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Stress-Strain Distribution in the Contact Surface of a Two-Layered RC Structural Element

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Summary

The contribution deals with the time-dependent analysis of the state of stress and strain distributions on the contact surface of a reinforced concrete two layered element (in the sense that „new material" means topping layer and „old material" means prestressed plate structure). We suppose that the structural system is linearly viscoelastic quasihomogenic isotropic continuum. The linear aging model for concrete creep is used. The theoretically derived solution procedures are examplified by particular numerical examples.

1. Mathematical Formulation of the Problem

We will deal in our analysis with the quasistatic problem of the two-layered planar composite structure with technologically conditioned defects. We suppose that the principles of the linear theory of viscoelasticity are valid. The mathematical representation of the discussed physical model is the operator equation [1]

$$\mathrm{L}\,(\mathbf{u}) = \mathbf{f} \tag{1}$$

in the domain $\Omega = \Omega_1 \cup \Omega_2$ $(\Omega \subset E_3)$ which is bounded per partes smooth surface $S = S_1 \cup S_2$ $(S_1 \cap S_2 = \varnothing)$, $\mathbf{u} = \mathbf{u}(\mathrm{P}, \mathrm{t}) = \{\mathrm{u}(x, y, z, t), \mathrm{v}(x, y, z, t), \mathrm{w}(x, y, z, t)\}$ is a sufficiently smooth vector-function, where $P \in \Omega$, $t \geq 0$. The solution of the Eq.(1) satisfied the following conditions

$$u^{\lambda} = 0 \qquad \text{on } S^{a}, S^{b} \tag{2}$$

$$\sigma^{\lambda\kappa} n_{\kappa} + \sigma^{(0)\kappa\mu} n_{\kappa}\, u^{\lambda}{}_{,\mu} = \bar{F}^{\lambda} \qquad \text{on } S - \left(S^{a} \cup S^{b}\right), \tag{3}$$

$$u^{\lambda}\big|_{\Omega_1} = u^{\lambda}\big|_{\Omega_2} \qquad \text{on } \Omega_1 \cap \Omega_2 . \tag{4}$$

This problem is equivalent to the problem of finding the element of energetic space which mininimalize energetical functional [1,2]

$$\mathrm{F}\,(\mathbf{u}) = (\mathbf{u}, \mathbf{u})_{L(.)} - 2(\mathbf{u}, \mathbf{f}) \tag{5}$$

The basic idea of FEM is to represent the displacement functions $\mathbf{u} = (u, v, w)$ within the element by continuum shape functions of the form

$$u(x,y,z,t) = \sum_{i=1}^{L} N_i(x,y,z)u_i(t), \ldots \tag{6}$$

where N_i $(i = 1,2,\ldots L)$ are the shape functions associated with the L nodes and u_i $(i = 1,2,\ldots L)$ are nodal values of the displacement $u(x,y,z,t)$, etc. The strain rate field within the element can be defined as [3]

$$\dot{\varepsilon}(x,y,z,t) = \mathbf{B}(x,y,z)\dot{\delta}(t), \tag{7}$$

where $\mathbf{B}$ is a matrix of gradients of the shape functions and $\dot{\delta}$ is the nodal velocity vector. The equilibrium of the stress rates $\dot{\sigma}$ is given by

$$\sum_{\forall e} \int_{Ve} \mathbf{B}^T \dot{\sigma}\, dV = \dot{\mathbf{R}}^a, \tag{8}$$

where $\dot{\mathbf{R}}^a$ is the applied nodal load rate vector. The stress-strain relationship for inelastic rate processes may be given by the formula

$$\dot{\sigma} = \mathbf{D}(\dot{\varepsilon} - \dot{\eta}), \tag{9}$$

where $\mathbf{D}$ is elasticity matrix and $\dot{\eta}$ is the creep strain rate (for instance)

