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## **Evaluation of Long-Term Effects in the Steel-Concrete Composite Beams**

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### **Summary**

In the paper, some of results obtained by using the Age-Adjusted Effective Modulus Method to evaluate creep and shrinkage effects in the steel-concrete composite beams with rigid or deformable connections are presented. Aging coefficients for the creep, relaxation and shrinkage problems, evaluated in absence of connection and rebars, are proposed. General considerations and numerical comparisons performed in the presence of rebars in the slab, point out how the proposed approach can properly substitute the more popular Effective Modulus Method.

### **1. Introduction**

It is known as design criteria proposed by current codes for steel-concrete composite beams, beside the ultimate limit states, require to satisfy the serviceability limit states [1,2]. Indeed, deflection limits, mainly introduced to reduce concrete cracking and to preserve supported elements together with serviceability stress limits based on durability criteria, govern more and more the design of composite beams. A correct evaluation of creep and shrinkage effects on stress and deflection long-term response is then very important. In general, actual codes make reference to simplified algebraic methods for the practical evaluation of this response; they are the "Effective Modulus Method" (EM), the "Mean Stress Method" (MS) and, when an accurate evaluation of viscous effects is required, the "Age-Adjusted Effective Modulus Method" (AAEM).

In the paper, working in the ambit of the AAEM method, as Age-Adjusted Effective Modulus of the concrete component beam, is assumed the one computed in absence of shear connection and rebars. With reference to the CEB 90 Model Code [3], simplified expressions of the Aging coefficient are given. By some examples, the capability to obtain very accurate long-term solutions for composite beams with rigid or deformable connections also in presence of rebars is shown. With the proposed approach, besides to have very correct results without remarkable complications, it is possible to follow a more physical approach respect to the EM method. For these reasons we think that it can in general substitute adequately the popular EM method, which can be adopted to analyse composite beams when long-term effects are not important.

## 2. The Proposed Approach

It is known as the AAEM method for homogeneous structures, in the hypothesis of linear visco-elasticity, allows to achieve an exact solution for a linear combination of pure creep and relaxation problems. In all other cases, adopting an aging coefficient  $\chi$  evaluated in these hypotheses, an approximate solution can be reached. In particular for composite beams, the presence of a viscous material (the concrete) and an elastic material (the steel), involves a migration of stresses from a point of the structure to another one, with different variation laws respect to an homogeneous structure. From a theoretical point of view, it is therefore not appropriate to apply the AAEM method with the same  $\chi$  values adopted for homogeneous structures, since the response is influenced strongly by the presence of the steel beam. For this reason, Trost proposed two  $\chi$  coefficients in the presence of rigid connection: the former one, labelled  $\chi_N$ , related to the normal force in the concrete component beam; the latter one, named  $\chi_M$ , related to the bending moment in the same beam [4]. In the hypotheses of strong steel beam respect to the concrete slab and affinity of the shrinkage law with the creep law, Trost found for creep or relaxation problems simple relations to evaluate  $\chi_N$  in a rigorous way and  $\chi_M$  in approximate way. The complexity of the problem due to the presence of two coefficients  $\chi$  as well as the limits due to Trost's hypotheses have induced the authors to propose, for the composite beams with rigid or deformable connections the use of one approximate  $\chi$  value only [5,6]. For three elementary problems of creep, relaxation and shrinkage, these  $\chi$  values were evaluated in the hypothesis of no connection between concrete and steel beam and no rebar in the slab, providing an exact value for creep and relaxation problems and a numerical evaluation for the shrinkage problem. In particular, for the creep problem was shown as in absence of connection and rebar, introducing the coefficient  $\beta = (E_s J_s) / (E_c(t_0) J_c + E_s J_s)$ , where  $E_s J_s$  and  $E_c(t_0) J_c$  are the steel beam and concrete slab bending stiffnesses at initial time  $t_0$ , the stress evolution in the slab for a creep problem in the time interval  $[t_0, t]$  is the same of a pure relaxation problem of an homogeneous concrete structure with the fictitious coefficient  $\bar{\phi}(t, t_0) = \beta \phi(t, t_0)$ , instead of creep coefficient  $\phi(t, t_0)$ . Following the Bazant approach [7], it was then possible to compute the parameter  $\chi = \chi_M$  for this composite beam. In the hypothesis of no connection, few additional difficulties respect to an homogeneous structure are then introduced to compute the exact solution (with the EM method, where  $\chi = 1$ , or the MS method, where  $\chi = 0.5$ , the response is obviously approximate). Also the proposed method becomes approximate in the presence of an elastic or rigid connection. In particular for rigid connections,  $\chi$  is not constant in the slab but it is in any point a linear combination of  $\chi_M$  and  $\chi_N$  Trost's values.

