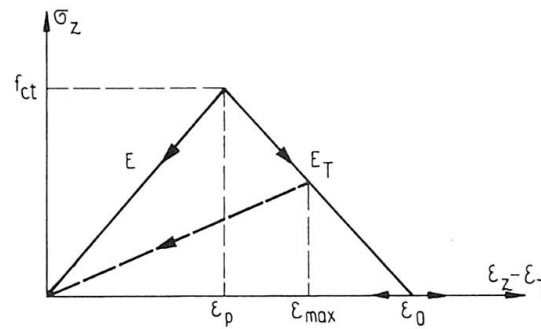


Fig.2 Assumed bilinear stress-strain relation for concrete in tension.

### 3. Characterization of Imposed Strains

Computations of the temperature fields in a wall with thickness  $2R=0.4$  m has been made by Hacon-T [5], a program for simulation of temperature in concrete. The initial uniform temperature of the concrete is taken to  $20^\circ\text{C}$ . The following data were used for the concrete: coefficient of heat transfer  $\lambda=2.1$  W/m·K, heat capacity  $c=1000$  J/kg·K and density  $\rho=2350$  kg/m<sup>3</sup>. Fig.3a shows the results for the case when the ambient temperature suddenly drops to zero. The computations were made for three different values of the surface heat transfer coefficient  $h$  corresponding to three values of wind velocity. Fig.3b corresponds to slow cooling, with a rate of decrease in ambient temperature equal to  $0.27^\circ\text{C/h}$ . It is seen that the temperature distribution in the wall is fairly uniform in most cases, except for the extreme situation in Fig.3a with  $h=1000$  W/m<sup>2</sup>·K. This value corresponds to a case with practically no surface resistance, which may occur in practice if the surface is directly exposed to wind with very high velocity or due to shrinkage process.

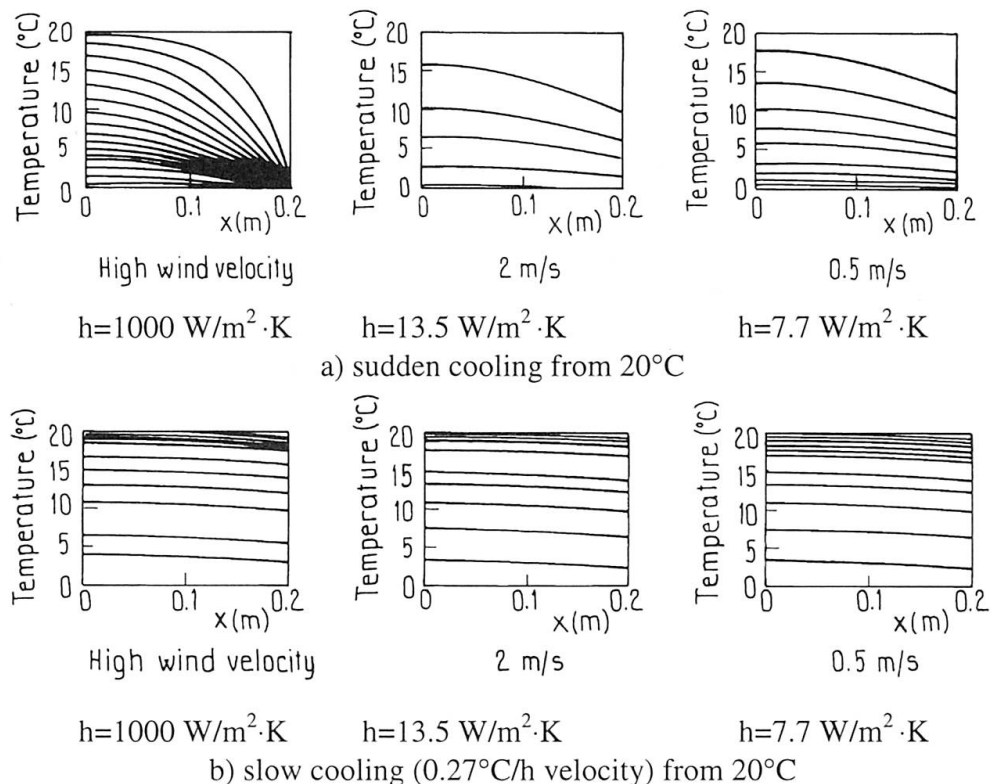


Fig.3 Temperature distribution in concrete wall for different cooling processes.