$$\dot{\eta} = \Re(\sigma, t, T).$$

We may transform the system of equilibrium equations into the matrix equation

$$\mathbf{K}\dot{\delta} = \dot{\mathbf{R}}^a + \dot{\mathbf{R}}^\eta \qquad \text{on } \Omega, \tag{10}$$

where $\mathbf{K} = \sum_{e} \int_{Ve} \mathbf{B}^T\mathbf{DB}\, dV$ is the assembled stiffness matrix and

$$\dot{\mathbf{R}}^\eta = \sum_{e} \int_{V^e} \mathbf{B}^T\mathbf{D}\, \dot{\eta}\, dV \tag{11}$$

is the vector of the creep process. After some rearrangements Eqs (10) and (11) can be written in he incremental form over a time interval

$$\mathbf{K}\Delta\delta = \Delta\mathbf{R}^a + \Delta\mathbf{R}^\eta \qquad \text{on } \Omega, \tag{12}$$

where

$$\Delta\mathbf{R}^\eta = \sum_{e} \int_{V^e} \mathbf{B}^T\mathbf{D}\, \Delta\eta\, dV. \tag{13}$$

Numerical schemes for the solutions of Eqs (12), (13) are discussed in [4].

2. Linear Aging Concrete Creep and Shrinkage Model

The characteristic of concrete that distinguishes it from the traditional viscoelastic material is the aging effect. Experimental tests [5] indicate that response on the increment of load is independent of all other part load increments, so that the principle of superposition is valid. Due to this fact the integral-type creep law has the form [6]

$$\varepsilon(t)=\int_0^t \prod(t,t')d\sigma, \tag{14}$$

where

$$\prod(t,t')=J(t,t')\begin{bmatrix} 1 & -\nu & -\nu & & & \\ -\nu & 1 & -\nu & & & \\ -\nu & -\nu & 1 & & & \\ & & & \frac{1}{2}(1+\nu) & & \\ & & & & \frac{1}{2}(1+\nu) & \\ & & & & & \frac{1}{2}(1+\nu) \end{bmatrix}$$

and

$$J(t,t')=\frac{1}{E(t')}+\sum_{j=1}^{n}\hat{A}_j^{-1}(t')(1-e^{-(t-t')/\tau_j}),\; E(t')=E_{28}\sqrt{\frac{t}{4+0.85t}},$$

where τ_j are constants (retardation times) and $\hat{A}_j$ are aging coefficients. For shrinkage of concrete we can assume the relation [6]

$$\varepsilon_{sh}(t)=0.0008c'(t-7)/(35+t-7) \tag{15}$$

in which t is in days and drying is assumed to begin at t_0 = 7 days, $c'=\prod_1^6 c_i$ is correction factor [6].

3. Numerical Example

This section deals with the numerical solution of some viscoelastic aging creep problems - two layered reinforced concrete plate with a prestressed lower layer. The analysed plate is simply supported in positions a and b (Fig. 2). The length of the system is 6000 mm, width 1190 mm. The thickness of the lower layer is 70 mm and upper concrete layer 170 mm, respectively. The cross-section of the given structure is in Fig.1

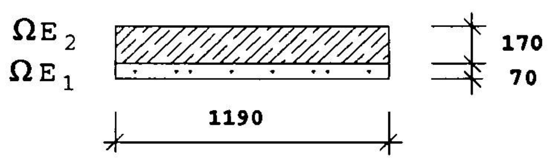

Fig. 1 Cross-section of the plate system

The plate carries a uniformly distributed load with the intensity q = 5,68 kN/m^2 (dead load) and a line load with the intensity p = 20,07 kN/m (Fig.2). The triangle represents the course of the prestressed force in the individual strings. The prestress on 8 wires ϕ 5mm is realised.

