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## Creep and Shear-Lag Effects in Composite Beams with Flexible Connection

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### 1. Problem statement

In modeling steel-concrete composite beams, two kinematical aspects should be considered: the deformability of the shear connection and the non-uniform distribution of the longitudinal displacements in the slab (shear-lag). The deformability of the shear connection allows a slip at the beam-slab interface, increasing the global flexibility of the structure, while the shear-lag effect implies a non-uniform distribution of stresses in the slab. Furthermore, the behaviour of the composite beam is strongly influenced by the concrete time-dependent effects [1]. Although the effects of creep, connection deformability and shear-lag have been extensively examined in literature, their interaction is not completely known.

For this purpose, a general analysis for composite beams has been developed to encompass shear-lag effect, flexible shear connection, creep and shrinkage of the concrete [2]. Starting from the definition of a suitable displacement field which takes into account slipping at beam-slab interface and slab shear deformation, a global balance condition is obtained by means of the virtual work principle. By assuming a linear elastic behaviour for steel beam and shear connection, and a linear viscoelastic behaviour for the concrete slab, the problem is governed by a coupled system of four integral-differential equations. The unknowns of the problem are the functions describing beam deflection, axial displacements of the steel beam and the concrete slab, and intensity (along the beam axis) of the shear-lag effect introduced by means of a suitable shape function for the shear warping of the slab cross section (depending on the point of the cross section only). In particular, the shape function is a quadratic function constant on the slab depth, null at the beam-slab interface and satisfying conditions ensuring local equilibrium at the slab free edges.

Given the generality of the creep function adopted, a closed form solution cannot be achieved for the system. In order to obtain an accurate numerical solution, the system is solved by introducing two discretizations: one for the time interval, which permits solving the integral-differential problem by a step-by-step procedure considering a set of simpler differential problems, and the other for the beam axis in order to apply the finite differences method.

### 2. Principal results

An extensive numerical parametric analysis, carried out for beams with different geometry and subjected to different restraints and load conditions, has made it possible to obtain some information on the complex time dependent behaviour of composite structures. In particular, the time evolution of the shear-lag and the mutual influence between shear-lag and connection deformability have been studied in detail. For the sake of brevity, only results related to an isolated case (but which can be qualitatively extended to a wide class of composite structure) are reported here.

Fig. 1 shows the numerical results obtained for a two-span continuous beam. The creep analysis was performed with the CEB creep function [3] by considering the following values for concrete strength and relative humidity:  $f_{ck}=30 \text{ MPa}$  and  $\text{RH}=50\%$ . The solution at loading time  $t_0=28 \text{ days}$  (elastic solution) is compared with the viscoelastic solution ( $t_\infty=25550 \text{ days}$ ). Furthermore, results obtained taking into account the shear-lag effect (curves denoted by SL) are compared with those obtained under the classical hypothesis adopted for composite beams with flexible shear connection, namely preservation of plane cross section for the steel beam and the concrete slab considered separately (curves denoted by P). The most important results are summarised in the sequel.

1. The beam axis deflections notably increase as a consequence of the time-dependent behaviour of the concrete, while they are less sensitive to the shear-lag effect (Fig. 1a).
2. The shear-lag effect, as is well known, strongly modifies the stress distribution in the slab only in the neighbourhood of the intermediate support, by significantly increasing the value which would be obtained by assuming the plane cross section hypothesis for concrete slab and steel beam (Fig. 1b).