Investigation of the effect of daily temperature cycles in summer and winter has been made. Simulation of the temperature fields in a wall with thickness  $R=0.6$  m is made for a daily variation of ambient temperature between  $12.9^\circ\text{C}$  and  $21.2^\circ\text{C}$  during summer.

The initial uniform temperature in the wall is set to  $17^\circ\text{C}$  during summer and the value of  $h$  was chosen to  $7.7\text{ W/m}^2\text{C}$  corresponding to a very low wind velocity. Fig.4 illustrates the range of temperature variation in the wall when exposed to a daily summer cycle. It is seen that the daily temperature cycle has almost no influence in the middle of the wall and causes only a few degrees change near the surface. During winter the temperature changes are even smaller.

The investigation leads to the conclusion that imposed thermal strain in general may occur both with an almost uniform distribution and with a highly non-uniform distribution over the cross section. For the purpose of discussion, it seems rational to idealize the thermal action as two types of processes, one where the imposed strain is *uniform* over the cross section, corresponding to most of the cooling processes in practice such as temperature change from summer to winter. The other one is with highly *non-uniform* distribution of the imposed strains corresponding to a shrinkage process characterized by a step change at the surface or a cooling process with high wind velocity.

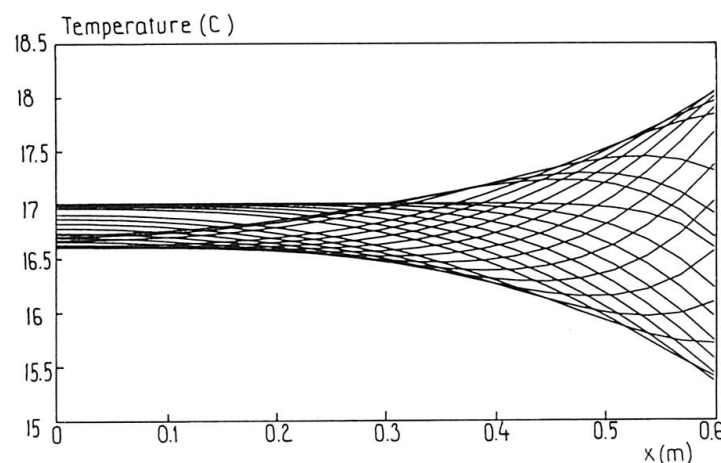


Fig.4 24 hours temperature cycle during summer in a wall with  $2R=1.2$  m.

#### 4. Influence of Internal and External Restraint

The simulations presented in this section are based on cooling processes of the type shown in Fig.3a for sudden cooling, with five different values of the temperature drop ranging from  $10^\circ\text{C}$  to  $35^\circ\text{C}$ . The calculations are made for two different values of the spring coefficient  $k$ : one with  $k\ell/2ER=225$ , i.e. very strong external restraint representing a wall with practically full restraint, and the other with  $k\ell/2ER=1$  representing a wall with a restraint stiffness equal to the axial stiffness of the wall itself. Two different concrete sections have been investigated: one with  $2R=0.4$  m and a thicker one with  $2R=1.2$  m. The temperature field in the wall is computed both with a high heat transfer coefficient ( $h=1000\text{ W/m}^2\text{K}$ , c.f. the left Fig.3a) and with a comparatively low heat transfer coefficient ( $h=7.7\text{ W/m}^2\text{K}$ , c.f. the right Fig.3a). The most important characteristics of the concrete used in the parametric investigation are:  $E=20\text{ GPa}$ ,  $\epsilon_p=0.000145$ . The amount of reinforcement is  $A_s=0.0004\text{ m}^2/\text{m}$  corresponding to  $\phi 10$  c 200 at each surface of the wall and with  $E_s=200\text{ GPa}$ . ( $\rho=0.002$ ,  $\alpha\rho=0.02$ , where  $\rho=A_s/A_c$  and  $\alpha=E_s/E$ ). The value of the ratio  $E_T/E$  is taken to  $-1.55$ . The computed ratio  $P_{z\max}/P_z^{\text{lim}}$  is given as a function of the parameter  $\epsilon_{T\text{tot}}/\epsilon_p$ , the ratio between the total induced thermal strain  $\epsilon_{T\text{tot}}$  (at the