With reference to the CEB 90 Model code and ACI 92 creep model, by performing a numerical analysis, we have shown as using the only  $\chi = \chi_M$  value of the beam without connection we get in general an accurate solution since  $\chi_N$  value is less influent on the solution of  $\chi_M$ , that is practically independent of the connection stiffness [6]. As a consequence, by assuming in the study of a generic composite beam  $\chi = \chi_M = \chi_N$ , it is natural to calibrate the  $\chi$  coefficient on the value determined in absence of connection, where  $\chi_N \rightarrow \chi_M$ , since it can be considered the most significant value for the beam. To use the AAEM method with the same simplicity of EM and MS methods, we provide simple relations in order to calculate the  $\chi$  coefficient. In particular for the pure creep problem, to evaluate the  $\chi = \chi_c(t_\infty, t_0) = \chi_\infty$  value, linked with the long-term solution, we propose an extension of the approximate expression given by Lacidogna for a homogeneous structure [8]:

$$\chi_c(t_\infty, t_0) = \chi_c(3 \cdot 10^4, t_0) = \frac{t_0^{0.5}}{n + t_0^{0.5}}, \quad (1)$$

where  $t_0$  is the initial time load,  $n$  is a corrective coefficient calibrated on the fictitious thickness  $h_0 = 2A_c/u$  expressed in centimeters ( $A_c$  is the area of the concrete slab while  $u$  represents his perimeter in contact with the atmosphere), the relative humidity R.H.(%), the characteristic strength of concrete  $f_{ck}$  (MPa) and the coefficient  $\beta$ . The coefficient  $n$  is calculated as summation of Lacidogna's term  $n_L$  and the corrective term  $n_C$ . The values of  $n_L$  and  $n_C$  are defined as:

$$n_L = f_a(h_0) \left[ 1 + \left( 1 - \frac{R.H.}{50} \right) f_b(h_0) \right] f_c(f_{ck}), \quad (2)$$

with

$$f_a(h_0) = \frac{0.28 h_0^{1/3}}{e^{(10^{-3} h_0)}}, \quad f_b(h_0) = -0.772 + 2.917 \cdot 10^{-3} h_0, \quad f_c(f_{ck}) = 0.772 + 0.0114 f_{ck}, \quad (3a, bc)$$

and

$$n_C(\beta, h_0) = 0.4133 (1-\beta)^3 + (0.2765 + 9.7545 \cdot 10^{-3} h_0 - 4.2689 \cdot 10^{-5} h_0^2) (1-\beta). \quad (3d)$$

These expressions provide accurate results when  $5 \leq h_0 \leq 160$  cm,  $50\% \leq R.H. \leq 80\%$  and  $3 \leq t_0 \leq 200$  days and imply 5% maximum error and 1% medium error. In figure 1a, a comparison between exact and proposed (dotted line)  $\chi_{c\infty}$  values is shown.

In the case of an imposed flexural distortion (very important relaxation problem to evaluate the stress state in a statically indeterminate composite beam with constant mechanics characteristics subjected to a settlement of the supports), in the hypothesis of no connection, the concrete beam is subjected to an effect of pure relaxation which does not depend on the presence of the steel beam. This allows to affirm that the  $\chi$  coefficient determined by means of the proposed approach is equal to that of an homogeneous structure subjected to a constant strain and vice versa. Analogous results is reached by working in the hypothesis of rigid connections because even in this case the strain law adopted for the determination of  $\chi$  coefficient for homogeneous structures is exactly respected with reference to concrete ( $\epsilon_c$  is constant in time in accordance with the second theorem of the linear viscoelasticity). The  $\chi_\infty$  values, denoted here as  $\chi_{r\infty}$ , can be then easily found by setting  $\beta=1$  in the eqn. (3d).