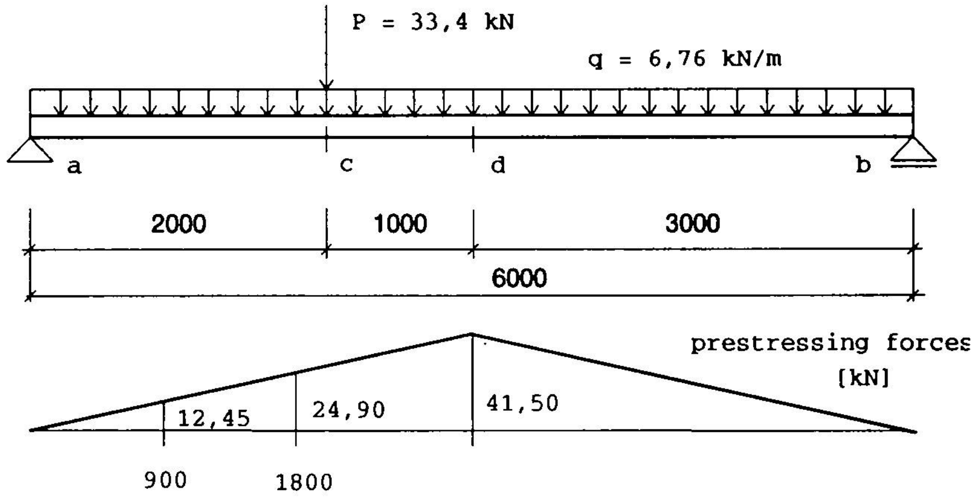

Fig. 2 Longitudinal section and values of prestressed forces

Characteristics of the materials are as follows: a lower concrete layer of type B 30 ($E_1 = 32500$ MPa) according to the code STN 73 1201 [7] and an upper layer B 20 ($E_2 = 27000$ MPa), respectively. The following cases were considered:

1) upper layer shrinking, lower layer non creeping □ s/nc
2) both layers creeping O c/c

The time intervals taken: 0, 5, 10, 20, 50, 100, 300, 500, 1000 and 3000 days. We assume a 28 day time interval for hardening of concrete. The top layer was laid down after 200 days. Between the layers the conditional technological horizontal defect range of the half-span was taken into account , too (case B). It means, that the following cases were solved:

A - without defect on section a - b
B - with defect on the section a - d.

Figs 3 and 4 represent the time - dependet vertical deflections in the half-span of the system.