end of the process) and the strain capacity  $\epsilon_p$  at tension failure.  $P_{z\max}$  is the maximum restraint force obtained during the process and  $P_z^{\lim}$  is the upper limit of the force according to Eq. (2). In Fig.6 results from computations for sudden cooling or strong internal restraint are shown. For the case with strong external restraint when the imposed thermal strain  $\epsilon_{T\text{tot}}$  is higher than the ultimate tensile strain  $\epsilon_0$  a through crack is developed. With the chosen value of  $E_T/E$ , this occurs for values of the ratio  $\epsilon_{T\text{tot}}/\epsilon_p$  higher than 1.65, c.f. For cases with weak external restraint a through crack is developed for values of  $\epsilon_{T\text{tot}}/\epsilon_p$  higher than 2.5.

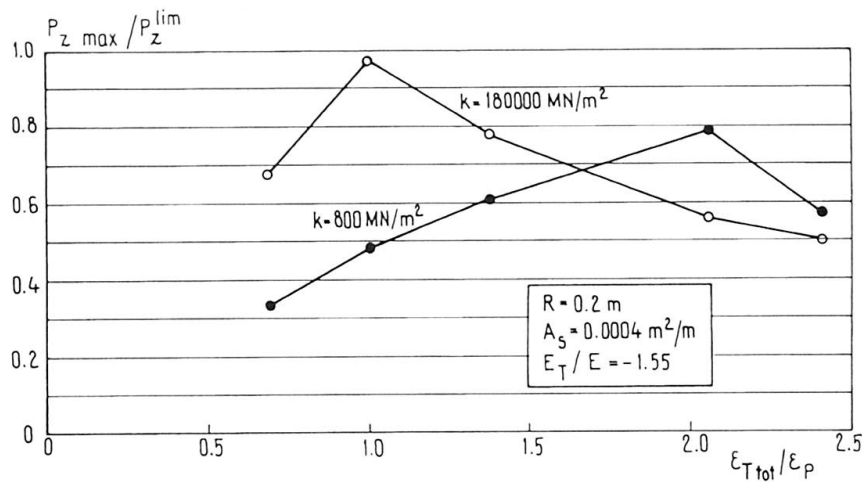


Fig.6 Force in restrained wall subjected to sudden cooling.

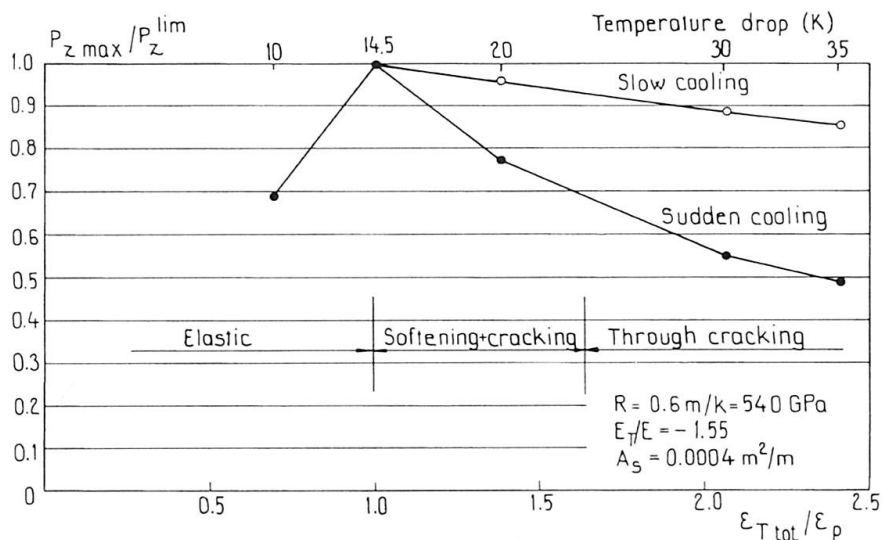


Fig.7 Force in restrained wall for strong external restraint with rapid and slow cooling.