- Influence of the shear connection stiffness ( $\rho$ ) on shear-lag is shown in Fig. 1c, where the elastic values of the stresses  $\sigma_{SL}$  and  $\sigma_P$  at the intermediate support cross section are compared. Increasing  $\rho$ , shear-lag stress  $\sigma_{SL}$  increases more than  $\sigma_P$  as shown by the dashed curve related to the ratio  $\sigma_{SL}/\sigma_P$ . The coupling between the shear-lag effect and the shear connection stiffness is thus evident.
- Fig. 1d shows the influence of creep on the shear-lag effect. The time evolution of the ratio between  $\Delta\sigma$  and  $\sigma_{SL}$  (see Fig. 1b) is reported for three different values of the shear connection stiffness. Such a ratio permits defining the slab effective width  $b_{eff}$  (adopted by the principal technical codes, e.g. ENV 1994-2) as

$$b_{eff} = \frac{1}{\sigma_{SL}} \int_{-b/2}^{b/2} \sigma_c dx = b - \frac{\Delta\sigma}{\sigma_{SL}} \int_{-b/2}^{b/2} f(x) dx$$

where  $b$  is the real value of the slab width and  $f(x)$  is a function depending on the cross section only. It is evident that such a ratio, even if it depends on the  $\rho$  value, remains almost constant in time showing a substantial uncoupling between creep and shear-lag effect.

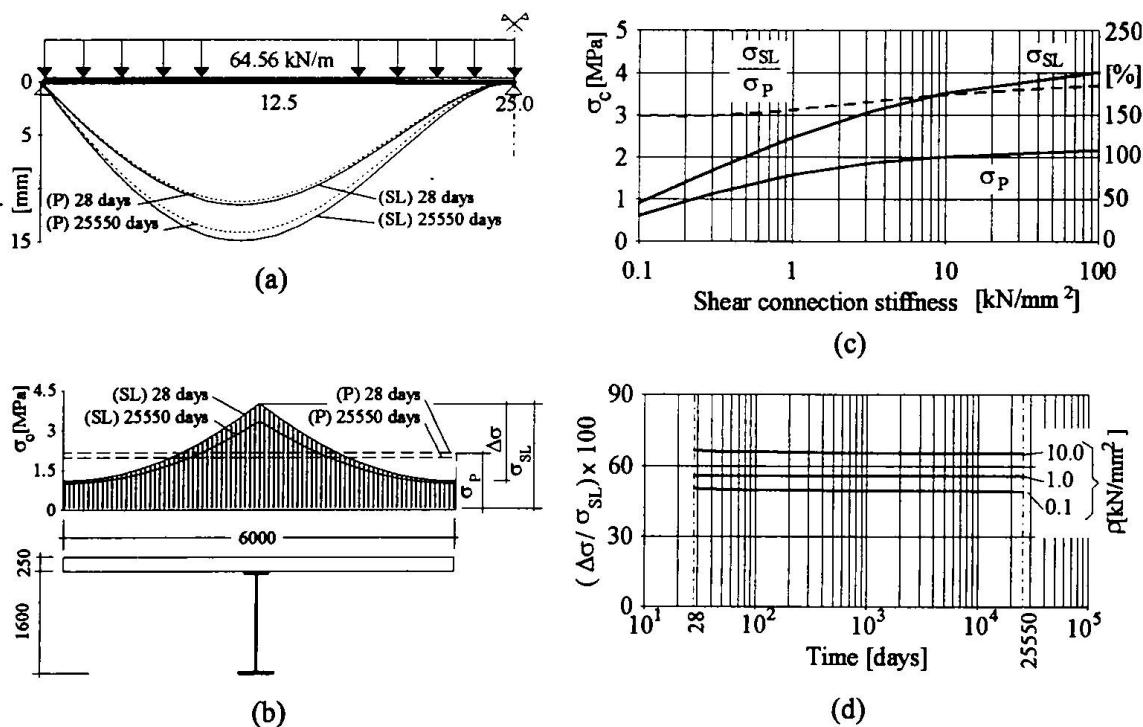


Fig. 1. (a) Influence of shear-lag and concrete creep on the beam deflections. (b) Concrete creep effect on the slab stress distribution. (c) Influence of the shear connection stiffness on the shear-lag effect. (d) Influence of the concrete creep on the shear-lag effect.