To evaluate slab shrinkage effects it should be observed that the laws of shrinkage evolution in time proposed by actual codes are not affine with the creep laws in general (if shrinkage is affine to creep,  $\chi_s$  shrinkage values can be calculated as in the case of constant load, by assuming  $\chi_s = \chi_c$ ). With reference to the CEB 90 model code where the shrinkage is not affine, analysing numerically the long-term response we have verified that also in this case the validity of the proposed method in the presence of shear connection is good. An approximate expression for the  $\chi = \chi_s(t_\infty, t_0) = \chi_{s\infty}$  coefficient is proposed here to evaluate the long-term effects. For this problem the significant parameters are the shrinkage initial time  $t_0$ , the relative humidity R.H., the characteristic strength of concrete  $f_{ck}$  and the fictitious thickness of the slab  $h_0$ . The stiffness of the steel beam and concrete slab does not influence practically the response. The  $\chi_\infty$  values, determined here with reference to a section in which  $\chi_N \rightarrow \chi_M$ , i. e. for a small stiffness of the steel beam respect to the concrete slab, with the same units seen above, take the form:

$$\chi_{s\infty} = \chi_s(3 \cdot 10^4, t_0) = \frac{4.364}{h_0} - \frac{8.9776}{h_0^2} + \left[ 0.1909 + \frac{3.8416}{9.5132 + h_0} - \frac{53.4992}{(9.5132 + h_0)^2} \right] \log(t_0) +$$

$$- 5.4306 \cdot 10^{-4} (R.H. - 75) - 8.956 \cdot 10^{-4} (f_{ck} - 30). \quad (4)$$

Figure 1b shows a comparison between numerical (continuum line) and proposed (dotted line)  $\chi_{s\infty}$  values. It is evident as the  $\chi_{s\infty}$  values can be very different from  $\chi_{c\infty}$  when the CEB 90 model is adopted.

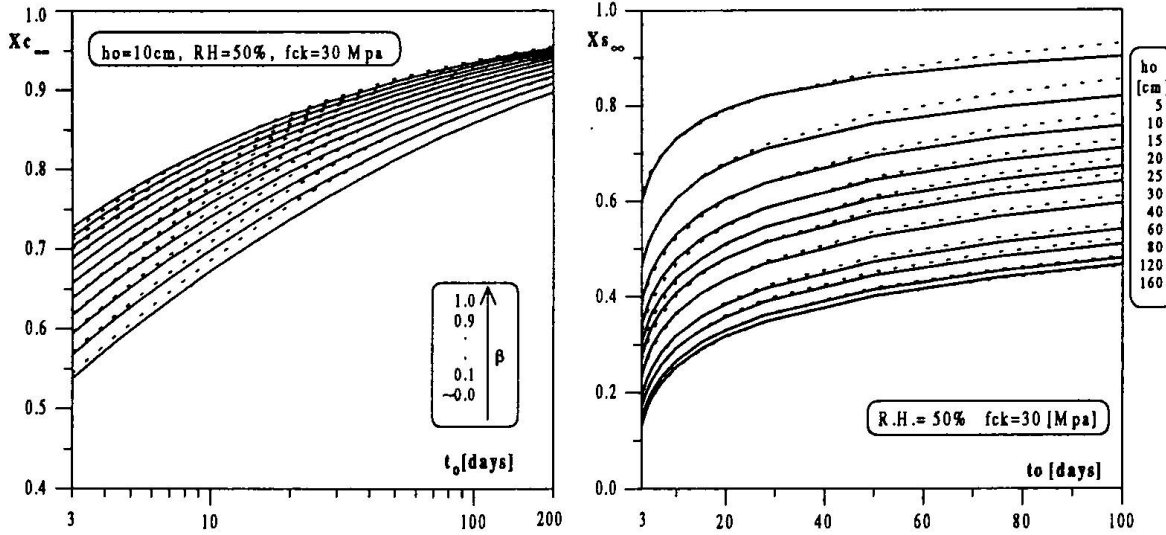


Fig. 1: a)  $\chi_{c\infty}$  values for the imposed load problem. b)  $\chi_{s\infty}$  values for the shrinkage problem.