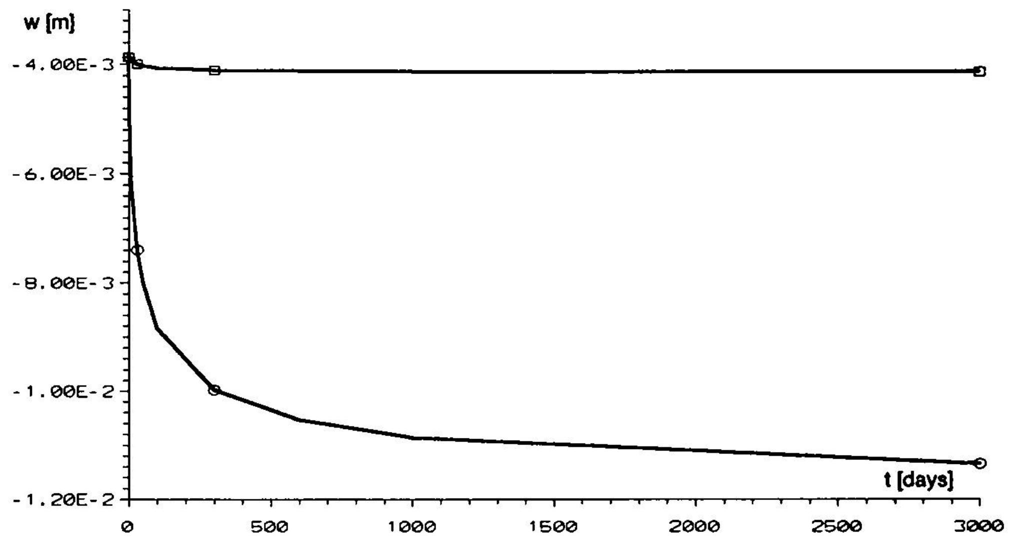

Fig. 3 Vertical deflections w(d,t) for case A, □ s/nc, O c/c

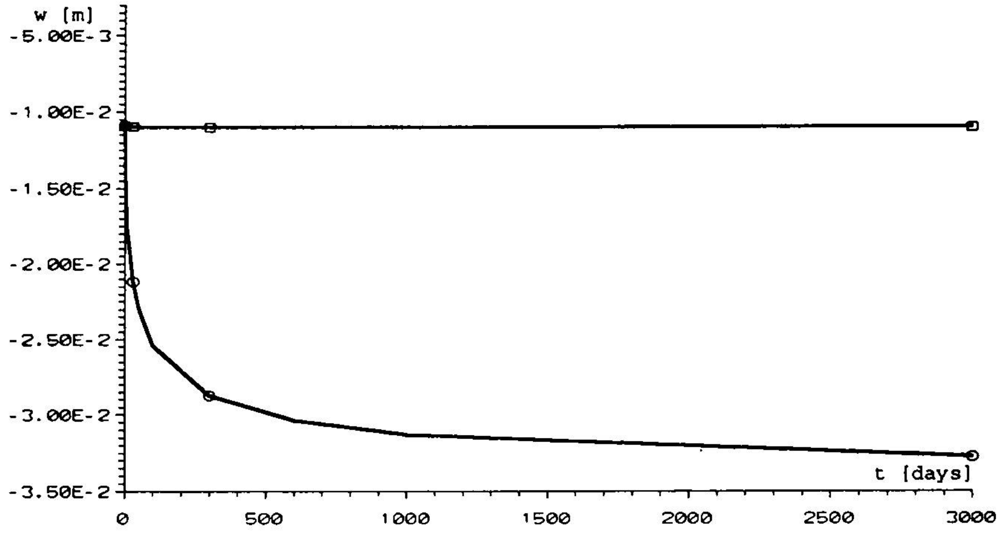

Fig. 4 Vertical deflections w(d,t) for case B, □ s/nc, O c/c

Normal and shear stresses for individual cases (A and B) under line - load (position c - where max. bending moment was expected) were calculated. Courses of normal stresses σ_x for cases A and B are given in Figs 5 and 6.