### 3. References

- 1 Dezi, L., and Tarantino, A.M. (1993), "Creep in composite continuous beams. I: Theoretical treatment." *J. Struct. Engng. ASCE*, 119(7), 2095-2111.
- 2 Dezi, L., Leoni, L., and Tarantino, A.M. (1997), "Time dependent analysis of shear-lag effect in composite beams." (Submitted for the publication on *J. Struct. Engng. ASCE*).
- 3 "CEB FIP model code 1990." (1988). *C.E.B. Bulletin d'information n. 190*, C.E.B. F.I.P. Comité Euro-International du Beton, Paris, France.

## Calculation of Stresses for Composite Structures

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### Summary

The calculations of the stresses for the statically indeterminate composite structures as general, are presented. The approximate methods EM and AAEM and exact method are applied. The stresses, for the example of statically indeterminate composite structures due to uniformly distributed load and the shrinkage of concrete are determined. Using the limiting concrete creep functions the upper and lower limits of the stresses are determined.

### 1. The exact method (TM)

The cross section of the composite structures contain concrete (b), prestressing steel (p), steel member (n) and reinforcing steel (m). Concrete is considered as a linear viscoelastic material. The relaxation of the presstresing steel is taken into account.

$$\sigma_b = E_{b0} \hat{R}'(\varepsilon - \varepsilon_s), \quad \sigma_p = E_p \hat{R}'_p \varepsilon \quad (1.1)$$

Other kinds of steel: steel member (n) and reinforcing steel(m) obey Hook's law :

$$\sigma_k = E_k \varepsilon \quad k=n,m. \quad (1.2)$$

The exact method, established by Lazic , using linear integral operators, is applied. Starting from the integral stress-strain relationship the expressions for stress and strain, in the exact method, are derived without mathematical negligence. Calculation of statically indeterminate composite structures is same as calculation of the corresponding structures whose material is homogeneous and elastic except that in composite structures we solve integral equations.

### 2. The approximate methods (AAEM, EM )

The algebraic stress-strain relationship for concrete contain two independent parameters: the reduced creep coefficient  $\varphi(t, t_0)$  and the aging coefficient  $\chi = \chi(t, t_0)$  (AAEM). When  $\chi=1$  the same equations represent the EM method.

$$\begin{aligned} \sigma_b &= E_{b0} \zeta_b (\varepsilon - \varepsilon_s) - \rho_b \sigma_b, \quad \sigma_b = \sigma_b(t_0, t_0), \\ \zeta_b &= \frac{1}{1 + \chi \varphi_r}, \quad \rho_b = (1 - \chi) \varphi_r. \end{aligned} \quad (2.1)$$

When the relaxation of prestressing steel is introduced, the algebraic stress-strain

relationship for the prestressing steel may be written as:

$$\sigma_p = E_p \zeta_p \epsilon \quad (2.2)$$

Calculation of statically indeterminate composite structures is same as calculation of the corresponding structures whose material is homogeneous and elastic.

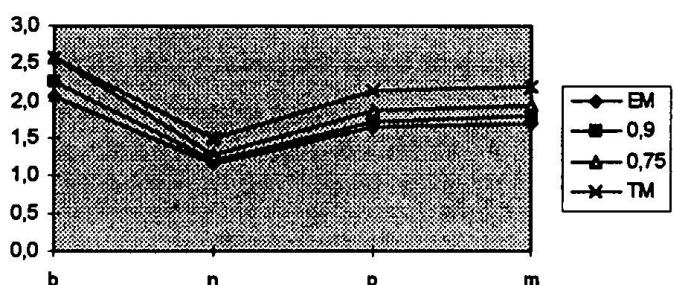
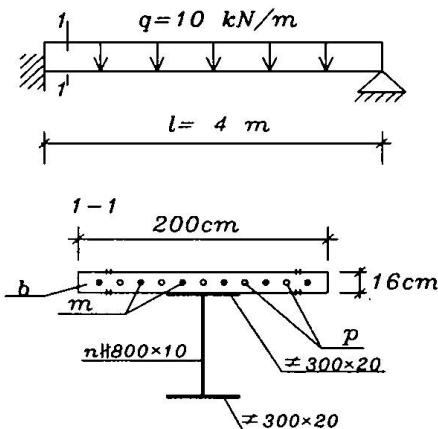
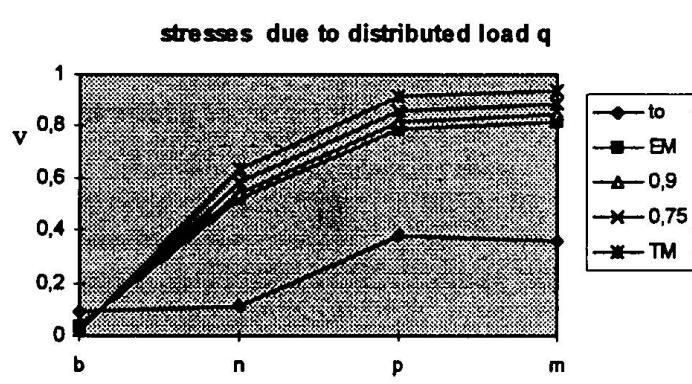
The redistribution of stresses for the example of composite structures due to uniformly distributed load and the shrinkage of concrete is calculated. Values of stresses are shown on the graphs 1,2 as follows.