### 3. Applicability of the Method

When the  $\chi_{\infty}$  values for the three fundamental cases are known, the study of composite beams with rigid or deformable shear connections in the hypothesis of uncracked concrete can be performed in a very simple way. However, it is important to observe as this formulation is particularly suitable to study beams where the slab is used to be completely in compression (for example simply supported bridges or beams with prestressed slab).

#### 3.1 Rigid connections

For rigid connections, the classical assumption that the whole cross section remains plane can be adopted and the long-term analysis can be performed at the cross section level. Following the AAEM approach, the stress-strain law for the concrete can be posed in the form:

$$\epsilon_c(t) - \epsilon_n(t) = \frac{\sigma_c(t)}{E_{cadj}} + \sigma_c(t_0) \left( \frac{1}{E_{ceff}} - \frac{1}{E_{cadj}} \right) = \frac{\sigma_c(t)}{E_{cadj}} + \bar{\epsilon}(t), \quad (5)$$

where  $\bar{\epsilon}(t)$  can be considered as an imposed strain linked to the viscosity effects in the interval  $[t_0, t]$ , while the quantities

$$E_{ceff} = \frac{E_c(t_0)}{1 + \phi(t, t_0)}, \quad E_{cadj} = \frac{E_c(t_0)}{1 + \chi(t, t_0)\phi(t, t_0)}, \quad (6)$$

as known, are the "Effective Modulus" and "Age-Adjusted Effective Modulus" respectively,  $\epsilon_c(t)$ ,  $\epsilon_n(t)$  are the elastic and inelastic strain at time  $t$  and  $\sigma_c(t_0)$ ,  $\sigma_c(t)$  the elastic stress in the concrete

at times  $t_0$  and  $t$ . To evaluate the response at time  $t_0$  where the concrete has modulus  $E_c(t_0)$  and at infinite time where it has modulus  $E_{cadj}$  evaluated with  $\chi = \chi_\infty$  for any elementary problem, it appears natural to introduce the modular ratios  $n_0 = E_s/E_c(t_0)$  and  $n_\infty = E_s/E_{cadj}$ , where  $E_s$  is the steel Young modulus, and to work with the transformed cross sections at times  $t_0$ ,  $t_\infty$ . In the presence of rigid connections and constant cross section, for the elementary cases of constant external sustained load, relaxation and shrinkage in the slab, with the above quantities, a long-term solution characterized by the same difficulties of application of the EM method can be reached [5,6]. Then, by applying the principle of effect superposition, it is possible to solve a large number of actual problems. We underline that the proposed approach, through the evaluation of the imposed strains  $\bar{\epsilon}(t)$ , allows a correct interpretation of viscous problem, since it not consists in the only change of the concrete modulus as in the EM method.