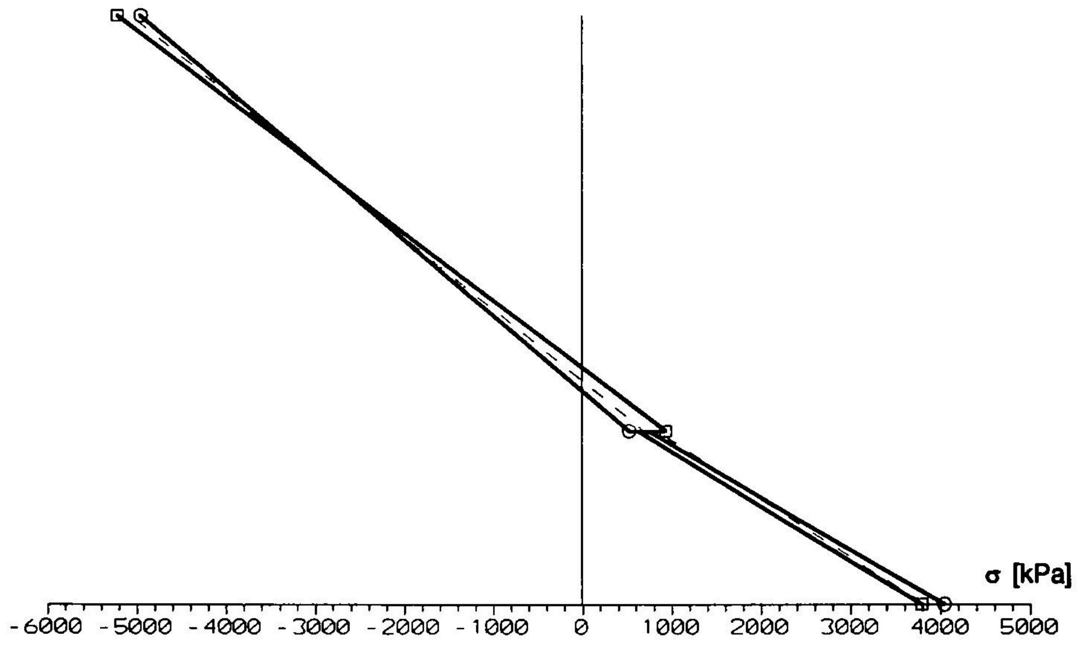

Fig. 5 Normal stresses $\sigma_x(c,t)$ for case A; □ s/nc, O c/c. The full lines represent values $\sigma_x(c, t \to \infty)$ and dashed line $\sigma_x(c, t \equiv 0)$

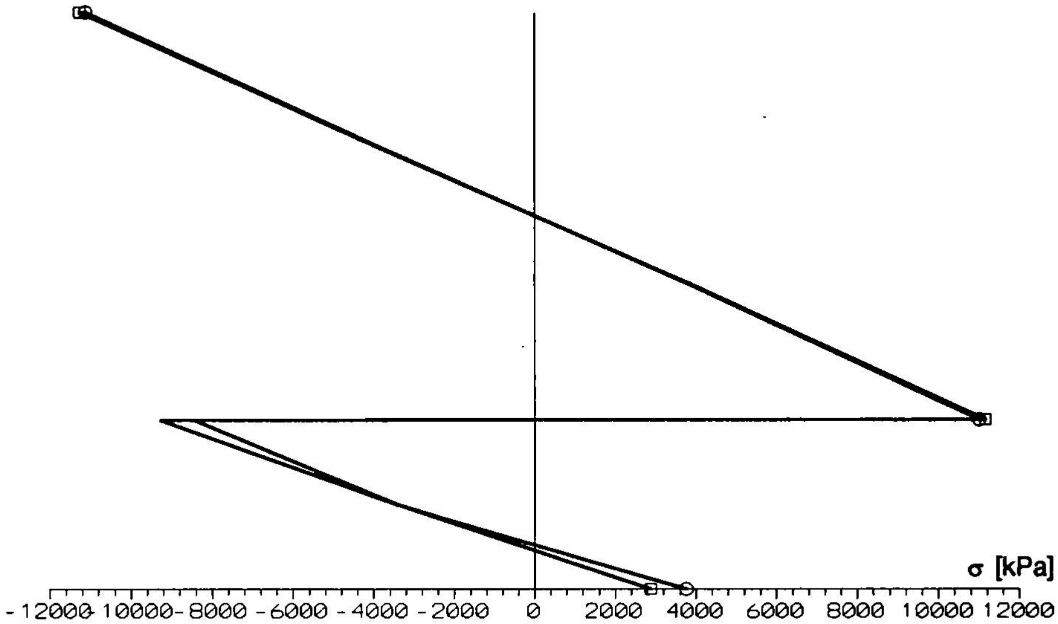

Fig. 6 Normal stresses $\sigma_x(c,t)$ for case B; □ s/nc, O c/c. The full lines represent values $\sigma_x(c,t\to\infty)$ and dashed line $\sigma_x(c,t\equiv 0)$

4. Conclusions

- on the interface of the layers cumulation of the shear stresses on the perimeter of the structure occured
- creep - deflection was 3 time larger than classical elastic deflection, that positively influence the course of the stresses
- the presence of technologically conditioned defects on the interface of the layers negatively influenced the stiffness of the system, in spite of the prestress on the lower layer
- a very significant element of the layered system is the technological surfacing, of the interface of the layers.

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