Data: Concrete (b)  $E_b = 30 \text{ GPa}$ ,  $\varphi_b = 3,5$ ,  $\epsilon_s = -30 \cdot 10^{-5}$

Prestressing steel (p):  $E_p = 210 \text{ GPa}$ ,  $F_p = 100 \text{ cm}^2$ ,  $\zeta_p = 8\%$

Steel member (n):  $E_n = 200 \text{ GPa} = E_s$

Reinforced steel (m) :  $E_m = 200 \text{ GPa}$ ,  $F_m = 80 \text{ cm}^2$



### 3. Conclusion

The redistribution of stresses for the composite section during in time, occurs due to viscoelastic properties of concrete and relaxation of prestressing steel. Stresses of concrete are reduced and stresses of steel parts are increased. Using the concrete creep function of the aging theory in the exact method and the hereditary function in the EM method the upper and the lower limits of the stresses are determined. We choose the aging coefficient  $\chi$  in the AAEM method to lie within these limits. As we can see in graphs this conditions for the values of coefficient  $\chi$  from 0,75 to 0,9 are fulfilled.

## Stochastic Long-Term Analysis of Composite Girders

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### Summary

The creep properties of concrete significantly influence the long-term behavior of steel-concrete incomplete composite girders. In this paper, a stochastic creep analysis based on the First-Order Second-Moment Method are carried out considering the uncertainties of creep properties. The results are compared with those obtained from the Monte Carlo simulation. The effect of variability of material properties on the long-term behavior of incomplete composite girders are exhibited.

### 1. Introduction

The creep properties of concrete significantly influence the long-term behavior of steel-concrete incomplete composite girders. In the design of those structures, the deterministic creep coefficient, such as the ACI-209 model, the CEB-FIP-90 model is utilized to estimate long-term effects. These creep properties are subjected to some amount of variability. Therefore, it is not so easy to correctly predict the long-term behavior of these girders. In this study, a stochastic creep FEM analysis based on the First-Order Second-Moment Method are carried out considering the uncertainties of creep properties. The results are compared with those obtained from the Monte Carlo simulation.

### 2. Stochastic FEM Analysis based on the F.O.S.M

The incomplete composite girder in this FEM analysis consists of a concrete beam element, a steel beam element and a continuous spring element which connects concrete and steel. Using the age adjusted effective modulus method in constitutive law on the concrete, the creep stiffness equation of the incomplete composite girder is expressed as following.

$$[K]\{U\} = \{F\} + \{G\} \quad (1)$$

where

$[K]$ :creep stiffness matrix of composite beam,  $\{U\}$ :creep displacement vector  
 $\{F\}$ :external force vector,  $\{G\}$ : creep force vector

The sensitivity displacement is derived from Eq.(1) as

$$[K] \frac{\partial \{U\}}{\partial X_i} = - \frac{\partial [K]}{\partial X_i} \{U\} + \frac{\partial \{G\}}{\partial X_i} \quad (2)$$

$(i = 1 \sim m)$

where  $X_i$  is probabilistic variables such as the relative humidity, affecting creep behavior of concrete. The value  $m$  is the number of the probabilistic variable. The variances of deflection and stress of the concrete slab and steel beam can be evaluated from Eq.(2).