The reinforcement is fully needed only after a through crack has developed. The design of the reinforcement should be based on the maximum restraint force  $P_{z\max}$  developed in processes where a through crack occurs. Fig.6 shows that both for strong external restraint and weak external restraint the force actually arising when a through crack develops is only about 0.5-0.7 of the maximum possible force  $P_z^{\lim}$  leading to a considerable reduction of the amount of reinforcement. The reason for this is the successive growth of softening and cracking through the section created by the non-uniform distribution of imposed strain in this case.



In Fig.7 similar results are displayed for a slower cooling process with a heat transfer coefficient  $h=7.7 \text{ W/m}^2\text{K}$ . A comparison is made with the case with rapid cooling with a surface heat transfer coefficient  $h=1000 \text{ W/m}^2\text{K}$ . The slow cooling process gives higher values of the actual force for high values of  $\epsilon_{\text{Ttot}}/\epsilon_p$ , since the diminished temperature gradient during slow cooling leads to a more uniform stress state in the section. Consequently, the tensile stresses are primarily governed by the external restraint acting on the wall. Similar results for weak external restraint also show that the ratio  $P_{\text{zmax}}/P_z^{\text{lim}}$  is higher for slow than for rapid cooling for large values of  $\epsilon_{\text{Ttot}}/\epsilon_p$ .

## 5. Conclusions

The minimum reinforcement needed to control cracking due to imposed strain in concrete structures depends on the magnitude and distribution of imposed strain, the degree of restraint and the deformation capacity of the material. In the present paper these factors have been analyzed for a simple reference structure in the form of a wall with axial restraint. For the purpose of discussion, imposed strain may be idealized in two distinct cases, one where the imposed strain is *uniform* over the cross section and one where it is highly *non-uniform* with strong gradients.

For the case with uniformly distributed imposed strain applies the following:

- is mainly relevant for cooling processes in practice such as a temperature change from summer to winter

- even for full restraint, concrete can normally survive an imposed strain of 0.1-0.15 ‰ corresponding to a temperature change of 10-15°C without cracking

For the case with non-uniformly distributed imposed strain applies the following:

- is valid for situations with rapid changes in temperature as well as for imposed strain related to drying shrinkage

- a 30% reduction of the required minimum reinforcement is possible for cases where a through crack appears independently of the magnitude of external restraint

- for cases where the internal restraint is dominating and where the external restraint is limited, usually no through cracks will be created and thus the crack reinforcement may be limited, the distribution of cracks will take place anyway as in plain concrete without reinforcement

- the amount of reinforcement needed to distribute cracks is smaller for non-uniform strains such as drying shrinkage than for uniformly distributed strains characteristic for most of the cooling processes.

## 6. References

- [1] CEB, 'Design Manual on Cracking and Deformations', No 158, 1985.
- [2] Jaccoud J. P., 'Armature minimale pour le contrôle de la fissuration des structures en béton', École Polytechnique Fédérale de Lausanne, Thèse No 666, 1987.
- [3] Thelandersson S., Mårtensson A., Dahlblom O., 'Tension Softening and Cracking in Drying Concrete', Materials and Structures, (1988), 21, pp 416/424.
- [4] Nagy A., 'Thermally Induced Softening and Cracking in Concrete with Minimal Reinforcement', accepted for publication in RILEM Journal Materials and Structures. 1996.
- [5] Dahlblom O., 'HACON-T - A Program for Simulation of Temperature in Hardening Concrete', Vattenfall (Swedish State Power Board) R,D&D. U 1990/31. ISSN 1100-5130.