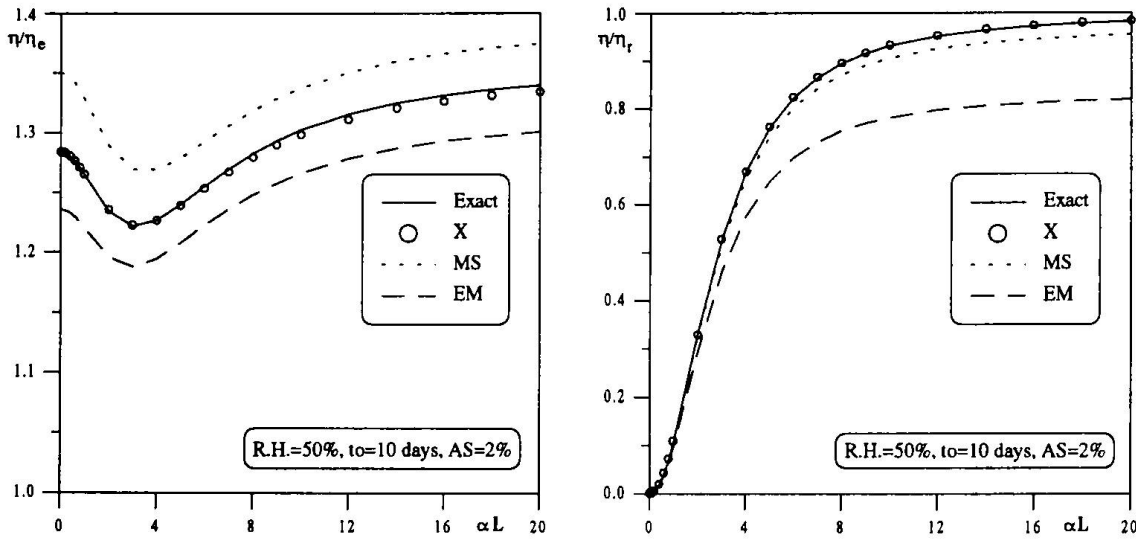


Fig. 2: a) Sustained load problem.

b) Shrinkage in the slab problem.

### 3.2 Deformable connections

To evaluate creep and shrinkage effects in a generic composite beam with deformable connections, it is not possible to work only at the section level but the whole beam has to be considered. In order to solve this problem, in general it is convenient to use a computer program. In this case, a remarkable simplification respect to an algorithm that utilizes a step-by-step procedure can be obtained. To solve for instance a pure creep problem, instead of 30+50 steps, by using the constitutive equation (5) and the  $\chi_{c_\infty}$  proposed coefficients, only two steps are necessary: one at time  $t_0$  and one at time  $t_\infty$ . A very easy method to solve, by means of the proposed  $\chi$  values, simply supported composite beams with deformable connections under sustained loads or shrinkage in the slab is presented in [9,10].

## 4. Examples and conclusive remarks

Fig. 2 shows a comparison for the  $\eta/\eta_e$  and  $\eta/\eta_r$  ratios obtained with the proposed  $\chi$  method, the exact solution and the EM, MS method varying the parameter  $\alpha L = \sqrt{KL^2(EJ)_r / [(EA)^*(EJ)_a]}$

that characterises the composite beams model with elastic connections introduced in [11]. In these diagrams,  $\eta$ ,  $\eta_e$  are the long-term and initial deflections, evaluated in the midspan section of a simple supported composite beam with deformable connections and length  $L$ , while  $\eta_r$  is the long-term deflection computed in the hypothesis of rigid connection.

The quantity  $(EA)^*$  is equal to  $(E_s A_s E_c A_c)/(E_s A_s + E_c A_c)$ , with  $E_s A_s$  and  $E_c A_c$  axial stiffnesses of steel and transformed concrete component beams at time  $t_0$ , while  $(EJ)_r$  and  $(EJ)_a$  are the bending stiffnesses of the transformed cross section determined at time  $t_0$  in the hypothesis of rigid connection or no connection respectively and  $K$  the connection stiffness. These ratios are computed for a sustained uniform load problem (fig. 2a) and a shrinkage slab problem (fig. 2b) adopting an IPE 300 steel beam and a 80x15 cm concrete slab with  $f_{ck} = 30$  MPa,  $h_0 = 30$  cm and a percentage of symmetric rebar  $AS=2\%$ . We can see as the proposed solutions is very adequate independently of the connection stiffness (when  $\alpha L > 20$  the connection can be considered rigid). Analogous results have been obtained in terms of stresses. In particular, adopting the  $\chi$  proposed values, computed in absence of connection and rebars, we have seen that the presence of a symmetric or asymmetric rebar with a percentage  $AS=0+3\%$  does not change the response accuracy. Further comparisons with EM solutions have allowed to point out the advantage of the proposed approach, in particular for the shrinkage problem where significant errors may occur by applying the EM method. To conclude, we think that when the effects of viscous problems are important, the proposed approach can be properly adopted by designer to analyse a large number of practical problems.

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