### 3. Calculation and Results

The CEB-FIP-90 model has adopted as a creep coefficient, which mainly consists of 4 terms of the relative humidity, the mean compressive strength of concrete, the notational size of member and the age of concrete. Besides the creep coefficient the aging coefficient and the modulus elasticity of concrete at loading time also effect the age adjusted effective modulus in the analysis. In this study, the relative humidity, the compressive strength of concrete at the age of 28 days, the modulus of elasticity of concrete and the aging coefficient are regarded as probabilistic variable. The data of those values are the mean value and the coefficient of variation which represents the scatter. Other data are deterministic values.

The numerical calculations are carried out for the simple composite beam shown in Fig.1. The following numerical values are adopted: span length  $L=40m$ ; modulus elasticity of steel  $E_s=2.1 \times 10^5 \text{ MPa}$ ; uniformly distributed sustained load  $p=54.145 \text{ kN/m}$ ; rigidity of connector  $Q_z=0.4 \text{ kN/mm/mm}$ ; loading time and final time for creep analysis is 14days, 10000days, respectively; mean relative humidity  $RH=60\%$ ; mean compressive strength of concrete at the age of 28days  $f_{ck}=30 \text{ MPa}$ ; mean aging coefficient  $\chi=0.76$ ; mean modulus elasticity of concrete  $E_c=2.85 \times 10^5 \text{ MPa}$ .

The comparisons of the variance of creep deflection and creep stress of concrete at the mid span are shown in Fig.2 and Fig.3 between this study and Monte Carlo simulations, where the number of sampling calculation is 1000, and every coefficient of the variation of relative humidity, compressive strength of concrete, aging coefficient, and modulus elasticity of concrete ranges from 10% to 40%. Results of this study show good agreements with those from Monte Carlo simulations.

### 4. Conclusion

The present paper expresses the incomplete composite analysis including the scatter of material properties of long-term behavior, which results in good agreement with the results evaluated from the Monte Carlo method.

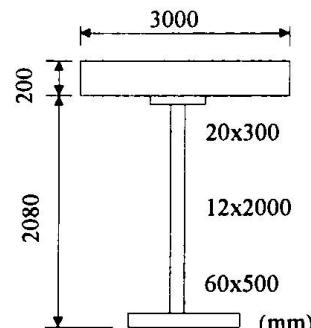


Fig.1 Cross Section

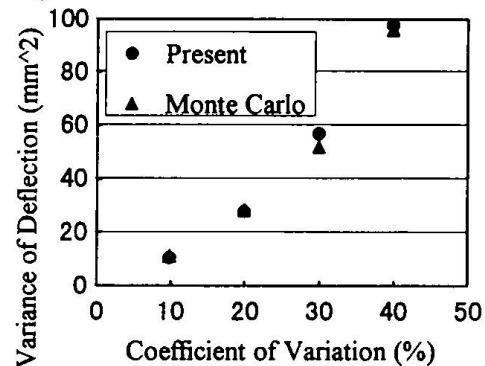


Fig.2 Comparison of Result(a)

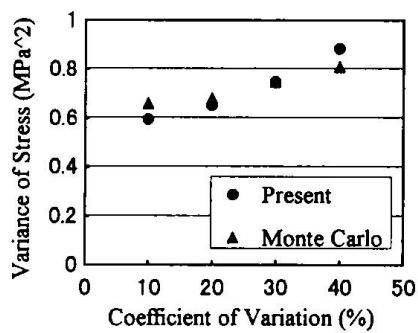


Fig.3 Comparison of Result(b)